4761 Mark Scheme January 2008

4761 Mechanics 1

Q 1		Mark	Comment	Sub
(i)	V.			
	15-1 m s 1 0 10 30 35	B1	Acc and dec shown as straight lines	
		B1 B1	Horizontal straight section All correct with v and times marked and at least one axis labelled. Accept (t, v) or (v, t) used.	3
(ii)	Distance is found from the area area is $\frac{1}{2} \times 10 \times 15 + 20 \times 15 + \frac{1}{2} \times 5 \times 15$ (or $\frac{1}{2} \times (20 + 35) \times 15$)	M1 A1	At least one area attempted or equivalent uvast attempted over one appropriate interval. Award for at least two areas (or equivalent) correct Allow if a trapezium used and only 1 substitution error. FT their diagram.	
	= 412.5 so distance is 412.5 m	A1	cao (Accept 410 or better accuracy)	3
		6		
2 (i)	$\begin{pmatrix} 6 \\ 9 \end{pmatrix} = 1.5 \mathbf{a} \text{ giving } \mathbf{a} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \text{ so } \begin{pmatrix} 4 \\ 6 \end{pmatrix} \text{ m s}^{-2}$	M1	Use of N2L with an attempt to find a . Condone spurious notation.	
		A1	Must be a vector in proper form. Penalise only once in paper.	2
(ii)	Angle is $\arctan\left(\frac{6}{4}\right)$	M1	Use of arctan with their $\frac{6}{4}$ or $\frac{4}{6}$ or equiv. May use F .	_
	= 56.309 so 56.3° (3 s. f.)	F1	FT their a provided both cpts are +ve and non-zero.	2
(iii)	Using $\mathbf{s} = t\mathbf{u} + 0.5t^2\mathbf{a}$ we have	M1	Appropriate single <i>uvast</i> (or equivalent sequence of <i>uvast</i>). If integration used twice condone omission of r (0) but not v (0).	
	$\mathbf{s} = 2 \begin{pmatrix} -2 \\ 3 \end{pmatrix} + 0.5 \times 4 \begin{pmatrix} 4 \\ 6 \end{pmatrix}$	A1	FT their a only	
	$so \begin{pmatrix} 4 \\ 18 \end{pmatrix} m$	A1	cao. isw for magnitude subsequently found. Vector must be in proper form (penalise only once in paper).	
		7	only ones in papery.	3

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Q 3		Mark	Comment	Sub
(i)	$m \times 9.8 = 58.8$ so $m = 6$	M1 A1	T = mg. Condone sign error. cao. CWO.	2
(ii)	Resolve $\rightarrow 58.8\cos 40 - F = 0$	M1	Resolving their tension. Accept $s \leftrightarrow c$. Condone sign errors but not extra forces.	
	F = 45.043 so 45.0 N (3 s. f.)	B1 A1	(their 7) x cos 40 (or equivalent) seen Accept ± 45 only.	3
(iii)	Resolve \uparrow $R + 58.8 \sin 40 - 15 \times 9.8 = 0$ R = 109.204 so 109 N (3 s. f.)	M1 A1 A1	Resolving their tension. All forces present. No extra forces. Accept $s\leftrightarrow c$. Condone errors in sign. All correct cao	3
		8		0
Q 4		Mark	Comment	Sub
(i)	Resultant is $ \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} $	M1	Adding the vectors. Condone spurious notation. Vector must be in proper form (penalise	
	Magnitude is $\sqrt{(-2)^2 + 3^2 + 6^2} = \sqrt{49} = 7 \text{ N}$	M1 F1	only once in the paper). Accept clear components. Pythagoras on their 3 component vector. Allow e.g. – 2² for (– 2)² even if evaluated as – 4. FT their resultant.	4
(ii)	F + 2G + H = 0	M1	Either F + 2 G + H = 0 or F + 2 G = H	
	So $\mathbf{H} = -2\mathbf{G} - \mathbf{F} = -\begin{pmatrix} -12\\4\\8 \end{pmatrix} - \begin{pmatrix} 4\\1\\2 \end{pmatrix}$	A1	Must see attempt at H = − 2 G − F	
	$= \begin{pmatrix} 8 \\ -5 \\ -10 \end{pmatrix}$	A1	cao. Vector must be in proper form (penalise only once in the paper).	
				3
		7		

Q 5		Mark	Comment	Sub
	a = 12 - 6t $a = 0 gives t = 2$	M1 A1 F1	Differentiation, at least one term correct. Follow their a	
	$x = \int (2+12t-3t^{2}) dx$ $2t + 6t^{2} - t^{3} + C$	M1 A1	Integration indefinite or definite, at least one term correct. Correct. Need not be simplified. Allow as definite integral. Ignore <i>C</i> or limits	
	x = 3 when $t = 0$	M1	Allow $x = \pm 3$ or argue it is \int_{0}^{2} from A then ± 3	
	so $3 = C$ and			
	$x = 2t + 6t^2 - t^3 + 3$	A1	Award if seen WWW or $x = 2t + 6t^2 - t^3$ seen with +3 added later.	
	x(2) = 4 + 24 - 8 + 3 = 23 m	B1	FT their <i>t</i> and their <i>x</i> if obtained by integration but not if -3 obtained instead of +3. [If 20 m seen WWW for displacement award SC6] [Award SC1 for position if constant acceleration used for displacement and then +3 applied]	8
		8		

Q 6		Mark	Comment	Sub
(i)	3.5 = 0.5 + 1.5T	M1	Suitable <i>uvast</i> , condone sign errors.	
	so <i>T</i> = 2 so 2 s	A1	cao	
	35+05			
	$s = \frac{3.5 + 0.5}{2} \times 2$	M1	Suitable <i>uvast</i> , condone sign errors.	
	so s = 4 so 4 m	F1	FT their <i>T</i> . [If <i>s</i> found first then it is cao. In this case when finding <i>T</i> , FT their <i>s</i> , if used.]	4
(ii) (A)	N2L \downarrow : $80 \times 9.8 - T = 80 \times 1.5$	M1	Use of N2L. Allow weight omitted and use of $F = mga$ Condone errors in sign but do not allow extra forces.	7
	T = 664 so 664 N	B1 A1	weight correct (seen in (A) or (B)) cao	
(B)	N2L \downarrow : $80 \times 9.8 - T = 80 \times (-1.5)$	M1	N2L with all forces and using $F = ma$. Condone errors in sign but do not allow extra forces.	
	T = 904 so 904 N	A1	cao [Accept 904 N seen for M1 A1]	5
(iii)	N2L \uparrow : 2500 – 80×9.8 – 116 = 80 <i>a</i>	M1	Use of N2L with $F = ma$. Allow 1 force missing. No extra forces. Condone errors in sign.	
	$a = 20 \text{ so } 20 \text{ m s}^{-2} \text{ upwards.}$	A1 A1 A1	±20, accept direction wrong or omitted upwards made clear (accept diagram)	
		AI	upwarus made ciear (accept diagram)	4
(iv)	N2L \uparrow on equipment: $80-10\times9.8=10a$	M1	Use of N2L on equipment. All forces. $F = ma$.	
	a = -1.8	A1	No extra forces. Allow sign errors. Allow ± 1.8	
	N2L ↑	M1	N2L for system or for man alone. Forces correct (with no extras); accept sign errors; their ±1.8 used	
	either all: $T - (80+10) \times 9.8 - 116 = 90 \times (-1.8)$			
	or on man: $T - (80 \times 9.8) - 116 - 80$ = $80 \times (-1.8)$ T = 836 so 836 N	A1	cao [NB The answer 836 N is independent of the value taken for <i>g</i> and hence may be obtained if all weights are omitted.]	
		17		4
		17		

Q 7		Mark	Comment	Sub
(i)	Horiz $21t = 60$	M1	Use of horizontal components and $a = 0$ or $s = vt - 0.5at^2$ with $v = 0$.	
	so $\frac{20}{7}$ s (2.8571)	A1	Any form acceptable. Allow M1 A1 for answer seen WW.	
			[If $s = ut + 0.5at^2$ and $u = 0$ used without justification award M1 A0] [If $u = 28$ assumed to find time then award SC1]	
	either $0 = u - 9.8 \times \frac{20}{7}$	M1	Use of $v = u + at$ (or $v^2 = u^2 + 2as$) with $v = 0$.	
	or $-u = u - 9.8 \times \left(\frac{40}{7}\right)$		or Use of $v = u + at$ with $v = -u$ and appropriate t .	
	or $40 = u \times \frac{20}{7} - 4.9 \left(\frac{20}{7}\right)^2$		or Use of $s = ut + 0.5at^2$ with $s = 40$ and appropriate t	
	so <i>u</i> = 28 so 28 m s ⁻¹	E1	Condone sign errors and, where appropriate, $u \leftrightarrow v$. Accept signs not clear but not errors. Enough working must be given for 28 to be properly shown. [NB $u = 28$ may be found first and used to find time]	4
(ii)	$y = 28t - 0.5 \times 9.8t^2$	E1	Clear & convincing use of $g = -9.8$ in $s = ut + 0.5at^2$ or $s = vt - 0.5at^2$ NB: AG	1
(iii)	Start from same height with same (zero) vertical speed at same time, same	E1	For two of these reasons	
	acceleration Distance apart is $0.75 \times 21t = 15.75t$	M1	0.75×21 <i>t</i> seen or 21 <i>t</i> and 5.25 <i>t</i> both seen with intention to subtract. Need simplification - LHS alone insufficient.	
		A1	CWO.	3
(iv) (A)	either Time is $\frac{20}{7}$ s by symmetry so $15.75 \times \frac{20}{7} = 45$ so 45 m	B1 B1	Symmetry or <i>uvast</i> FT their (iii) with $t = \frac{20}{7}$	
	or Hit ground at same time. By symmetry one travels 60 m so the other travels 15 m in this	B1		
	time ($\frac{1}{4}$ speed) so 45 m.	B1	[SC1 if 90 m seen]	2
(B)	see next page			

Q7	continued			
(B)			[SC1 if either and or methods mixed to give $\pm 30 = 28t - 4.9t^2$ or $\pm 10 = 4.9t^2$]	
	either Time to fall is $40-10=0.5\times9.8\times t^2$	M1 A1	Considering time from explosion with $u = 0$. Condone sign errors. LHS. Allow ± 30	
	t = 2.47435 need $15.75 \times 2.47435 = 38.971$ so	A1 A1	All correct cao	
	39.0 (3sf) or	F1	FT their (iii) only.	
	Need time so $10 = 28t - 4.9t^2$ $4.9t^2 - 28t + 10 = 0$	M1 M1*	Equating $28t - 4.9t^2 = \pm 10$ Dep. Attempt to solve quadratic by a method that could give two roots.	
	so $t = \frac{28 \pm \sqrt{28^2 - 4 \times 4.9 \times 10}}{9.8}$ so 0.382784 or 5.33150	A1	Larger root correct to at least 2 s. f. Both method marks may be implied from two correct roots alone (to at least 1 s. f.). [SC1 for either root seen WW]	
	Time required is 5.33150 $-\frac{20}{7}$ = 2.47435 need 15.75×2.47435 = 38.971 so 39.0 (3sf)	M1 F1	FT their (iii) only.	5
(v)	Horiz $(x =) 21t$	B1		
(*)	Elim t between $x = 21t$ and $y = 28t - 4.9t^2$	M1	Intention must be clear, with some attempt made.	
	so $y = 28\left(\frac{x}{21}\right) - 4.9\left(\frac{x}{21}\right)^2$	A1	<i>t</i> completely and correctly eliminated from their expression for <i>x</i> and correct <i>y</i> . Only accept wrong notation if subsequently explicitly given correct value e.g. $\frac{x^2}{21}$ seen as $\frac{x^2}{441}$.	
	so $y = \frac{4x}{3} - \frac{0.1x^2}{9} = \frac{1}{90} (120x - x^2)$	E1	Some simplification must be shown.	
			[SC2 for 3 points shown to be on the curve. Award more only if it is made clear that (a) trajectory is a parabola (b) 3 points define a parabola]	
		19		4