## Exercise 3D

**1 a** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

when $\mathbf{r}_0 = 3\mathbf{j}$, $\mathbf{v} = 2\mathbf{i}$ and $t = 4$

$\mathbf{r} = 3\mathbf{j} + 4(2\mathbf{i})$

$= 8\mathbf{i} + 3\mathbf{j}$

**b** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

when $\mathbf{r}_0 = 2\mathbf{i} - \mathbf{j}$, $\mathbf{v} = -2\mathbf{j}$ and $t = 3$

$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3(-2\mathbf{j})$

$= 2\mathbf{i} - 7\mathbf{j}$

**c** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

when $\mathbf{r}_0 = \mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j}$ and $t = 6$

$\mathbf{r} = \mathbf{i} + 4\mathbf{j} + 6(-3\mathbf{i} + 2\mathbf{j})$

$= -17\mathbf{i} + 16\mathbf{j}$

**d** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

when $\mathbf{r}_0 = -3\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$ and $t = 5$

$\mathbf{r} = -3\mathbf{i} + 2\mathbf{j} + 5(2\mathbf{i} - 3\mathbf{j})$

$= 7\mathbf{i} - 13\mathbf{j}$

**2 a** $\mathbf{v} = \dfrac{\mathbf{b} - \mathbf{a}}{t}$

when $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 6\mathbf{i} + 13\mathbf{j}$ and $t = 2$

$$\mathbf{v} = \frac{6\mathbf{i} + 13\mathbf{j} - (2\mathbf{i} + 3\mathbf{j})}{2}$$

$$= \frac{4\mathbf{i} + 10\mathbf{j}}{2}$$

$$= 2\mathbf{i} + 5\mathbf{j}$$

**b** $\mathbf{v} = \dfrac{\mathbf{b} - \mathbf{a}}{t}$

when $\mathbf{a} = 4\mathbf{i} + \mathbf{j}$, $\mathbf{b} = 9\mathbf{i} + 16\mathbf{j}$ and $t = 5$

$$\mathbf{v} = \frac{9\mathbf{i} + 16\mathbf{j} - (4\mathbf{i} + \mathbf{j})}{5}$$

$$= \frac{5\mathbf{i} + 15\mathbf{j}}{5}$$

$$= \mathbf{i} + 3\mathbf{j}$$

**2 c** $\mathbf{v}=\frac{\mathbf{b}-\mathbf{a}}{t}$

when $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$, $\mathbf{b} = 9\mathbf{i} + 7\mathbf{j}$ and $t = 3$

$\mathbf{v}=\frac{9\mathbf{i}+7\mathbf{j}-(3\mathbf{i}-5\mathbf{j})}{3}$

$=\frac{6\mathbf{i}+12\mathbf{j}}{3}$

$=2\mathbf{i}+4\mathbf{j}$

**d** $\mathbf{v}=\frac{\mathbf{b}-\mathbf{a}}{t}$

when $\mathbf{a} = -2\mathbf{i} + 7\mathbf{j}$, $\mathbf{b} = 4\mathbf{i} - 8\mathbf{j}$ and $t = 3$

$\mathbf{v}=\frac{4\mathbf{i}-8\mathbf{j}-(-2\mathbf{i}+7\mathbf{j})}{3}$

$=\frac{6\mathbf{i}-15\mathbf{j}}{3}$

$=2\mathbf{i}-5\mathbf{j}$

**e** $\mathbf{v}=\frac{\mathbf{b}-\mathbf{a}}{t}$

when $\mathbf{a} = -4\mathbf{i} + \mathbf{j}$, $\mathbf{b} = -12\mathbf{i} - 19\mathbf{j}$ and $t = 4$

$\mathbf{v}=\frac{-12\mathbf{i}-19\mathbf{j}-(-4\mathbf{i}+\mathbf{j})}{4}$

$=\frac{-8\mathbf{i}-20\mathbf{j}}{4}$

$=-2\mathbf{i}-5\mathbf{j}$

**3 a** When $v = 10$, $\mathbf{d} = 3\mathbf{i} - 4\mathbf{j}$

The magnitude of **d** is $\sqrt{3^2+(-4)^2}=5$

So the unit vector in the direction of **d** is $\frac{1}{5}(3\mathbf{i}-4\mathbf{j})$

The velocity of the particle is $\mathbf{v}=10\times\frac{1}{5}(3\mathbf{i}-4\mathbf{j})$

$=6\mathbf{i}-8\mathbf{j}$

**b** When $v = 15$, $\mathbf{d} = -4\mathbf{i} + 3\mathbf{j}$

The magnitude of **d** is $\sqrt{(-4)^2+3^2}=5$

So the unit vector in the direction of **d** is $\frac{1}{5}(-4\mathbf{i}+3\mathbf{j})$

The velocity of the particle is $\mathbf{v}=15\times\frac{1}{5}(-4\mathbf{i}+3\mathbf{j})$

$=-12\mathbf{i}+9\mathbf{j}$

**3 c** When $v = 7.5$, $\mathbf{d} = -6\mathbf{i} + 8\mathbf{j}$

The magnitude of **d** is $\sqrt{(-6)^2 + 8^2} = 10$

So the unit vector in the direction of **d** is $\frac{1}{10}(-6\mathbf{i} + 8\mathbf{j})$

The velocity of the particle is $\mathbf{v} = 7.5 \times \frac{1}{10}(-6\mathbf{i} + 8\mathbf{j})$

$= -4.5\mathbf{i} + 6\mathbf{j}$

**d** When $v = 5\sqrt{2}$, $\mathbf{d} = \mathbf{i} + \mathbf{j}$

The magnitude of **d** is $\sqrt{1^2 + 1^2} = \sqrt{2}$

So the unit vector in the direction of **d** is $\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$

The velocity of the particle is $\mathbf{v} = 5\sqrt{2} \times \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$

$= 5\mathbf{i} + 5\mathbf{j}$

**e** When $v = 2\sqrt{13}$, $\mathbf{d} = -2\mathbf{i} + 3\mathbf{j}$

The magnitude of **d** is

$\sqrt{(-2)^2 + 3^2} = \sqrt{13}$

So the unit vector in the direction of **d** is $\frac{1}{\sqrt{13}}(-2\mathbf{i} + 3\mathbf{j})$

The velocity of the particle is $\mathbf{v} = 2\sqrt{13} \times \frac{1}{\sqrt{13}}(-2\mathbf{i} + 3\mathbf{j})$

$= -4\mathbf{i} + 6\mathbf{j}$

**f** When $v = \sqrt{68}$, $\mathbf{d} = 3\mathbf{i} - 5\mathbf{j}$

The magnitude of **d** is

$\sqrt{3^2 + (-5)^2} = \sqrt{34}$

So the unit vector in the direction of **d** is

$\frac{1}{\sqrt{34}}(3\mathbf{i} - 5\mathbf{j})$

The velocity of the particle is $\mathbf{v} = \sqrt{68} \times \frac{1}{\sqrt{34}}(3\mathbf{i} - 5\mathbf{j})$

$= 3\sqrt{2}\mathbf{i} - 5\sqrt{2}\mathbf{j}$

**3 g** When $v = \sqrt{60}$, $\mathbf{d} = -4\mathbf{i} - 2\mathbf{j}$
The magnitude of **d** is
$\sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$

So the unit vector in the direction of **d** is $\frac{1}{\sqrt{20}}(-4\mathbf{i} - 2\mathbf{j})$

The velocity of the particle is $\mathbf{v} = \sqrt{60} \times \frac{1}{\sqrt{20}}(-4\mathbf{i} - 2\mathbf{j})$
$= -4\sqrt{3}\mathbf{i} - 2\sqrt{3}\mathbf{j}$

**h** When $v = 15$, $\mathbf{d} = -\mathbf{i} + 2\mathbf{j}$
The magnitude of **d** is
$\sqrt{(-1)^2 + 2^2} = \sqrt{5}$
So the unit vector in the direction of **d** is
$\frac{1}{\sqrt{5}}(-\mathbf{i} + 2\mathbf{j})$

The velocity of the particle is $\mathbf{v} = 15 \times \frac{1}{\sqrt{5}}(-\mathbf{i} + 2\mathbf{j})$
$= -3\sqrt{5}\mathbf{i} + 6\sqrt{5}\mathbf{j}$

**4 a** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = 2\mathbf{i}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$ and $t = 4$
$\mathbf{r} = 2\mathbf{i} + 4(\mathbf{i} + 3\mathbf{j})$
$= 6\mathbf{i} + 12\mathbf{j}$

**b** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$ and $t = 5$
$\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 5(-2\mathbf{i} + \mathbf{j})$
$= -7\mathbf{i} + 4\mathbf{j}$

**c** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$ and $t = 3$
$4\mathbf{i} + 3\mathbf{j} = \mathbf{r}_0 + 3(2\mathbf{i} - \mathbf{j})$
$\mathbf{r}_0 = 4\mathbf{i} + 3\mathbf{j} - 3(2\mathbf{i} - \mathbf{j})$
$\mathbf{r}_0 = -2\mathbf{i} + 6\mathbf{j}$

**d** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r} = -2\mathbf{i} + 5\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$ and $t = 6$
$-2\mathbf{i} + 5\mathbf{j} = \mathbf{r}_0 + 6(-2\mathbf{i} + 3\mathbf{j})$
$\mathbf{r}_0 = -2\mathbf{i} + 5\mathbf{j} - 6(-2\mathbf{i} + 3\mathbf{j})$
$\mathbf{r}_0 = 10\mathbf{i} - 13\mathbf{j}$

**e** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = 2\mathbf{i} + 2\mathbf{j}$, $\mathbf{r} = 8\mathbf{i} - 7\mathbf{j}$ and $t = 3$
$8\mathbf{i} - 7\mathbf{j} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{v}$
$3\mathbf{v} = 6\mathbf{i} - 9\mathbf{j}$
$\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

**4 f** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = 10\mathbf{i} - 5\mathbf{j}$, $\mathbf{r} = -2\mathbf{i} + 9\mathbf{j}$
and $t = 4$
$-2\mathbf{i} + 9\mathbf{j} = 10\mathbf{i} - 5\mathbf{j} + 4\mathbf{v}$
$4\mathbf{v} = -12\mathbf{i} + 14\mathbf{j}$
$\mathbf{v} = -3\mathbf{i} + 3.5\mathbf{j}$
$$|\mathbf{v}| = \sqrt{(-3)^2 + 3.5^2}$$
$$= \frac{\sqrt{85}}{2}$$
$$= 4.61 \text{ m s}^{-1} \text{ (3 s.f.)}$$

**g** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = 4\mathbf{i} + \mathbf{j}$, $\mathbf{r} = 12\mathbf{i} - 11\mathbf{j}$
and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$
$12\mathbf{i} - 11\mathbf{j} = 4\mathbf{i} + \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$
$t(2\mathbf{i} - 3\mathbf{j}) = 8\mathbf{i} - 12\mathbf{j}$
$t = 4$ s

**h** $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
when $\mathbf{r}_0 = -2\mathbf{i} + 3\mathbf{j}$, $\mathbf{r} = 6\mathbf{i} - 3\mathbf{j}$
and $v = 4$ m s$^{-1}$
$6\mathbf{i} - 3\mathbf{j} = -2\mathbf{i} + 3\mathbf{j} + t\mathbf{v}$
$t\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$
$t|\mathbf{v}| = |8\mathbf{i} - 6\mathbf{j}|$
$$4t = \sqrt{8^2 + (-6)^2}$$
$$= 10$$
$t = 2.5$ s

**5 a** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = 5\mathbf{i}$, $\mathbf{a} = 3\mathbf{j}$ and $t = 4$
$\mathbf{v} = 5\mathbf{i} + 4(3\mathbf{j})$
$= (5\mathbf{i} + 12\mathbf{j})$ m s$^{-1}$
$$|\mathbf{v}| = \sqrt{5^2 + 12^2}$$
$$= 13 \text{ m s}^{-1}$$

**b** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$, $\mathbf{a} = \mathbf{i} - \mathbf{j}$ and $t = 3$
$\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 3(\mathbf{i} - \mathbf{j})$
$= (6\mathbf{i} - 5\mathbf{j})$ m s$^{-1}$
$$|\mathbf{v}| = \sqrt{6^2 + (-5)^2}$$
$$= \sqrt{61}$$
$$= 7.81 \text{ m s}^{-1} \text{ (3 s.f.)}$$

**5** **c** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = -2\mathbf{i} + \mathbf{j}$, $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j}$ and $t = 2$
$\mathbf{v} = -2\mathbf{i} + \mathbf{j} + 2(2\mathbf{i} - 3\mathbf{j})$
$= (2\mathbf{i} - 5\mathbf{j})\text{ m s}^{-1}$
$|\mathbf{v}| = \sqrt{2^2 + (-5)^2}$
$= \sqrt{29}$
$= 5.39\text{ m s}^{-1}$ (3 s.f.)

**d** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$, $\mathbf{a} = -\mathbf{i}$ and $t = 6$
$\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 6(-\mathbf{i})$
$= (-3\mathbf{i} - 2\mathbf{j})\text{ m s}^{-1}$
$|\mathbf{v}| = \sqrt{(-3)^2 + (-2)^2}$
$= \sqrt{13}$
$= 3.61\text{ m s}^{-1}$ (3 s.f.)

**e** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = -3\mathbf{i} + 4\mathbf{j}$, $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$ and $t = 5$
$\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} + 5(2\mathbf{i} + \mathbf{j})$
$= (7\mathbf{i} + 9\mathbf{j})\text{ m s}^{-1}$
$|\mathbf{v}| = \sqrt{7^2 + 9^2}$
$= \sqrt{130}$
$= 11.4\text{ m s}^{-1}$ (3 s.f.)

**6** $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
when $\mathbf{u} = 0$, $\mathbf{v} = 6\mathbf{i} - 8\mathbf{j}$ and $t = 5$
$6\mathbf{i} - 8\mathbf{j} = 5a$
$a = 1.2\mathbf{i} - 1.6\mathbf{j}$
By Newton's 2nd law $\mathbf{F} = m\mathbf{a}$
$\mathbf{F} = 4(1.2\mathbf{i} - 1.6\mathbf{j})$
$= (4.8\mathbf{i} - 6.4\mathbf{j})\text{ N}$

**7** By Newton's 2nd law $\mathbf{F} = m\mathbf{a}$
$2\mathbf{i} - \mathbf{j} = 2\mathbf{a}$
$\mathbf{a} = \mathbf{i} - 0.5\mathbf{j}$
$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
when $\mathbf{u} = \mathbf{i} + 3\mathbf{j}$ and $t = 3$
$\mathbf{s} = 3(\mathbf{i} + 3\mathbf{j}) + \frac{9}{2}(\mathbf{i} - 0.5\mathbf{j})$
$= 7.5\mathbf{i} + 6.75\mathbf{j}$
$|\mathbf{s}| = \sqrt{7.5^2 + 6.75^2}$
$= 10.1\text{ m}$ (3 s.f.)

**8** $\mathbf{r} = \mathbf{r_0} + \mathbf{v}t$
For $P$, $\mathbf{r_0} = 4\mathbf{i}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j}$
$\mathbf{r} = 4\mathbf{i} + t(\mathbf{i} + \mathbf{j})$ **(1)**
For $Q$, $\mathbf{r_0} = -3\mathbf{j}$
$\mathbf{r} = -3\mathbf{j} + t\mathbf{v}$ **(2)**

Since the particles meet at $t = 8$
$4\mathbf{i} + 8(\mathbf{i} + \mathbf{j}) = -3\mathbf{j} + 8\mathbf{v}$
$8\mathbf{v} = 12\mathbf{i} + 11\mathbf{j}$
$\mathbf{v} = 1.5\mathbf{i} + 1.375\mathbf{j}$
$|\mathbf{v}| = \sqrt{1.5^2 + 1.375^2}$
$= 2.03 \text{ m s}^{-1}$

**9** Taking the observation point as the origin:

**a** $\mathbf{r} = -500\mathbf{j} + t(2\mathbf{i} + 3\mathbf{j})$
$= 2t\mathbf{i} + (3t - 500)\mathbf{j}$

**b** At 2:05 pm $t = 300$ s, therefore
$\mathbf{r} = 2(300)\mathbf{i} + (3(300) - 500)\mathbf{j}$
$= 600\mathbf{i} + 400\mathbf{j}$
$|\mathbf{r}| = \sqrt{600^2 + 400^2}$
$= 721$ m (3 s.f.)

**10 a** For $F$
$\mathbf{r} = 400\mathbf{j} + t(7\mathbf{i} + 7\mathbf{j})$
$= 7t\mathbf{i} + (7t + 400)\mathbf{j}$
For $S$
$\mathbf{r} = 500\mathbf{i} + t(-3\mathbf{i} + 15\mathbf{j})$
$= (500 - 3t)\mathbf{i} + 15t\mathbf{j}$

**b** If $F$ and $S$ collide then
$7t = 500 - 3t$ and $15t = 7t + 400$ will give the same value of $t$.
$10t = 500$ and $8t = 400$
$t = 50$ and $t = 50$
So the ferry and the speedboat will collide 50 seconds after noon.
Point of collision is position of F and S at $t = 50$
$\mathbf{r} = 7(50)\mathbf{i} + (7(50) + 400)\mathbf{j}$ ***or*** $\mathbf{r} = (500 - 3(50))\mathbf{i} + 15(50)\mathbf{j}$
So, point of collision = $350\mathbf{i} + 750\mathbf{j}$

**11 a** $\mathbf{r}_A = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{r}_B = 5\mathbf{i} - 2\mathbf{j}$
$\mathbf{v}_A = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v}_B = -\mathbf{i} + 4\mathbf{j}$
For ship $A$
$\mathbf{r} = \mathbf{i} + 3\mathbf{j} + t(2\mathbf{i} - \mathbf{j})$
$= (2t + 1)\mathbf{i} + (3 - t)\mathbf{j}$
For ship $B$
$\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} + t(-\mathbf{i} + 4\mathbf{j})$
$= (5 - t)\mathbf{i} + (4t - 2)\mathbf{j}$

**11 b** The position of $B$ relative to $A$ is
$(5 - t)\mathbf{i} + (4t - 2)\mathbf{j} - ((2t + 1)\mathbf{i} + (3 - t)\mathbf{j})$
$= ((4 - 3t)\mathbf{i} + (5t - 5)\mathbf{j})$ km as required

**c** If $A$ and $B$ collide then the position of $B$ relative to $A$ will be $0\mathbf{i} + 0\mathbf{j}$
so $(4 - 3t)\mathbf{i} + (5t - 5)\mathbf{j}$ must equal $0\mathbf{i} + 0\mathbf{j}$ for some value of $t$
However, there is no value of $t$ for which $(4 - 3t) = 0$ ***and*** $(5t - 5) = 0$.
Therefore the ships will not collide.

**d** At 10 am $t = 2$, so the position of $B$ relative to $A$ is
$(4 - 3(2))\mathbf{i} + (5(2) - 5)\mathbf{j} = -2\mathbf{i} + 5\mathbf{j}$

$$|-2\mathbf{i} + 5\mathbf{j}| = \sqrt{(-2)^2 + 5^2}$$
$$= \sqrt{29}$$
$$= 5.39 \text{ km (3 s.f.)}$$

**12 a** $\mathbf{u}_A = -\mathbf{i} + \mathbf{j}$, $\mathbf{a}_A = 2\mathbf{i} - 4\mathbf{j}$ and $t = 3$
$\mathbf{u}_B = \mathbf{i}$, $\mathbf{a}_B = 2\mathbf{j}$ and $t = 3$
For $A$

$$\mathbf{v}_A = \mathbf{u}_A + \mathbf{a}_A t$$
$$= -\mathbf{i} + \mathbf{j} + 3(2\mathbf{i} - 4\mathbf{j})$$
$$= 5\mathbf{i} - 11\mathbf{j}$$
$$|\mathbf{v}_A| = \sqrt{5^2 + (-11)^2}$$
$$= \sqrt{146}$$
$$= 12.1 \text{ m s}^{-1} \text{ (3 s.f.)}$$

For $B$

$$\mathbf{v}_B = \mathbf{u}_B + \mathbf{a}_B t$$
$$= \mathbf{i} + 3(2\mathbf{j})$$
$$= \mathbf{i} + 6\mathbf{j}$$
$$|\mathbf{v}_B| = \sqrt{1^2 + 6^2}$$
$$= \sqrt{37}$$
$$= 6.08 \text{ m s}^{-1} \text{ (3 s.f.)}$$

**b** $\mathbf{u}_A = -\mathbf{i} + \mathbf{j}$, $\mathbf{a}_A = 2\mathbf{i} - 4\mathbf{j}$ and $t = 3$

$$\mathbf{s}_A = \mathbf{u}_A t + \frac{1}{2}\mathbf{a}_A t^2$$
$$= 3(-\mathbf{i} + \mathbf{j}) + \frac{9}{2}(2\mathbf{i} - 4\mathbf{j})$$
$$= 6\mathbf{i} - 15\mathbf{j}$$

$A$ has initial position vector $12\mathbf{i} + 12\mathbf{j}$,
therefore the particles collide at
$\mathbf{r} = 12\mathbf{i} + 12\mathbf{j} + 6\mathbf{i} - 15\mathbf{j}$
$= 18\mathbf{i} - 3\mathbf{j}$

**12 c** $\mathbf{u}_B = \mathbf{i}$, $\mathbf{a}_B = 2\mathbf{j}$ and $t = 3$

$$\mathbf{s}_B = \mathbf{u}_B t + \frac{1}{2}\mathbf{a}_B t^2$$
$$= 3\mathbf{i} + \frac{9}{2}(2\mathbf{j})$$
$$= 3\mathbf{i} + 9\mathbf{j}$$

The particles collide at $\mathbf{r} = 18\mathbf{i} - 3\mathbf{j}$.
So $B$ has starting position
$\mathbf{r}_B = 18\mathbf{i} - 3\mathbf{j} - (3\mathbf{i} + 9\mathbf{j})$
$= 15\mathbf{i} - 12\mathbf{j}$

## Challenge

Let the two aeroplanes be $A$ and $B$
$\mathbf{u}_A = 20\mathbf{i} - 100\mathbf{j}$, $\mathbf{a}_A = 6\mathbf{j}$
$\mathbf{u}_B = 70\mathbf{i} + 40\mathbf{j}$, $\mathbf{a}_A = -8\mathbf{j}$
Let the time between plane B flying over the control tower and the two planes passing over one another be $t_1$. When the planes pass directly over each other:

$$\mathbf{s}_A = \mathbf{u}_A T + \frac{1}{2}\mathbf{a}_A T^2$$
$$= (20\mathbf{i} - 100\mathbf{j})(t + t_1) + \frac{1}{2}(t + t_1)^2(6\mathbf{j})$$
$$= 20(t + t_1)\mathbf{i} - 100(t + t_1)\mathbf{j} + 3(t + t_1)^2\mathbf{j}$$
$$\mathbf{s}_B = \mathbf{u}_B T + \frac{1}{2}\mathbf{a}_B T^2$$
$$= (70\mathbf{i} + 40\mathbf{j})t_1 + \frac{1}{2}t_1^2(-8\mathbf{j})$$
$$= 70t_1\mathbf{i} + 40t_1\mathbf{j} - 4t_1^2\mathbf{j}$$

Since the aeroplanes pass directly over one another, $\mathbf{s}_A = \mathbf{s}_B$.

$$20(t + t_1) = 70t_1$$
$$50t_1 = 20t$$
$$t_1 = 0.4t$$

and

$$40t_1 - 4t_1^2 = -100(t + t_1) + 3(t + t_1)^2$$

So

$$40 \times 0.4t - 4(0.4t)^2 = -100(1.4t) + 3(1.4t)^2$$
$$16t - 0.64t^2 = -140t + 5.88t^2$$
$$6.52t^2 - 156t = 0$$
$$t(6.52t - 156) = 0$$
$$t \neq 0 \text{ so } t = \frac{156}{6.52}$$

$t = 23.9$ s (3 s.f.)