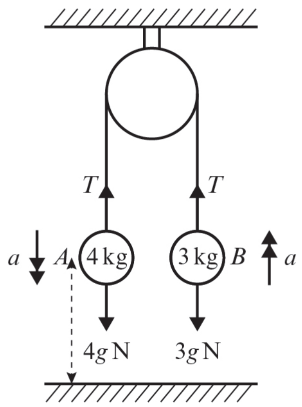
Exercise 4F

1 a



For A: $R(\downarrow)$, $4g - T = 4a$ **(1)**

For B: $R(\uparrow)$, $T - 3g = 3a$ **(2)**

(1) + **(2)**: $4g - 3g = 7a$

$\Rightarrow a = \dfrac{g}{7}$

Substituting into equation (**2**):

$T = 3a + 3g = \dfrac{3g}{7} + 3g = \dfrac{24g}{7}$

$= 33.6$ N (3 s.f.)

b $u = 0,\ a = \dfrac{g}{7}$, $s = 2$, m, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times \dfrac{g}{7} \times 2 = \dfrac{4g}{7} = 5.6$

$v = \sqrt{5.6} = 2.366...$

When A hits the ground it is travelling at 2.37 m s^{-1} (3 s.f.).

c For A: $(\downarrow)$

From part **b**, $v^2 = \dfrac{4g}{7}$

This represents the initial velocity of B when A hits the ground.

For B: $(\uparrow)$

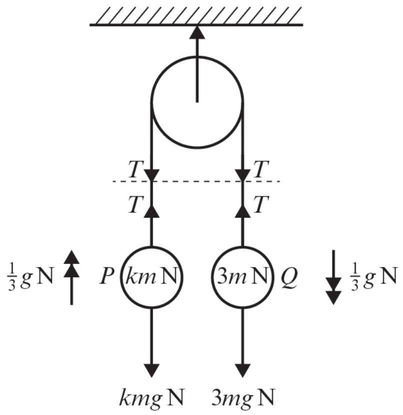
$u^2 = \dfrac{4g}{7}$, $v = 0$, $a = -g$, $s = ?$

$v^2 = u^2 + 2as$

$0 = \dfrac{4g}{7} - 2gs \Rightarrow s = \dfrac{2}{7}$

The height above the initial position is $2\frac{2}{7}$ m.

2



a For Q, $R(\downarrow)$: $3mg - T = 3m \times \frac{1}{3}g = mg$

$2mg = T$

The tension in the string is $2mg$ N.

b For P, $R(\uparrow)$: $T - kmg = km \times \frac{1}{3}g$

$3T - 3kmg = kmg$
$3T = 4kmg$
Substituting for T: $6mg = 4kmg$
$k = \frac{6mg}{4mg}$
The value of k is 1.5.

c Because the pulley is smooth, there is no friction, so the magnitude of acceleration of P = the magnitude of acceleration of Q.

d Up is positive.
While Q is descending, the distance travelled by $P = s_1$
$u = 0,\ a = \frac{1}{3}g,\ t = 1.8,\ s = s_1$

$s = ut + \frac{1}{2}at^2$

$s_1 = (0 \times 1.8) + \left(\frac{1}{2} \times \frac{g}{3} \times 1.8^2\right) = \frac{3.24g}{6} = 0.54g$ **(1)**

Speed of P at this time $= v_1$
Using $v^2 = u^2 + 2as$

After Q hits the ground, P travels freely under gravity and rises by a further distance s_2
$v = 0,\ u = v_1,\ a = -g,\ s = s_2$
$v^2 = u^2 + 2as$
$0^2 = 0.36g^2 - 2gs_2$

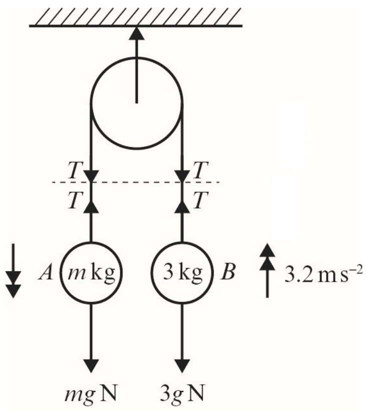
$s_2 = \frac{0.36g^2}{2g} = 0.18g$ **(2)**

(1) + **(2)**: Total distance travelled by P from its initial position $= s_1 + s_2$

2 d P and Q are at the same height initially, so P starts at height s_1 above the plane.
Its final position = initial position + total distance travelled
$$= s_1 + (s_1 + s_2) = 2s_1 + s_2 = 2 \times 0.54g + 0.18g = 1.26g$$

P reaches a maximum height of $1.26g$ m above the plane, as required.

3



a Since the pulley is smooth, |acceleration of A| = |acceleration of B|
For A: $s = 2.5$, $u = 0$, $t = 1.25$, $a = ?$ (down is positive)
$$s = ut + \tfrac{1}{2}at^2$$
$$2.5 = (0 \times 1.25) + \tfrac{1}{2}a \times 1.25^2$$
$$a = \frac{2.5 \times 2}{1.25^2} = 3.2$$
The initial acceleration of B is 3.2 m s^{-2} as required.

b For B, $R(\uparrow)$: $T - 3g = 3a$
$T = 3(a + g) = 3(3.2 + 9.8) = 39$
The tension in the string is 39 N.

c For A, $R(\downarrow)$: $mg - T = ma$
$T = m(g - a) = m(9.8 - 3.2) = 6.6m$
Substituting for T:
$39 = 6.6m$
$$m = \frac{39}{6.6} = \frac{390}{66} = \frac{65}{11} \text{ as required}$$

d Because the string is inextensible, the tension on both sides of the pulley is the same.

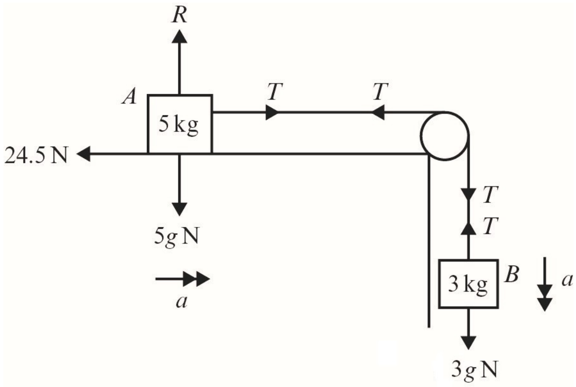
e The string will become taut again when B has risen to its maximum height and then descended to the point where A is just beginning to rise again.
If B reaches the maximum height t seconds after A hits the ground, it will also take t seconds to return to the same position as it is moving under gravity alone throughout this period. The total time of travel will be $2t$.
For B, taking up as positive, while the string is taut:
$u = 0,\ a = 1.4,\ s = 2.5,\ m,\ v = v_1$
$v^2 = u^2 + 2as$
$v_1^2 = 0^2 + 2 \times 3.2 \times 2.5 = 16$
Once the string is slack: $u = v_1 = 4$, $v = 0$, $a = -9.8$, $t = ?$
$v = u + at$
$0 = 4 - 9.8t$

3 e $t = \frac{4}{9.8} = \frac{40}{98} = \frac{20}{49}$

At this point B descends under gravity. After a further t seconds the string once again becomes taut.

The string becomes taut again $2t = \frac{40}{49}$ s after A hits the ground.

4



a For A: $R(\rightarrow)$, $T - 24.5 = 5a$ **(1)**

For B: $R(\downarrow)$, $3g - T = 3a$

$29.4 - T = 3a$ **(2)**

$(\mathbf{1}) + (\mathbf{2})$: $29.4 - 24.5 = 8a$

$4.9 = 8a$

$0.6125 = a$

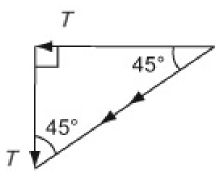
The acceleration of the system is $0.613\ \text{m s}^{-2}$ (3 s.f.)

b $T - 24.5 = 5 \times 0.6125$

$T = 27.5625$

The tension in the string is 27.6 N (3 s.f.)

c



By Pythagoras,

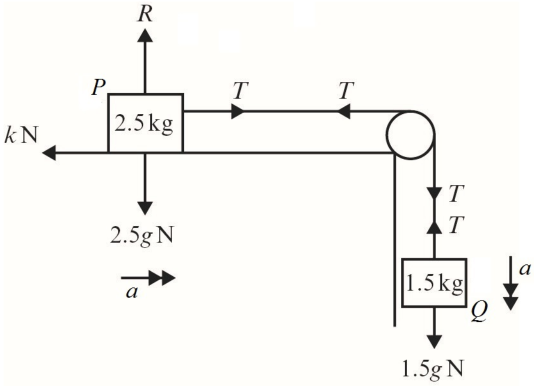
$F^2 = T^2 + T^2 = 2T^2$

$F = T\sqrt{2} = 27.5625 \times \sqrt{2}$

$= 38.979\ldots$

The magnitude of the force exerted on the pulley is 39 N (2 s.f.)

5



a **i** For Q: $s = 0.8$, $u = 0$, $t = 0.75$, $a = ?$ (down is positive)

$s = ut + \frac{1}{2}at^2$

$0.8 = (0 \times 0.75) + \frac{1}{2}a \times 0.75^2$

$a = \frac{0.8 \times 2}{0.75^2} = 2.844...$

The acceleration of Q is 2.84 m s^{-2} (3 s.f.)

ii For Q, $R(\downarrow)$: $1.5g - T = 1.5a$

$T = 1.5(g - a) = 1.5(9.8 - 2.84) = 10.44$

The tension in the string is 10.4 N (to 3 s.f.), as required.

iii For P, R ($\rightarrow$): $T - k = 2.5a$

Substituting:

$10.4 - k = 2.5 \times 2.84$

$k = 10.4 - 7.1$

The value of k is 3.3 N

b Because the string is inextensible, the tension in all parts of it is the same.