Exercise 4D

1 a $F = (\mathbf{i} + 4\mathbf{j})$, $m = 2$, $\mathbf{a} = ?$

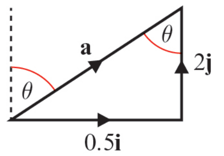
$F = m\mathbf{a}$

$(\mathbf{i} + 4\mathbf{j}) = 2\mathbf{a}$

$\mathbf{a} = \dfrac{(\mathbf{i} + 4\mathbf{j})}{2}$

The acceleration of the particle is $(0.5\mathbf{i} + 2\mathbf{j})$ m s^{-2}.

b



$|\mathbf{a}| = \sqrt{0.5^2 + 2^2} = \sqrt{4.25}$

The magnitude of the acceleration is 2.06 m s^{-2}.

Using Z angles (see diagram), bearing = θ

$\tan\theta = \dfrac{0.5}{2}$

$\theta = 14°$

The bearing of the acceleration is 014°.

2 $F = (4\mathbf{i} + 3\mathbf{j})$, $\mathbf{a} = (20\mathbf{i} + 15\mathbf{j})$, $m = ?$

$F = m\mathbf{a}$

$(4\mathbf{i} + 3\mathbf{j}) = m \times (20\mathbf{i} + 15\mathbf{j})$

$m = \dfrac{(4\mathbf{i} + 3\mathbf{j})}{(20\mathbf{i} + 15\mathbf{j})} = \dfrac{1}{5}$

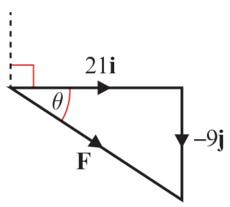
The mass of the particle is 0.2 kg.

3 a $\mathbf{a} = (7\mathbf{i} - 3\mathbf{j})$, $m = 3$, $F = ?$

$F = m\mathbf{a}$

$= 3 \times (7\mathbf{i} - 3\mathbf{j})$

$= (21\mathbf{i} - 9\mathbf{j})$



b

$|\mathbf{F}| = \sqrt{21^2 + 9^2} = \sqrt{522}$

The force has a magnitude of 22.8 N (3 s.f.)

$\tan\theta = \dfrac{9}{21}$

$\theta = 23.19\ldots°$

But bearing = $90° + \theta$ (see diagram)

The force acts at a bearing of 113° (to the nearest degree).

4 a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$, $m = 0.25$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 0.25\mathbf{a}$

$(-\mathbf{i} + 8\mathbf{j}) = 0.25\mathbf{a}$

$$\mathbf{a} = \frac{(-\mathbf{i}+8\mathbf{j})}{0.25}$$

The acceleration is $(-4\mathbf{i} + 32\mathbf{j})$ m s^{-2}.

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$, $m = 6$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = 6\mathbf{a}$

$(5\mathbf{i} - \mathbf{j}) = 6\mathbf{a}$

$$\mathbf{a} = \frac{(5\mathbf{i}-\mathbf{j})}{6}$$

The acceleration is $\left(\frac{5}{6}\mathbf{i} - \frac{1}{6}\mathbf{j}\right)$ m s^{-2}.

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$, $m = 15$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$(-40\mathbf{i} - 20\mathbf{j}) + (25\mathbf{i} + 10\mathbf{j}) = 15\mathbf{a}$

$(-15\mathbf{i} - 10\mathbf{j}) = 15\mathbf{a}$

$$\mathbf{a} = \frac{(-15\mathbf{i}-10\mathbf{j})}{15}$$

The acceleration is $\left(-\mathbf{i} - \frac{2}{3}\mathbf{j}\right)$ m s^{-2}.

d $\mathbf{F}_1 = 4\mathbf{j}$, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$, $m = 1.5$

$F = \mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a}$

$4\mathbf{j} + (-2\mathbf{i} + 5\mathbf{j}) = 1.5\mathbf{a}$

$(-2\mathbf{i} + 9\mathbf{j}) = 1.5\mathbf{a}$

$$\mathbf{a} = \frac{(-2\mathbf{i}+9\mathbf{j})}{1.5}$$

The acceleration is $\left(-\frac{4}{3}\mathbf{i} + 6\mathbf{j}\right)$ m s^{-2}.

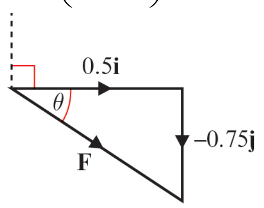
5 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$F = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$F = m\mathbf{a}$

$$8\mathbf{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$\mathbf{a} = \begin{pmatrix} 0.5 \\ -0.75 \end{pmatrix}$$



5 a $|\mathbf{a}| = \sqrt{0.5^2 + 0.75^2} = \sqrt{0.8125}$

$\tan\theta = \dfrac{0.75}{0.5}$

$\theta = 56°$

But bearing = 90° + θ (see diagram)

The acceleration has a magnitude of 0.901 m s^{-2} and acts at a bearing of 146°.

b $s = 20$, $u = 0$, $a = 0.901$

$s = ut + \frac{1}{2}at^2$

$20 = (0 \times t) + \left(\frac{1}{2} \times 0.901 \times t^2\right)$

$t^2 = \dfrac{20 \times 2}{0.901} = 44.39$

The particle takes 6.66 s to travel 20 m.

6 $\mathbf{R} = (2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j})$

Since $\mathbf{R}$ is parallel to $(-\mathbf{i} + 4\mathbf{j})$,

$\mathbf{R} = (-k\mathbf{i} + 4k\mathbf{j})$ where k is a constant

$(2\mathbf{i} + 3\mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = (-k\mathbf{i} + 4k\mathbf{j})$

Collecting **i** terms: $2 + p = -k$

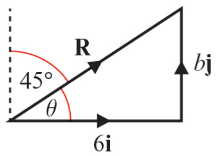
so $k = -2 - p$

Collecting **j** terms: $3 + q = 4k$

Substituting for k: $3 + q = 4(-2 - p)$

so $3 + q = -8 - 4p$

$4p + q + 11 = 0$



7 a $\theta = 90° - 45°$ (see diagram)

$\tan 45° = \dfrac{b}{6}$

$b = 6 \times \tan 45° = 6 \times 1$

The value of b is 6.

b $|\mathbf{R}| = \sqrt{6^2 + 6^2} = \sqrt{72}$

The magnitude of **R** is $6\sqrt{2}$ N (8.49 N to 3.s.f)

c $F = 6\sqrt{2}$, $m = 4$, $a = ?$

$F = ma$

$6\sqrt{2} = 4a$

The magnitude of the acceleration of the particle is $\dfrac{3\sqrt{2}}{2}$ m s^{-2} (2.12 m s^{-2} to 3 s.f.)

7 **d** $t = 5, u = 0, a = \frac{3\sqrt{2}}{2}, s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 5) + \left(\frac{1}{2} \times \frac{3\sqrt{2}}{2} \times 5^2\right)$$

$$s = \frac{75\sqrt{2}}{4}$$

In the first 5 s the particle travels $\frac{75\sqrt{2}}{4}$ m (26.5 m to 3 s.f.).

8 **a** Since particle is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$(-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
Collecting **i** terms: $-3 + 1 + p = 0$
Collecting **j** terms: $7 - 1 + q = 0$

The value of p is 2, and the value of q is -6.

b When $\mathbf{F}_2$ is removed, resultant force, $F = \mathbf{F}_1 + \mathbf{F}_3$
$F = (-3\mathbf{i} + 7\mathbf{j}) + (2\mathbf{i} - 6\mathbf{j}) = (-\mathbf{i} + \mathbf{j})$
The magnitude of this force is $\sqrt{1^2 + 1^2} = \sqrt{2}$

$s = 12, t = 10, u = 0, a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$12 = (0 \times 20) + \left(\frac{1}{2} \times a \times 10^2\right)$$

$$12 = 50a$$

$$a = \frac{12}{50} = \frac{6}{25}$$

$$F = \sqrt{2},\ a = \frac{6}{25}$$

$$F = ma$$

$$\sqrt{2} = m \times \frac{6}{25}$$

The mass of the particle is $\frac{25\sqrt{2}}{6}$ kg.

9 Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$F = (5\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = 6\mathbf{i}$
Since this has only a single component, the magnitude of the force is 6 N.
$a = 7$
$F = ma$
$6 = m \times 7$
$m = 6 \div 7$
The mass of the particle is 0.86 kg.

10 a $\mathbf{R} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix}$

Since $\mathbf{R}$ is parallel to $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix}$ where k is a constant

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} k \\ -2k \end{pmatrix}$

Collecting **i** terms: $2 + p = k$
Collecting **j** terms: $5 + q = -2k$
Substituting for k: $5 + q = -2(2 + p)$
so $5 + q = -4 - 2p$
$2p + q + 9 = 0$

b $p = 1$
From **a** above, $k = 2 + p$
so $k = 2 + 1 = 3$
so $\mathbf{R} = \begin{pmatrix} k \\ -2k \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$|\mathbf{R}| = \sqrt{3^2 + (-6)^2} = \sqrt{45}$

$a = 15\sqrt{5}$, $F = \sqrt{45}$

$F = ma$

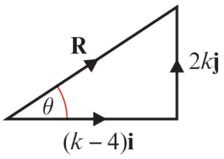
$\sqrt{45} = m \times 15\sqrt{5}$

$m = \dfrac{\sqrt{45}}{15\sqrt{5}} = \dfrac{\sqrt{9 \times 5}}{15\sqrt{5}} = \dfrac{3\sqrt{5}}{15\sqrt{5}} = \dfrac{1}{5} = 0.2$

The mass of the particle is 0.2 kg.

Challenge

Resultant force, $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$\mathbf{R} = -4\mathbf{i} + (k\mathbf{i} + 2k\mathbf{j})$



$F = ma$
$m = 0.5,\ a = 8\sqrt{17}$
So magnitude of the resultant force $= 0.5 \times 8\sqrt{17} = 4\sqrt{17}$
$|\mathbf{R}|^2 = (k-4)^2 + (2k)^2$
$\left(4\sqrt{17}\right)^2 = 16 \times 17 = k^2 - 8k + 16 + 4k^2$
$272 = 5k^2 - 8k + 16$
$5k^2 - 8k - 256 = 0$
$k = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$k = \dfrac{8 \pm \sqrt{8^2 - 4 \times 5 \times (-256)}}{2 \times 5} = \dfrac{8 \pm \sqrt{5184}}{10} = \dfrac{8 \pm 72}{10}$
$k = -6.4$ or 8
Since k is given as a positive constant, the value of k is 8.