MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

MECHANICS 1, M1

Practice Paper M1-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Unless otherwise specified, the value of g should be taken to be exactly 9.8 $\mathrm{m\,s^{-2}}$.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 A particle is projected vertically upwards from ground level and it returns to the ground 10 seconds later. Air resistance is negligible.

Find

(i) the speed of projection, [3]

(ii) the maximum height above the ground reached by the particle. [2]

2 A block of mass 2 kg slides on a smooth horizontal table. It is attached to a hanging object of mass 0.5 kg by a light inelastic string that passes over a small smooth pulley. The part of the string between the block and the pulley is horizontal. The string is taut with the block held on the table. The block is released.

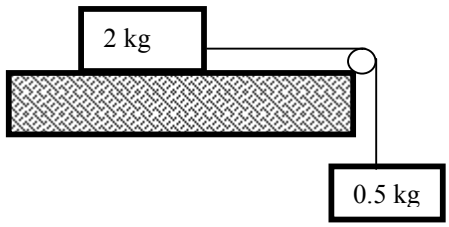

Fig. 2

Find

(i) the acceleration of the block, [3]

(ii) the tension in the string. [2]

3 A block of mass 2 kg is in equilibrium on a slope of 40^{o}.

(i) Draw a diagram showing all the forces acting on the block. [1]

(ii) Show that the resistance on the block is 12.60 N, correct to 2 decimal places. [2]

(iii) Find, correct to 2 decimal places, the normal contact force exerted on the block by the plane. [2]

4 A particle is in equilibrium under the action of three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$.
$\mathbf{F_1} = 10\mathbf{i} + 3\mathbf{j}$ and $\mathbf{F_2} = 5\mathbf{i} - 12\mathbf{j}$.

(i) Find $\mathbf{F_3}$. [2]

(ii) Calculate the magnitude of $\mathbf{F_1}$ and the angle it makes with the $\mathbf{i}$ direction. [4]

5 A boat moves through still water. Its initial velocity is $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ m s^{-1}. It has uniform acceleration $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ m s^{-2}, where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents the unit vector due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents the unit vector due north. The boat starts from the point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Find

(i) the velocity and speed of the boat after 2 seconds, [4]

(ii) the displacement of the boat after 4 seconds. [3]

6 A stone is thrown from the top of a cliff, 50 m above sea level. The initial velocity of the stone is 15 ms^{-1} at an angle of θ to the horizontal, where $\cos\theta = 0.8$ and $\sin\theta = 0.6$. Air resistance is negligible.

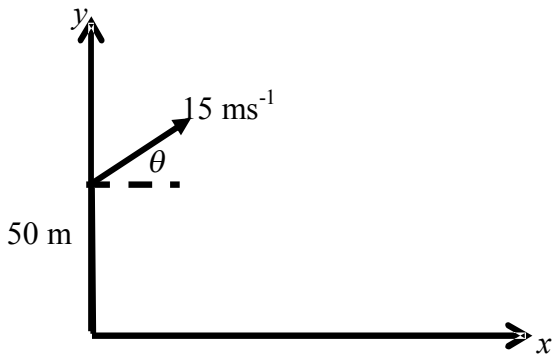

Fig. 6

(i) Show that after t seconds the vertical height, y m, of the stone above sea level is given by the equation

$$y = 9t - 4.9t^2 + 50.$$ [3]

(ii) Find the horizontal distance, x m, travelled after t seconds. [2]

(iii) Hence show that, referred to the axes shown in Fig. 6, the trajectory of the stone is given by the equation

$$y = \frac{3}{4}x - \frac{49}{1440}x^2 + 50.$$ [3]

Section B (36 marks)

7 A car is towing a trailer. They are connected by a light horizontal towbar. The mass of the car and trailer are 1000 kg and 200 kg respectively. The resistance to motion of the car is 800 N and of the trailer is 150 N. The driving force exerted by the car is D N.

(a) The car travels along a horizontal road.

(i) Draw a force diagram showing all the forces acting in a horizontal direction on the car and another diagram showing all the forces acting in a horizontal direction on the trailer. [2]

(ii) The car is moving at a constant speed. Write down the value of D and find the value of the pulling force of the towbar on the trailer. [2]

(iii) At a later time the car accelerates uniformly from 10 m s^{-1} to 20 m s^{-1} over 15 seconds. What is the driving force now? [4]

(b) The car is then driven **up** a slope of angle θ where $\sin\theta = 0.1$. The resistances to motion remain as above. The car slows with constant deceleration of 0.5 m s^{-2}.

(i) Draw a force diagram, showing all the forces acting on the car and another diagram showing all the forces acting on the trailer. [3]

(ii) Find the value of D and the tension in the towbar. [7]

8 John can run at 8 m s^{-1} and he is entered for a 100 metre race.

He suggests a model in which he accelerates uniformly to the maximum speed of 8 ms^{-1} in 4 seconds and then runs at maximum speed for the rest of the distance. Fig. 8 is the speed-time graph for this model.

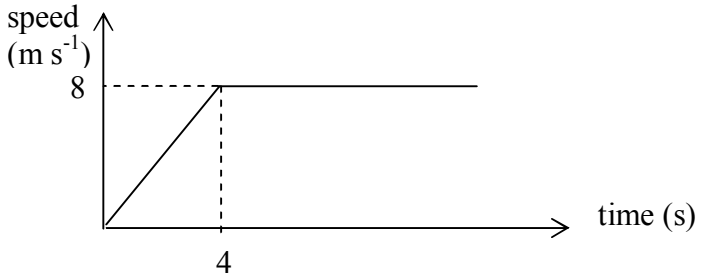

Fig. 8

(i) Find the distance run in the first 4 seconds and hence the time taken to run the race. [5]

John's teacher suggests a model such that for the first four seconds his speed, v m s^{-1} at time t seconds, is given by the formula

$$v = \frac{1}{4}\left(6t^2 - t^3\right).$$

(ii) **(*A*)** Find v when $t = 4$.

(*B*) Find an expression for the acceleration as a function of t.
For what values of t is the acceleration zero? [4]

(*C*) Show that with this improved model the time taken to complete the race is the same as that for John's model in part **(i)** . [6]

(iii) Compare the two models. [2]