Mark Scheme January 2010

4761 Mechanics 1

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		1	I	
1 (i)	$0 < t < 2, v = 2$ $2 < t < 3.5 \ v = -5$	B1 B1	Condone '5 downwards' and ' – 5 downwards'	2
(ii)	$\begin{array}{c} s \\ 2 \\ \hline $		Condone intent – e.g. straight lines free-hand and scales not labelled; accept non-vertical sections at $t = 2 \& 3.5$.	
		B1	Only horizontal lines used and 1 st two parts present. BOD <i>t</i> -axis section. One of 1 st 2 sections correct. FT (i) and allow if answer correct with (i) wrong All correct. Accept correct answer with (i) wrong. FT (i) only if 2 nd section –ve in (i)	2
(iii)	(A) upwards; (B) and (C) downwards	E1	All correct. Accept +/- ve but not towards/away from O Accept forwards/backwards. Condone additional wrong statements about position.	1
				5
2 (i)		M1 A1	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ If vector \mathbf{a} seen, isw.	
				2
(ii)	either $\mathbf{r} = \begin{pmatrix} -1\\2 \end{pmatrix} + \begin{pmatrix} 2\\-3 \end{pmatrix} \times 4 + \frac{1}{2}\mathbf{a} \times 4^{2}$ $\mathbf{r} = \begin{pmatrix} 27\\14 \end{pmatrix} \text{ so } \begin{pmatrix} 27\\14 \end{pmatrix} \text{ m}$	M1 A1 A1	For use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with their a. Initial position may be omitted. FT their a. Initial position may be omitted. cao. Do not condone magnitude as final answer.	
	or	M1 A1 A1	Use of $\mathbf{s} = 0.5t(\mathbf{u} + \mathbf{v})$ Initial position may be omitted. Correct substitution. Initial position may be omitted. cao Do not condone mag as final answer. SC2 for $\binom{28}{12}$	3

(iii)	Using N2L			
	$\mathbf{F} = 5\mathbf{a} = \begin{pmatrix} 12.5 \\ 15 \end{pmatrix} \text{ so } \begin{pmatrix} 12.5 \\ 15 \end{pmatrix} \mathbf{N}$	M1	Use of $\mathbf{F} = m\mathbf{a}$ or $\mathbf{F} = mg\mathbf{a}$.	
	(13) (13)	F1	FT their a only. Do not accept magnitude as final ans.	
				2
				7
3 (i)	$ \mathbf{F} = \sqrt{(-1)^2 + 5^2}$	M1	Accept $\sqrt{-1^2 + 5^2}$ even if taken to be $\sqrt{24}$	
	$=\sqrt{26} = 5.0990 = 5.10 (3 \text{ s. f.})$	A1		
	Angle with \mathbf{j} is $\arctan(0.2)$	M1	accept $arctan(p)$ where $p = \pm 0.2$ or ± 5 o.e.	
	so 11.309 so 11.3° (3 s. f.)	A1	cao	
				4
(ii)	$ \binom{-2}{3b} = 4 \binom{-1}{5} + \binom{2a}{a} $	M1	$\mathbf{H} = 4\mathbf{F} + \mathbf{G}$ soi	
	(3b) (5) (a)	M1	Formulating at least 1 scalar equation from their	
			vector equation soi	
	a = 1, b = 7 (2)	A1	a correct or G follows from their wrong a	
	so $\mathbf{G} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{H} = \begin{pmatrix} -2 \\ 21 \end{pmatrix}$	A1	Н сао	
	or $\mathbf{G} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{H} = -2\mathbf{i} + 21\mathbf{j}$			4
				8
4(i)	20cos 15 = 19.3185			
4(1)	so 19.3 N (3 s. f.) in direction BC	B1	Accept no direction. Must be evaluated	1
(ii)	Let the tension be T $T \sin 50 = 19.3185$	M1	Accept $\sin \leftrightarrow \cos$ but not (i) $\times \sin 50$	
	so <i>T</i> = 25.2185 so 25.2 N (3 s. f.)	F1	FT their 19.3 only. cwo	
				2
(iii)	$R + 20 \sin 15 - 2.5g - 25.2185 \times \cos 50 = 0$	M1	Allow 1 force missing or 1 tension not resolved. FT <i>T</i> .	
	cos 30 – 0		No extra forces. Accept mass used.	
		B1	Accept sin ↔ cos. Weight correct	
		A1	All correct except sign errors. FT their T	
	R = 35.5337 so 35.5 N (3 s. f.)	A1	cao. Accept 35 or 36 for 2. s.f.	4
(iv)	The horizontal resolved part of the 20 N	E1	Accept no reference to vertical component but do	
(17)	force is not changed.		not accept 'no change' to both components.	
			No need to be explicit that value of tension in AB depends only on horizontal component of force at C	
			depends only on normalities component of force at C	1
				8

5(i)	a = 6t - 12	M1	Differentiating	
- (-)		A1	cao	
				2
(ii)	We need $\int_{1}^{3} (3t^2 - 12t + 14) dt$	M1	Integrating. Neglect limits.	
` ,	• 1	A1	At least two terms correct. Neglect limits.	
	$= \left[t^3 - 6t^2 + 14t\right]_1^3$	AI	At least two terms correct. Neglect limits.	
	either $= (27 - 54 + 42) - (1 - 6 + 14)$	M1	Dep on 1 st M1. Use of limits with attempt at	
	(27 31 : 12)	1111	subtraction seen.	
	= 15 - 9 = 6 so 6 m	A1	cao	
	or $s = t^3 - 6t^2 + 14t + C$			
	s = 0 when $t = 1$ gives	M1	Dep on 1 st M1. An attempt to find C using $s(1) = 0$	
	0 = 1 - 6 + 14 + C so $C = -9Put t = 3 to give$		and then evaluating $s(3)$.	
	s = 27 - 54 + 42 - 9 = 6 so 6 m.	A1	cao	
				4
(iii)	v > 0 so the particle always travels in the			
()	same (+ve) direction	E1	Only award if explicit	
	As the particle never changes direction,	E1	Complete argument	
	the final distance from the starting point is the displacement.			
	•			2
				8
6 (i)	Component of weight down the plane is			
. ,	$1.5 \times 9.8 \times \frac{2}{7} = 4.2 \text{ N}$	M1	Use of mgk where k involves an attempt at	
		E1	resolution Accept $1.5 \times 9.8 \times \frac{2}{7} = 4.2$ or $14.7 \times \frac{2}{7} = 4.2$ seen	
			Accept $1.3 \times 9.6 \times \frac{7}{7} = 4.2$ of $14.7 \times \frac{7}{7} = 4.2$ seen	2
(ii)	Down the plane. Take F down the plane.			
	4.2 $-$ 6.4 $+$ $F = 0$	M1	Allow sign errors. All forces present. No extra	
	an E = 2.2 Enjoying is 2.2 N darrow de-	A 1	forces.	
	so $F = 2.2$. Friction is 2.2 N down the plane	A1	Must have direction. [Award 1 for 2.2 N seen and 2 for 2.2 N down plane seen]	
			^	2
(iii)	F up the plane N2L down the plane	M1	N2L. $F = ma$. No extra forces. Allow weight term	
	4.2 – $F = 1.5 \times 1.2$	1411	missing or wrong	
		A1	Allow only sign errors	
	so $F = 4.2 - 1.8 = 2.4$ Friction is 2.4 N up the plane	A1 A1	± 2.4 cao. Accept no reference to direction if $F = 2.4$.	
	Thenon is 2.4 iv up the plane	AI	cao. Accept no reference to unection if $F=2.4$.	4
(iv)	$2^2 = 0.8^2 + 2 \times 1.2 \times s$	M1	Use of $v^2 = u^2 + 2as$ or sequence	
	s = 1.4 so 1.4 m	A1 A1	All correct in 1 or 2-step method	
	5 – 1.4 50 1.4 III	AI		3

(v)	Diagrams	B1	Frictions and coupling force correctly labelled	
			with arrows.	
		B1	All forces present and properly labelled with	
			arrows.	
	either			
	Up the plane	M1	N2L. $F = ma$. No extra forces. Condone sign errors.	
	$10 - 3.5 \times 9.8 \times \frac{2}{7} - (2.3 + 0.7) = 3.5a$		Allow total/part weight or total/part friction omitted (but not both). Allow mass instead of weight and mass/weight not or wrongly resolved.	
	$a = -0.8 \text{ so } 0.8 \text{ m s}^{-2}$.	B1	Correct overall mass and friction	
	down the plane For barge B up the plane	A1	Clear description or diagram	
	$T - 2 \times 9.8 \times \frac{2}{7} - 0.7 = 2 \times (-0.8)$	M1	N2L on one barge with their $\pm a$ (\neq 1.2 or 0). All forces present and weight component attempted. No extra forces. Condone sign errors.	
	T = 4.7 so 4.7 N. Tension	A1	cao	
	or (separate equations of motion)		In eom for A or B allow weight or friction missing and also allow mass used instead of weight and wt not or wrongly resolved. In other equn weight component attempted and friction term present.	
	Barge A	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
	Barge B	M1	N2L. Do not allow $F = mga$. No extra forces. Condone sign errors.	
		M1	Solving a pair of equns in a and T	
	$a = -0.8 \text{ so } 0.8 \text{ m s}^{-2}$.			
	down the plane	A1	Clear description or diagram	
	T = 4.7 so 4.7 N. Tension	A1	cao cwo	
				7
				18
7 (i)	y(0) = 1	B1		1
	Either			1
(ii)	$\frac{1}{2}(20+5)-5=7.5$	M1	Use of symmetry e.g. use of $\frac{1}{2}(20+5)$	
(11)	$\binom{2}{2}(20+3)-3=7.3$			
		A1 A1	12.5 o.e. seen 7.5 cao	
	or	M1	Attempt at y' and to solve $y' = 0$	
	or	A1	Attempt at y and to solve $y = 0$ $k(15 - 2x)$ where $k = 1$ or $\frac{1}{100}$	
			100	
	$y(7.5) = \frac{1}{100} (100 + 15 \times 7.5 - 7.5^2)$	A1 M1	7.5 cao, seen as final answer FT their 7.5	
	$=\frac{25}{16}$ (1.5625) so 1.5625 m	E1	AG	
			[SC2 only showing 1.5625 leads to $x = 7.5$]	5

(iii)	$4.9t^2 = \frac{25}{16} \ (1.5625)$	M1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of	
(111)	$4.9t - \frac{1}{16} (1.3023)$	1011	$\pm 10, \pm 9.8, \pm 9.81$. If sequence of <i>suvat</i> used,	
			complete method required.	
		A1	In any method only error accepted is sign error	
	$t^2 = 0.31887$ so $t = \pm 0.56469$	711	in any method only error decepted is sign error	
	Hence 0.565 s (3 s. f.)	E1	AG. Condone no reference to –ve value. www.	
			0.565 must be justified as answer to 3 s. f.	
				3
(iv)	$\dot{x} = \frac{12.5}{0.56469} = 22.1359$	M1	or 25 / (2×0.56469)	
	0.56469	D.1	XX 610.5	
	so 22.1 m s ⁻¹ (3 s. f.))	B1	Use of 12.5 or equivalent	
	SO 22.1 m s (3 s. 1.))	E1	22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii).	
	Either		penanse if penanty affeaty given in (iii).	
		M1		
	Time is $\frac{20}{12.5} \times 0.56469$ s			
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or			
	Time is $\frac{20}{22.1359}$ s	M1		
	22.1269			
	= 0.903507 so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or 7.5	M1		
	$(iii) + \frac{7.5}{\text{their } \dot{x}}$	IVII		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	80 0.904 8 (3 8. 1.)	AI	Cao Accept 0.91 (2 8. 1.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$	M1	Must have attempts at both components	
	$\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or	M1	Or equiv. $u = 0$. Condone use of	
	$\dot{y} = 0 + 9.8 \times 0.5646$		±10, ±9.8, ±9.81.	
			Accept wrong s (or t in alternative method)	
	$=\frac{245}{8}$ (30.625) or $\dot{y} = \pm 5.539$	A1	Or equivalent. May be implied. Could come from	
			(iii) if $v^2 = u^2 + 2as$ used there. Award marks	
			again.	
	so $v = \sqrt{490 + 30.625} = 22.8172$ m s ⁻¹			
	so 22.8 m s ⁻¹ (3 s. f.)	A1	cao. www	
				18
				10