

## Exercise 1A

1 a i  $x = 0$  gives  $h = 0.36 \times 0 - 0.003 \times (0)^2 = 0$

Height = 0 m

ii  $x = 100$  m gives  $h = 0.36 \times 100 - 0.003 \times (100)^2 = 36 - 30$

Height = 6 m

b  $x = 200$  m gives  $h = 0.36 \times 200 - 0.003 \times (200)^2 = 72 - 120$

Height = -48 m

c The model is not valid for this distance as it predicts the ball would be 48 m below ground level: the ball has already hit the ground at this point.

2 a 90 m (as this is the height when  $t = 0$ )

b i  $t = 3$  gives  $h = -5 \times (3)^2 + 15 \times 3 + 90 = -45 + 45 + 90$

Height above sea level = 90 m

ii  $t = 5$  gives  $h = -5 \times (5)^2 + 15 \times 5 + 90 = -125 + 75 + 90$

Height above sea level = 40 m

c  $t = 20$  gives  $h = -5 \times (20)^2 + 15 \times 20 + 90 = -2000 + 300 + 90 = -1610$

Height = 1610 m below sea level

d The prediction is incorrect because this height is below sea level, where the model is probably no longer valid, because forces acting on the ball will be different.

3 a When  $h = 4$ ,

$4 = 2 + 1.1x - 0.1x^2$  or rearranging,  $0 = -0.1x^2 + 1.1x - 2$ , so using the quadratic formula,

$$\begin{aligned}
 x &= \frac{-1.1 \pm \sqrt{(1.1)^2 - 4 \times (-0.1) \times (-2)}}{2 \times (-0.1)} \\
 &= \frac{-1.1 \pm \sqrt{0.41}}{-0.2} \\
 &\approx \frac{-1.1 \pm 0.6403}{-0.2}
 \end{aligned}$$

So  $x = 2.30$  or  $8.70$  (to 3 s.f.)

The ball is 4 m above the ground after it has travelled both 2.30 m and 8.70 m horizontally.

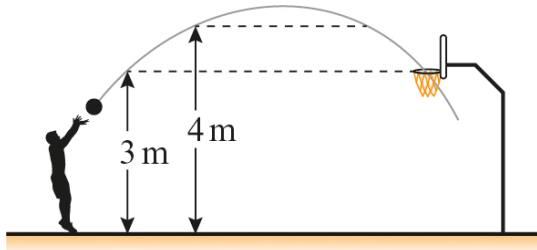
3 b When  $h = 3$ ,

$3 = 2 + 1.1x - 0.1x^2$  or rearranging,  $0 = -0.1x^2 + 1.1x - 1$ , and dividing by  $-0.1$ ,

$$0 = x^2 - 11x + 10 = (x - 1)(x - 10)$$

So  $x = 1$  or  $10$ , and the ball is at height  $3$  m after it has travelled both  $1$  m and  $10$  m horizontally.

At the shorter distance, the ball will be travelling upward (see diagram), so  $k = 10$ .



c If  $x > 10$  m the equation is no longer valid as the ball will have gone through (or past) the net, and there would possibly be new forces acting on the ball.

4 a When  $t = 1$ ,  $d = 13.2$

So, substituting,  $d = kt^2$  becomes  $13.2 = k \times 1^2$  and therefore  $k = 13.2$ .

Our completed equation is  $d = 13.2t^2$ . When  $t = 10$ ,

$$d = 13.2 \times 10^2 = 1320$$

The distance travelled is  $1320$  m.

b Clearly, the model is valid for positive values of  $t$  only. We are also unsure of what happens after  $t = 10$ , and we therefore can't use this model past that point. The model is valid for  $0 \leq t \leq 10$ .

5  $h \geq 0$  means that  $0.36x - 0.003x^2 = x(0.36 - 0.003x) \geq 0$

We are assuming that  $x \geq 0$  after the ball is struck, so we need the bracket to be non-negative:

$$0.36 \geq 0.003x$$

$$x \leq \frac{0.36}{0.003} = 120$$

So the model is valid for  $0 \leq x \leq 120$ .

6 When the stone enters the sea,  $h = 0$

$$0 = -5t^2 + 15t + 90, \text{ or, dividing by } -5, 0 = t^2 - 3t - 18 = (t - 6)(t + 3)$$

So,  $t = -3$  or  $6$  and the stone hits the sea  $6$  seconds after it is thrown, since the model is valid only for the time **after** the stone is thrown at  $t = 0$ .

Therefore the model is valid for  $0 \leq t \leq 6$ .