GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4761:** Mechanics 1

Mark Scheme for June 2011

4761   **Mark Scheme**   **June 2011**

comment

You should expect to follow through from one part to another unless the scheme says otherwise but not follow through within a part unless the scheme specifies this.

Each script must be viewed as a whole at some stage so that

(i) a candidate's writing of letters, digits, symbols on diagrams etc can be better interpreted;

(ii) repeated mistakes can be recognised (e.g. calculator in wrong angle mode throughout – penalty 1 in the script and FT except given answers).

You are advised to 'set height' in *scoris*, particularly for question 7(ii). Questions 5 and 8(v) also spread onto two pages.

Q 1		mark	notes
	$v^2 = 11^2 + 2 \times (-9.8) \times 2.4$	M1	Use of $v^2 = u^2 + 2as$ or complete sequence of correct *suvat*. Accept sign errors in substitution.
		A1	All correct
	$v = 8.6$ so 8.6 m s^{-1}.	A1	cao [Award all marks if 8.6 seen WWW] Do not condone ±8.6.
		3	

Q 2		mark	comment
	either for u first: $8 = \frac{1}{2}(u + 2.25) \times 32$	M1	Using $s = \frac{1}{2}(u + v)t$
	$u = -1.75$ so 1.75m s^{-1}	A1	
	$2.25 = -1.75 + 32a$	M1	Use of any appropriate *suvat* with **their** values and correct signs
	$a = 0.125$ so 0.125 m s^{-2}	F1	Sign must be consistent with their u, FT from their value of u
	Directions of u and a are defined	F1	Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
		5	
	Or for a first: $8 = 2.25 \times 32 - \frac{1}{2} \times a \times 32^2$	M1	Using $s = vt - \frac{1}{2}at^2$
	$a = 0.125$ so 0.125 m s^{-2}	A1	
	$2.25 = u + 32 \times 0.125$	M1	Use of any appropriate *suvat* with **their** values and correct signs
	$u = -1.75$ so 1.75m s^{-1}	F1	Sign must be consistent with their a, FT from their value of a
	Directions of u and a are defined	F1	Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
		5	
	Or *using simultaneous equations* Set up one relevant equation with a and u.	M1	Using **one** of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$
	Set up second relevant equation with a and u.	M1	Using **another** of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$
	Solving to find $u = -1.75$ so 1.75m s^{-1}	A1	
	Solving to find $a = 0.125$ so 0.125 m s^{-2}	F1	FT from their value of u or a, whichever found first
	Directions of u and a are defined	F1	Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.
		5	
		5	

Q 3		m a r k	Notes
(i)	$-6 = -2 \times 3$ so $y = 3 \times 3 = 9$ and $z = -4 \times 3 = -12$	M1 A1 2	May be implied Both correct [Award 2 for both correct answers seen WW]
(ii)	$\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} = 5\mathbf{a}$ $\mathbf{a} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ so accn is $\begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix}$ m s^{-2} Magnitude is $\sqrt{0.2^2 + (-0.4)^2 + (-1)^2}$ $= 1.09544\ldots$ so 1.10 m s^{-2}, (3 s. f.)	M1 B1 A1 M1 F1 5	Use of Newton's 2nd Law in vector form for all 3 cpts of attempted resultant Treat use of wrong vectors as MR. Correct LHS The acceleration may be written as a magnitude in a given direction. FT **their** values. Condone missing brackets. Condone no – signs. Accept 1.1. Accept surd form. Must come from a vector with 3 non-zero components for **a**
		7	

Q 4		mark	Comment
(i)		B1	Any one force in correct direction correctly labelled with arrow **or** all forces with correct directions and arrows. A force may be replaced by its components if labelled correctly eg $m$gcos20°, $m$gsin20°.
		B1	All correct (Accept words for labels and weight as W, mg, 147 (N)) No extra or duplicate forces. Do not allow force **and** its components unless components are clearly distinguished, eg by broken lines.
		2	
(ii)	**Either** Up the plane Pcos20 – 15 × 9.8 ×sin20 = 0	M1	Attempt to resolve at least one force up plane. Accept mass not weight. No extra forces. If other directions used, all forces must be present but see below for resolving vertically and horizontally.
		A1	Accept only error as consistent s $\leftrightarrow$ c .
	P = 53.50362… so 53.5 (3 s. f.)	A1	Cao
		3	
	Or Vertically and horizontally $R\cos 20° = 15g$, $R\sin 20° = P$ Eliminate R	M1	Attempt to resolve all forces both horizontally and vertically and attempt to combine into a single equation. No extra forces. Accept s $\leftrightarrow$ c . Accept mass not weight.
	$P = \dfrac{15g}{\cos 20°} \times \sin 20°$	A1	Accept only error as consistent s $\leftrightarrow$ c .
	$P = 53.5$ (3.s.f.)	A1	Cao
		3	
	Or Triangle of forces Triangle drawn and labelled	M1	All sides must be labelled and in correct orientation; three forces only; condone no arrows
	$\dfrac{P}{15g} = \tan 20°$	A1	Oe
	$P = 53.5$ (3.s.f.)	A1	Cao
		3	
		5	

Q 5		mark	notes
	Usual notation **either** consider height: Attempt to substitute for u and a in $s = ut + \frac{1}{2}at^2$	M1	Accept: g as g, ± 9.8, ± 9.81, ± 10; $u = 30$; $s \leftrightarrow c$.
	$y = 30\sin 35\, t - 4.9t^2$	A1	Derivation need not be shown
	Need $y = 0$ for time of flight T	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ ($= 3.511692\ldots$)	A1	cao. Any form. May not be explicit.
	Or Consider time to top Attempt to substitute for u and a in $v = u + at$	M1	Accept: g as g, ± 9.8, ± 9.81, ± 10; $u = 30$; $s \leftrightarrow c$.
	$v = 30\sin 35 - 9.8t$	A1	Derivation need not be shown
	Need $v = 0$ and to double for time of flight T	B1	
	giving $T = \dfrac{30\sin 35}{4.9}$ ($= 3.511692\ldots$)	A1	cao. Any form. May not be explicit.
	then $x = 30\cos 35\, T$	M1	Accept $s \leftrightarrow c$ if consistent with above
	so $x = 30\cos 35 \times \dfrac{30\sin 35}{4.9}$ ($= 86.29830\ldots$)	F1	FT for their time Condone consistent $s \leftrightarrow c$ error (which could lead to correct answer here).
	Required time for sound is $x/343$	M1	FT from their x
	Total time is $3.511692\ldots + 0.251598\ldots = $ 3.76329… so 3.76 s (3 s. f.)	A1	cao following fully correct working throughout question.
		8	

Q6		mark	notes
(i)	**Either** using *suvat*: Use of $\mathbf{v} = \mathbf{u} + t\mathbf{a}$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ Use of $\mathbf{r} = (\mathbf{r}_0 +)\, t\mathbf{u} + \frac{1}{2}\, t^2\mathbf{a}$ $+\, 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$ 5	 M1 A1 M1 B1 A1	Column vectors may be used throughout; lose 1 mark once if **j** components put at top or if fraction line included. . Notation used must be clear. substitution required. Must be vectors. substitution required. $\mathbf{r}_0$ not required. Must be vectors. May be seen on either side of a meaningful equation for **r** Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \frac{1}{2} \times 2 \times t^2\, \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12c in marking instructions)
	Or using integration: $\mathbf{v} = \int \mathbf{a}\, dt$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ $\mathbf{r} = \int \mathbf{v}\, dt$ $+\, 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$ 5	 M1 A1 M1 B1 A1	 Attempt at integration. Condone no '+**c**'. Must be vectors. cao Integrate their **v** but must contain 2 components. Must be vectors. May be seen on either side of a meaningful equation for **r** Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \frac{1}{2} \times 2 \times t^2\, \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12e in marking instructions)
		5	
(ii)	$\mathbf{v}(2.5) = 4\mathbf{i} - 5\mathbf{j}$ Angle is $(90+)\arctan\dfrac{5}{4}$ $= 141.34019\ldots$ so $141°$ (3 s. f.) 3	B1 M1 A1	FT **their v** Award for arctan attempted oe. FT **their** values. Allow argument to be $\pm$ (**their i** cpt)/(**their j** cpt) or $\pm$ (**their j** cpt)/(**their i** cpt). Allow this mark if bearing of position vector attempted. cao
		8	

Q7		mark	notes
(i)	$\dfrac{-20}{2} = -10$ -10 m s^{-2}	M1 A1 2	Use of a suitable triangle to attempt at $\Delta v / \Delta t$ for suitable interval. Accept wrong sign. cao. Allow both marks if correct answer seen.
(ii) (A) **(B)**	Signed area under graph $\frac{1}{2} \times 2 \times 20 = 20$	M1 A1	Using the relevant area or other complete method
	either using areas Signed area $2 \le t \le 5$ is $\frac{1}{2} \times \big((5-2) + (4.5-2.4) \big) \times (-4) = -10.2$ Signed area $5 \le t \le 6$ is $\frac{1}{2} \times 1 \times 8 = 4$ Total displacement is 13.8 m	 B1 B1 B1	 Allow + 10.2. cao but FT from their 20 in part (A)
	or using *suvat* From $t = 0$ to $t = 2.4$: 19.2 From $t = 4.5$ to $t = 6$: 3.0 From $t = 2.4$ to $t = 4.5$: -8.4 Total : 13.8	 B1 B1 B1 5	 Both required and both must be correct.
(iii)	$a = 4t - 14$ $a(0.5) = -12$ so -12 m s^{-2}	M1 A1 A1 3	Differentiate. Do not award for division by *t*.
(iv)	Model A gives -4 m s^{-1} For model B we need *v* when $a = 0$ $v\left(\frac{7}{2}\right) = -4.5$ so model B is 0.5 m s^{-1} less	B1 M1 A1 F1 4	May be implied by other working Using (iii) or an argument based on symmetry or sketch graph that $a = 0$ when $t = 3.5$ Accept values without more or less

(v)			Do not penalise poor notation
	Displacement is $\displaystyle\int_0^6 \left(2t^2 - 14t + 20\right)\,\mathrm{d}t$	M1	Limits not required.
	$= \left[\dfrac{2t^3}{3} - 7t^2 + 20t\right]_0^6$	A1	Limits not required. Accept 2 terms correct.
		M1	Substitute limits
	$= 12$ so 12 m.	A1	cao. Accept bottom limit not substituted.
		4	
		18	

Q 8		m a r k	notes
(i)	25 N	B1 1	Condone no units. Do not accept -25 N.
(ii)	50 cos25 $= 45.31538\ldots$ so 45.3 N (3 s. f.)	M1 A1 2	Attempt to resolve 50 N. Accept $s \leftrightarrow c$. No extra forces. cao but accept – 45.3.
(iii)	Resolving vertically $R + 50 \sin 25 - 8 \times 9.8 = 0$ $R = 57.26908\ldots$ so 57.3 N (3 s. f.)	M1 A1 A1 3	All relevant forces with resolution of 50 N. No extras. Accept $s \leftrightarrow c$. All correct.
(iv)	Newton's 2nd Law in direction DC $50 \cos 25 - 20 = 18a$ $a = 1.4064105\ldots$ so 1.41 m s^{-2} (3 s. f.)	M1 A1 A1 3	Newton's 2nd Law with $m = 18$. Accept $F = mga$. Attempt at resolving 50 N. Allow 20 N omitted and $s \leftrightarrow c$. No extra forces. Allow only sign error and $s \leftrightarrow c$. cao
Q8	continued		
(v)	Resolution of weight down the slope	B1	$mg\sin5°$ where $m = 8$ or 10 or 18, wherever first seen
	either Newton's 2nd Law down slope overall $18 \times 9.8 \times \sin5 – 20 = 18a$ $a = - 0.2569\ldots$ Newton's 2nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives For D: $10 \times 9.8 \times \sin 5 - 15 - T = 10a$ (For C: $8 \times 9.8 \times \sin 5 - 5 + T = 8a$) $T = - 3.888\ldots = - 3.89$ N (3 s. f.) The force is a thrust	M1 A1 M1 F1 A1 A1	$F = ma$. Must have 20 N and $m = 18$. Allow weight not resolved and use of mass. Accept $s \leftrightarrow c$ and sign errors (including inconsistency between the 15 N and the 5 N). cao $F = ma$. Must consider the motion of either C or D and include: component of weight, resistance and T. No extra forces. Condone sign errors and $s \leftrightarrow c$. Do not condone inconsistent value of mass. FT only applies to a, and only if direction is consistent. '+T' if T taken as a thrust '-T' if T taken as a thrust If T taken as thrust, then $T = +3.89$. Dependent on T correct

or Newton's 2nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives	M1	$F = ma$. Must consider the motion of C and include: component of weight, resistance and T. No extra forces. Condone sign errors and s $\leftrightarrow$ c . Do not condone inconsistent value of mass.
	M1	$F = ma$. Must consider the motion of D and include: component of weight, resistance and T. No extra forces. Condone sign errors and s $\leftrightarrow$ c . Do not condone inconsistent value of mass.
For C: $8 \times 9.8 \times \sin 5 - 5 + T = 8a$	A1	Award for either the equation for C or the equation for D correct. '-T' if T taken as a thrust
For D: $10 \times 9.8 \times \sin 5 - 15 - T = 10a$		'+T' if T taken as a thrust
$a = -0.2569\ldots$ $T = -3.888\ldots = $ - 3.89 N (3s.f.)	A1	First of a and T found is correct. If T taken as thrust, then $T = +3.89$.
	F1	The second of a and T found is FT
The force is a thrust	A1	Dependent on T correct
then After 2 s: $v = 3 + 2 \times a$	M1	Allow sign of a not followed. FT their value of a. Allow change to correct sign of a at this stage.
$v = 2.4860303..$ so 2.49 m s⁻¹ (3 s. f.)	F1	FT from magnitude of **their** a but must be consistent with its direction.
	9	
	18	