

Exercise 3B

$$1 \text{ a } \mathbf{a} + \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{i} - \mathbf{j} \\ = 6\mathbf{i} + 2\mathbf{j}$$

$$\text{b } 3\mathbf{a} + \mathbf{b} = 3(2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) \\ = 6\mathbf{i} + 9\mathbf{j} + 4\mathbf{i} - \mathbf{j} \\ = 10\mathbf{i} + 8\mathbf{j}$$

$$\text{c } 2\mathbf{a} - \mathbf{b} = 2(2\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - \mathbf{j}) \\ = 4\mathbf{i} + 6\mathbf{j} - 4\mathbf{i} + \mathbf{j} \\ = 7\mathbf{j}$$

$$\text{d } 2\mathbf{b} - \mathbf{a} = 2(4\mathbf{i} - \mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) \\ = 8\mathbf{i} - 2\mathbf{j} - 2\mathbf{i} - 3\mathbf{j} \\ = 6\mathbf{i} - 5\mathbf{j}$$

$$\text{e } 3\mathbf{a} - 2\mathbf{b} = 3(2\mathbf{i} + 3\mathbf{j}) - 2(4\mathbf{i} - \mathbf{j}) \\ = (6\mathbf{i} + 9\mathbf{j}) - (8\mathbf{i} - 2\mathbf{j}) \\ = (6 - 8)\mathbf{i} + (9 + 2)\mathbf{j} \\ = -2\mathbf{i} + 11\mathbf{j}$$

$$\text{f } \mathbf{b} - 3\mathbf{a} = (4\mathbf{i} - \mathbf{j}) - 3(2\mathbf{i} + 3\mathbf{j}) \\ = (4\mathbf{i} - \mathbf{j}) - (6\mathbf{i} + 9\mathbf{j}) \\ = (4 - 6)\mathbf{i} + (-1 - 9)\mathbf{j} \\ = -2\mathbf{i} - 10\mathbf{j}$$

$$\text{g } 4\mathbf{b} - \mathbf{a} = 4(4\mathbf{i} - \mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) \\ = (16\mathbf{i} - 4\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) \\ = (16 - 2)\mathbf{i} + (-4 - 3)\mathbf{j} \\ = 14\mathbf{i} - 7\mathbf{j}$$

$$\text{h } 2\mathbf{a} - 3\mathbf{b} = 2(2\mathbf{i} + 3\mathbf{j}) - 3(4\mathbf{i} - \mathbf{j}) \\ = (4\mathbf{i} + 6\mathbf{j}) - (12\mathbf{i} - 3\mathbf{j}) \\ = (4 - 12)\mathbf{i} + (6 + 3)\mathbf{j} \\ = -8\mathbf{i} + 9\mathbf{j}$$

$$2 \text{ a } \mathbf{a} + \lambda\mathbf{b} = (2\mathbf{i} + 5\mathbf{j}) + \lambda(3\mathbf{i} - \mathbf{j}) \\ = (2 + 3\lambda)\mathbf{i} + (5 - \lambda)\mathbf{j} \\ \text{Parallel to } \mathbf{i}, \text{ so } 5 - \lambda = 0, \lambda = 5.$$

$$\text{b } \mu\mathbf{a} + \mathbf{b} = \mu(2\mathbf{i} + 5\mathbf{j}) + (3\mathbf{i} - \mathbf{j}) \\ = (2\mu + 3)\mathbf{i} + (5\mu - 1)\mathbf{j} \\ \text{Parallel to } \mathbf{j}, \text{ so } 2\mu + 3 = 0, \mu = -\frac{3}{2}$$

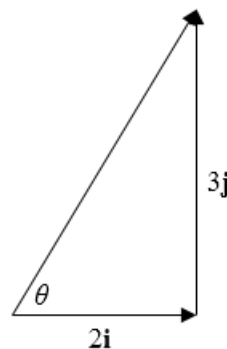
$$3 \text{ a } \mathbf{c} + \lambda\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) + \lambda(\mathbf{i} - 2\mathbf{j}) \\ = (3 + \lambda)\mathbf{i} + (4 - 2\lambda)\mathbf{j} \\ \text{Parallel to } \mathbf{i} + \mathbf{j}, \text{ so } 3 + \lambda = 4 - 2\lambda \\ 3\lambda = 1, \lambda = \frac{3}{2}$$

$$\text{b } \mu\mathbf{c} + \mathbf{d} = \mu(3\mathbf{i} + 4\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \\ = (3\mu + 1)\mathbf{i} + (4\mu - 2)\mathbf{j} \\ \text{Parallel to } \mathbf{i} + 3\mathbf{j}, \text{ so } 4\mu - 2 = 3(3\mu + 1) \\ 4\mu - 2 = 9\mu + 3 \\ 5\mu = -5, \mu = -1$$

$$\text{c } \mathbf{c} - s\mathbf{d} = (3\mathbf{i} + 4\mathbf{j}) - s(\mathbf{i} - 2\mathbf{j}) \\ = (3 - s)\mathbf{i} + (4 + 2s)\mathbf{j} \\ \text{Parallel to } 2\mathbf{i} + \mathbf{j}, \text{ so} \\ 3 - s = 2(4 + 2s) \\ 3 - s = 8 + 4s \\ -5 = 5s, s = -1$$

$$\text{d } \mathbf{d} - t\mathbf{c} = (\mathbf{i} - 2\mathbf{j}) - t(3\mathbf{i} + 4\mathbf{j}) \\ = (1 - 3t)\mathbf{i} + (-2 - 4t)\mathbf{j} \\ \text{Parallel to } -2\mathbf{i} + 3\mathbf{j}, \text{ so} \\ -2(-2 - 4t) = 3(1 - 3t) \\ 4 + 8t = 3 - 9t \\ 1 = -17t, t = -\frac{1}{17}$$

$$4 \text{ a } 2\mathbf{i} + 3\mathbf{j}$$



$$|2\mathbf{i} + 3\mathbf{j}| = \sqrt{2^2 + 3^2} \\ = \sqrt{13} \\ = 3.61 \text{ (3 s.f.)}$$

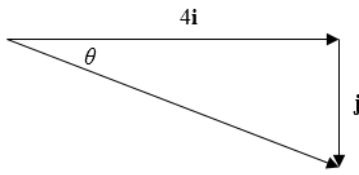
$$\tan \theta = \frac{3}{2}$$

$$\theta = 56.3\dots$$

Therefore the bearing is

$$90 - 56.3\dots = 33.7\dots$$

$$= 034^\circ \text{ (to the nearest degree)}$$

4 b $4\mathbf{i} - \mathbf{j}$ 

$$\begin{aligned} |4\mathbf{i} - \mathbf{j}| &= \sqrt{4^2 + (-1)^2} \\ &= \sqrt{17} \\ &= 4.12 \text{ (3 s.f.)} \end{aligned}$$

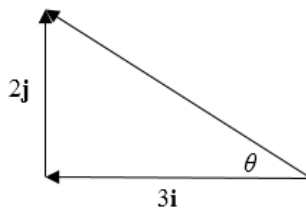
$$\tan \theta = \frac{1}{4}$$

$$\theta = 14.0\dots$$

Therefore the bearing is

$$90 + 14.0\dots = 104\dots$$

$$= 104^\circ \text{ (to the nearest degree)}$$

c $-3\mathbf{i} + 2\mathbf{j}$ 

$$\begin{aligned} |-3\mathbf{i} + 2\mathbf{j}| &= \sqrt{(-3)^2 + 2^2} \\ &= \sqrt{13} \\ &= 3.61 \text{ (3 s.f.)} \end{aligned}$$

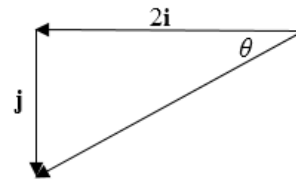
$$\tan \theta = \frac{2}{3}$$

$$33.6\dots$$

Therefore the bearing is

$$270 + 33.6\dots = 303.6\dots$$

$$= 304^\circ \text{ (to the nearest degree)}$$

4 d $-2\mathbf{i} - \mathbf{j}$ 

$$\begin{aligned} |-2\mathbf{i} - \mathbf{j}| &= \sqrt{(-2)^2 + (-1)^2} \\ &= \sqrt{5} \\ &= 2.24 \text{ (3 s.f.)} \end{aligned}$$

$$\tan \theta = \frac{1}{2}$$

$$\theta = 26.5\dots$$

Therefore the bearing is

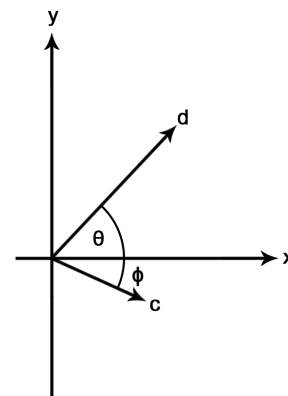
$$270 - 26.5\dots = 243.4\dots$$

$$= 243^\circ \text{ (to the nearest degree)}$$

Challenge

$$\mathbf{c} = 3\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{d} = (3 + \lambda)\mathbf{i} + (4\lambda - 5)\mathbf{j} = \mathbf{a} + \lambda\mathbf{b}$$



$$\arctan \frac{2}{3} = \theta$$

$$\arctan \frac{4\lambda - 5}{3 + \lambda} = \phi$$

$$\phi + \theta = 90^\circ$$

Rearranging and solving for lambda,

$$\frac{4\lambda - 5}{3 + \lambda} = -1.5$$

$$\lambda = 3.8$$