Exercise 4B

1 a $(-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = (3\mathbf{i} + 2\mathbf{j})$
The resultant force is $(3\mathbf{i} + 2\mathbf{j})$ N.

b $\begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

The resultant force is $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ N.

c $(\mathbf{i} + \mathbf{j}) + (5\mathbf{i} - 3\mathbf{j}) + (-2\mathbf{i} - \mathbf{j}) = (4\mathbf{i} - 3\mathbf{j})$
The resultant force is $(4\mathbf{i} - 3\mathbf{j})$ N.

d $\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

The resultant force is $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ N.

2 a $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$\Rightarrow (2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) + \mathbf{F}_3 = 0$

$\Rightarrow \mathbf{F}_3 = -(2\mathbf{i} + 7\mathbf{j}) - (-3\mathbf{i} + \mathbf{j})$

$= -2\mathbf{i} - 7\mathbf{j} + 3\mathbf{i} - \mathbf{j}$

$= \mathbf{i} - 8\mathbf{j}$

b $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$\Rightarrow (3\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) + \mathbf{F}_3 = 0$

$\Rightarrow \mathbf{F}_3 = -(3\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j})$

$= -3\mathbf{i} + 4\mathbf{j} - 2\mathbf{i} - 3\mathbf{j}$

$= -5\mathbf{i} + \mathbf{j}$

3 Since object is in equilibrium:

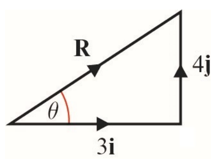
$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} a \\ 2b \end{pmatrix} + \begin{pmatrix} -2a \\ -b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\begin{pmatrix} -a \\ b \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$a = 3$ and $b = 4$

4 a $(3\mathbf{i} + 4\mathbf{j})$



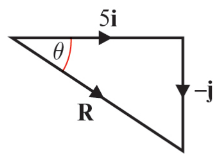
4 a i $\sqrt{3^2+4^2}=\sqrt{25}$

The resultant force is 5 N.

ii $\tan\theta=\frac{4}{3}$

The force makes an angle of 53.1° with **i**.

b (5**i** – **j**)



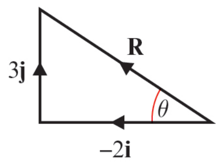
i $\sqrt{5^2+1^2}=\sqrt{26}$

The resultant force is $\sqrt{26}$ N.

ii $\tan\theta=\frac{1}{5}$

The force makes an angle of 11.3° with **i**.

c (−2**i** + 3**j**)



i $\sqrt{2^2+3^2}=\sqrt{13}$

The resultant force is $\sqrt{13}$ N.

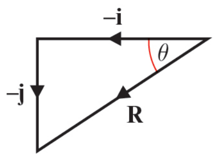
ii $\tan\theta=\frac{3}{2}$

$\theta=56.3°$ This is the angle made with the negative **i** vector

Angle made with the positive **i** vector = 180 − θ

The force makes an angle of 123.7° with **i**.

d



i $\sqrt{1^2+1^2}=\sqrt{2}$

The resultant force is $\sqrt{2}$ N.

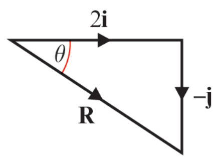
4 d ii $\tan\theta = \frac{1}{1}$

$\theta = 45°$. This is the angle made with the negative **i** vector.
The obtuse angle made with the positive **i** vector = $180 - \theta$
The force makes an angle of 135° with **i**.

5 a i $(-2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = (2\mathbf{i} - \mathbf{j})$
The resultant vector is $(2\mathbf{i} - \mathbf{j})$ N.

ii $\sqrt{2^2 + 1^2} = \sqrt{5}$

The magnitude of the resultant vector is $\sqrt{5}$ N.



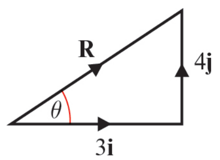
iii $\tan\theta = \frac{1}{2}$

$\theta = -26.6°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 116.6°.

b i $(-2\mathbf{i} + \mathbf{j}) + (2\mathbf{i} - 3\mathbf{j}) + (3\mathbf{i} + 6\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j})$
The resultant vector is $(3\mathbf{i} + 4\mathbf{j})$ N

ii $\sqrt{3^2 + 4^2} = \sqrt{25}$

The resultant force is 5 N.



iii $\tan\theta = \frac{4}{3}$

$\theta = 53.1°$ This is the angle made from **east**, with **anticlockwise** defined as positive.
The **bearing** is the angle made from **north**, with **clockwise** defined as positive = $90 - \theta$
The force acts at a bearing of 36.9°.

6 Since the object is in equilibrium:
$(a\mathbf{i} - b\mathbf{j}) + (b\mathbf{i} + a\mathbf{j}) + (-4\mathbf{i} - 2\mathbf{j}) = 0$
Considering **i** components:
$a + b - 4 = 0$
so $b = 4 - a$ **(1)**
Considering **j** components:
$-b + a - 2 = 0$
Substituting $b = 4 - a$ from **(1)**:
$-(4 - a) + a - 2 = 0$
$2a = 2 + 4 = 6$
$a = 3$ **(2)**

Substituting **(2)** into **(1)**:
$b = 4 - 3 = 1$
The values of a and b are 3 and 1, respectively.

7 Since the object is in equilibrium:
$(2a\mathbf{i} + 2b\mathbf{j}) + (-5b\mathbf{i} + 3a\mathbf{j}) + (-11\mathbf{i} - 7\mathbf{j}) = 0$

Considering **i** components:
$2a - 5b - 11 = 0$ **(1)**

Considering **j** components:
$2b + 3a - 7 = 0$ **(2)**

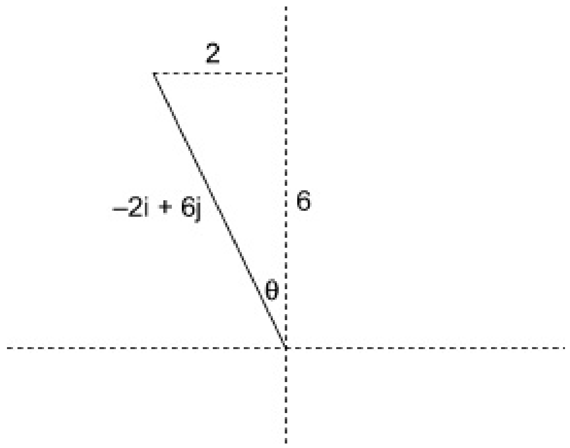
equation **(1)** × 3 → $6a - 15b - 33 = 0$ **(3)**
equation **(2)** × 2 → $6a + 4b - 14 = 0$ **(4)**

Subtracting (**4**) from (**3**):
$-15b - 33 - 4b - (-14) = 0$
$-19b = 33 - 14$
$b = -1$
Substituting this value into equation (**1**):
$2a - 5(-1) - 11 = 0$
$2a = 11 - 5 = 6$
The values of a and b are 3 and −1, respectively.

8 a $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$
$\Rightarrow (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) = 0$
$(-3 + 1 + p)\mathbf{i} + (7 - 1 + q)\mathbf{j} = 0$
$p = 2,\ q = -6$

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$
$= (-3\mathbf{i} + 7\mathbf{j}) + (\mathbf{i} - \mathbf{j})$
$= -2\mathbf{i} + 6\mathbf{j}$
$|\mathbf{R}| = \sqrt{(-2)^2 + 6^2}$
$= \sqrt{4 + 36}$
$= \sqrt{40}$
$= 6.32$ N

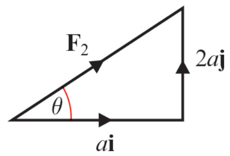
8 c



$\tan\theta = \frac{2}{6}$

$\theta = 18°$

9 a $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$



$\tan\theta = \frac{2a}{a} = 2$

$\mathbf{F}_2$ makes an angle of 63.4° with $\mathbf{i}$.

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (3\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + 2a\mathbf{j})$

$\mathbf{i}$ vector $= 3 + a$ $\qquad$ $\mathbf{j}$ vector $= -2 + 2a$

In the vector $(13\mathbf{i} + 10\mathbf{j})$:
$\mathbf{i}$ vector $= 13$ $\qquad$ $\mathbf{j}$ vector $= 10$

Let θ_1 = the angle of vector $\mathbf{R}$ and θ_2 = the angle of vector $(13\mathbf{i} + 10\mathbf{j})$

Since the vectors are parallel, $\theta_1 = \theta_2$ so $\tan\theta_1 = \tan\theta_2$:

$\tan\theta_1 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{-2+2a}{3+a}$ $\qquad$ $\tan\theta_2 = \frac{\mathbf{j}\text{ vector}}{\mathbf{i}\text{ vector}} = \frac{10}{13}$

$\Rightarrow \frac{-2+2a}{3+a} = \frac{10}{13}$

$(-2 + 2a) \times 13 = (3 + a) \times 10$

$16a = 56$

$a = 3.5$

10 a Since the particle P is in equilibrium:

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$$\begin{pmatrix}-7\\-4\end{pmatrix}+\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

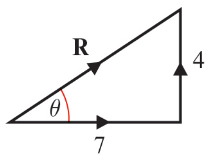
$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\4\end{pmatrix}+\begin{pmatrix}-4\\-2\end{pmatrix}$$

$$\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$$

The values are $a = 3$, $b = 2$

b $\mathbf{R} = \mathbf{F}_2 + \mathbf{F}_3$

$$\mathbf{R}=\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}7\\4\end{pmatrix}$$



i $|\mathbf{R}| = \sqrt{7^2 + 4^2} = \sqrt{65}$

The magnitude of **R** is $\sqrt{65}$ N.

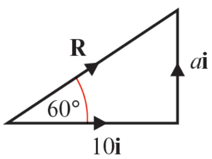
ii $\tan\theta = \dfrac{4}{7}$

$\theta = 29.7\ldots°$

R acts at 30° above the horizontal (to 2 s.f.)

Challenge

Redrawing the diagram as a closed triangle:



$\tan 60 = \dfrac{a}{10}$

$a = 10\tan 60 = 10 \times \sqrt{3}$

$$\mathbf{R}=\begin{pmatrix}10\\a\end{pmatrix}=\begin{pmatrix}10\\10\sqrt{3}\end{pmatrix}$$

$|\mathbf{R}| = \sqrt{10^2 + (10\sqrt{3})^2} = \sqrt{100+300} = \sqrt{400}$

The value of a is 17.3 (to 3 s.f.), and the magnitude of the resultant force is 20 N.