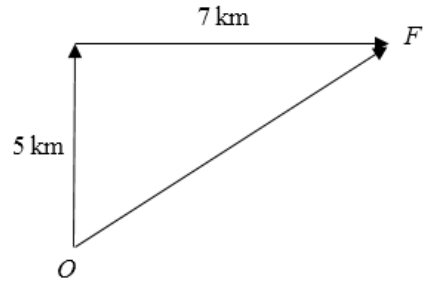
Exercise 3A

1 Let the initial position of the bird be O and the final position be F



By Pythagoras' theorem

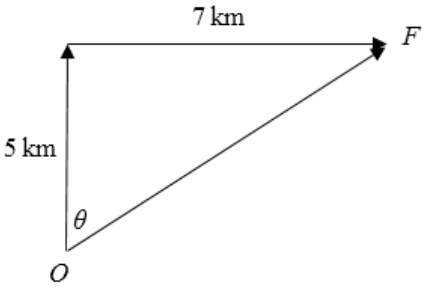
$OF^2 = 5^2 + 7^2$

$= 74$

$OF = \sqrt{74}$

$= 8.60$ km (3 s.f.)

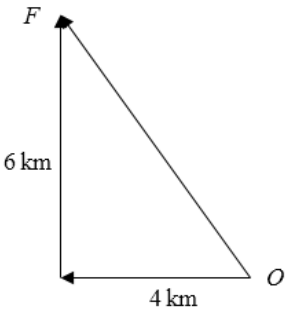
Let the bearing of F from O be θ



$\tan\theta = \frac{7}{5}$

$\theta = 054°$ (to the nearest degree)

2 Let the initial position of the girl be O and the final position be F



The total distance cycled by the girl is

$4 + 6 = 10$ km

Her displacement from O is $|OF|$

$OF^2 = 4^2 + 6^2$

$= 52$

$OF = \sqrt{52}$

$= 7.21$ km (3 s.f.)

3 a $\mathbf{v}_1 = 4\mathbf{i}$

$\mathbf{v}_2 = 5\mathbf{i} + 2\mathbf{j}$

$\mathbf{v}_3 = -3\mathbf{i} + \mathbf{j}$

$\mathbf{v}_4 = 2\mathbf{i} + 3\mathbf{j}$

$\mathbf{v}_5 = -2\mathbf{i} - \mathbf{j}$

$\mathbf{v}_6 = -3\mathbf{j}$

b i $\mathbf{v}_1 + \mathbf{v}_2 = 4\mathbf{i} + 5\mathbf{i} + 2\mathbf{j}$

$= 9\mathbf{i} + 2\mathbf{j}$

ii $\mathbf{v}_4 + \mathbf{v}_5 = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{i} - \mathbf{j}$

$= 2\mathbf{j}$

iii $\mathbf{v}_6 + \mathbf{v}_1 + \mathbf{v}_5 = -3\mathbf{j} + 4\mathbf{i} - 2\mathbf{i} - \mathbf{j}$

$= 2\mathbf{i} - 4\mathbf{j}$

4 a $|3\mathbf{i} + 4\mathbf{j}| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

b $|6\mathbf{i} - 8\mathbf{j}| = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

c $|5\mathbf{i} + 12\mathbf{j}| = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

d $|2\mathbf{i} + 4\mathbf{j}| = \sqrt{2^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 4.47$ (3 s.f.)

e $|3\mathbf{i} - 5\mathbf{j}| = \sqrt{3^2 + 5^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$= 5.83$ (3 s.f.)

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4 f $|4\mathbf{i}+7\mathbf{j}| = \sqrt{4^2+7^2}$

$= \sqrt{16+49}$

$= \sqrt{65}$

$= 8.06$ (3 s.f.)

g $|-3\mathbf{i}+5\mathbf{j}| = \sqrt{3^2+5^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

$= 5.83$ (3 s.f.)

h $|-4\mathbf{i}+-\mathbf{j}| = \sqrt{4^2+1^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

$= 4.12$ (3 s.f.)

5 a $\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}, \mathbf{b} = \begin{pmatrix}3\\-4\end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix}5\\-1\end{pmatrix}$

$\mathbf{a}+\mathbf{b} = \begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}3\\-4\end{pmatrix}$

$= \begin{pmatrix}5\\-1\end{pmatrix}$

$|\mathbf{a}+\mathbf{b}| = \sqrt{5^2+(-1)^2}$

$= \sqrt{26}$

b $2\mathbf{a}-\mathbf{c} = 2\begin{pmatrix}2\\3\end{pmatrix}-\begin{pmatrix}5\\-5\end{pmatrix}$

$= \begin{pmatrix}-1\\7\end{pmatrix}$

$|2\mathbf{a}-\mathbf{c}| = \sqrt{(-1)^2+7^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

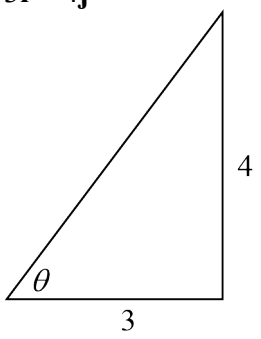
c $3\mathbf{b}-2\mathbf{c} = 3\begin{pmatrix}3\\-4\end{pmatrix}-2\begin{pmatrix}5\\-1\end{pmatrix}$

$= \begin{pmatrix}-1\\-10\end{pmatrix}$

$|3\mathbf{b}-2\mathbf{c}| = \sqrt{(-1)^2+(-10)^2}$

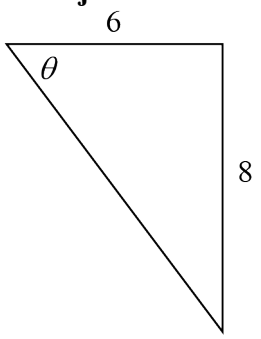
$= \sqrt{101}$

6 a $3\mathbf{i}+4\mathbf{j}$



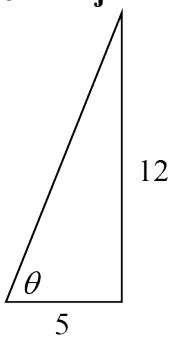
$\tan^{-1}\left(\frac{4}{3}\right) = 53.1°$ above (3 s.f.)

b $6\mathbf{i}-8\mathbf{j}$



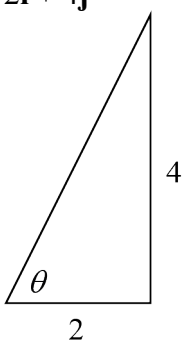
$\tan^{-1}\left(\frac{8}{6}\right) = 53.1°$ below (3 s.f.)

c $5\mathbf{i}+12\mathbf{j}$



$\tan^{-1}\left(\frac{12}{5}\right) = 67.4°$ above (3 s.f.)

d $2\mathbf{i}+4\mathbf{j}$



$\tan^{-1}\left(\frac{4}{2}\right) = 63.4°$ above (3 s.f.)

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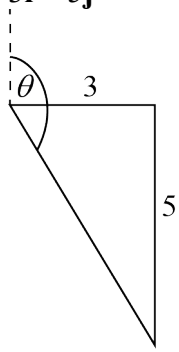
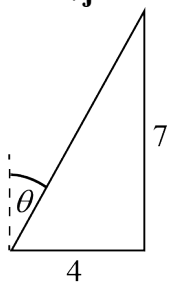
7 a $3\mathbf{i} - 5\mathbf{j}$



$90° + \tan^{-1}\left(\frac{5}{3}\right) = 90° + 59°$

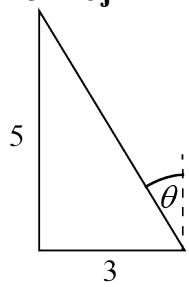
$= 149°$ (3 s.f.) to the right

b $4\mathbf{i} + 7\mathbf{j}$



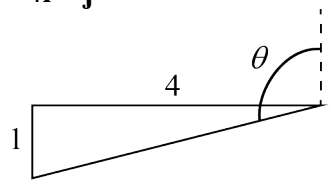
$\tan^{-1}\left(\frac{4}{7}\right) = 29.7°$ (3 s.f.) to the right

c $-3\mathbf{i} + 5\mathbf{j}$



$\tan^{-1}\left(\frac{3}{5}\right) = 31.0°$ (3 s.f.) to the left

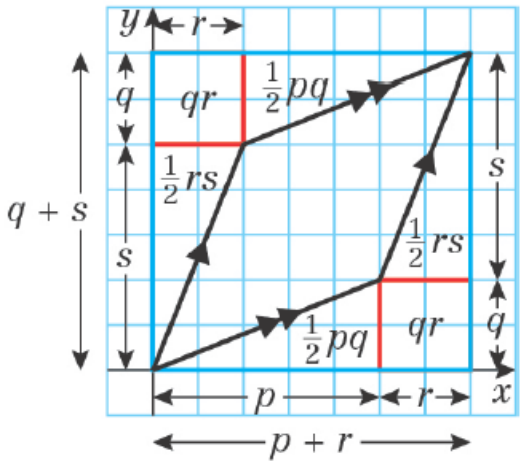
7 d $-4\mathbf{i} - \mathbf{j}$



$90° + \tan^{-1}\left(\frac{1}{4}\right) = 90° + 14°$

$= 104°$ (3 s.f.) to the left

Challenge



Area of parallelogram

= area of large blue rectangle − 2(area of small red rectangle) − 2(area of triangle 1) − 2(area of triangle 2)

$= (p + r)(q + s) - 2(qr) - 2(\frac{1}{2}pq) - 2(\frac{1}{2}rs)$

$= pq + ps + qr + rs - 2qr - pq - rs = ps - qr$

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