

June  
2016 M1 UK Solutions Kprime2

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1. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin  $O$ .]

Two cars  $P$  and  $Q$  are moving on straight horizontal roads with constant velocities. The velocity of  $P$  is  $(15\mathbf{i} + 20\mathbf{j}) \text{ m s}^{-1}$  and the velocity of  $Q$  is  $(20\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-1}$

- (a) Find the direction of motion of  $Q$ , giving your answer as a bearing to the nearest degree.

(3)

At time  $t = 0$ , the position vector of  $P$  is  $400\mathbf{i}$  metres and the position vector of  $Q$  is  $800\mathbf{j}$  metres. At time  $t$  seconds, the position vectors of  $P$  and  $Q$  are  $\mathbf{p}$  metres and  $\mathbf{q}$  metres respectively.

- (b) Find an expression for

(i)  $\mathbf{p}$  in terms of  $t$ ,

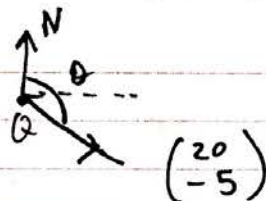
(ii)  $\mathbf{q}$  in terms of  $t$ .

(3)

- (c) Find the position vector of  $Q$  when  $Q$  is due west of  $P$ .

(4)

1.(a)



$$\theta = 90^\circ + \arctan\left(\frac{5}{20}\right) = 104.03\dots$$

$$\therefore \theta = 104^\circ \text{ (nearest degree)}$$

(b) Consider  $P$ :

$$\underline{r}_0 = 400\mathbf{i} \quad \underline{v}_P = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

$$\text{@ time } t, \quad \underline{r} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} + t \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

$$\Rightarrow \underline{p} = \begin{pmatrix} 400 + 15t \\ 20t \end{pmatrix}$$



Question 1 continued

$$(i) \underline{p} = (400 + 15t) \underline{i} + 20t \underline{j}$$

(ii) Consider  $\theta$ :

$$\underline{r}_0 = \begin{pmatrix} 0 \\ 800 \end{pmatrix} \quad \underline{v}_a = \begin{pmatrix} 20 \\ -5 \end{pmatrix}$$

$$\Rightarrow \underline{r} = \begin{pmatrix} 0 \\ 800 \end{pmatrix} + t \begin{pmatrix} 20 \\ -5 \end{pmatrix}$$

$$\therefore \underline{q} = \begin{pmatrix} 20t \\ 800 - 5t \end{pmatrix} \Rightarrow \underline{q} = 20t \underline{i} + (800 - 5t) \underline{j}$$

$$(c) \begin{pmatrix} 20t \\ 800 - 5t \end{pmatrix} \cdot \begin{pmatrix} 400 + 15t \\ 20t \end{pmatrix}$$

$$\Rightarrow 800 - 5t = 20t$$

$$\therefore 25t = 800$$

$$\therefore t = 32$$

$$t = 32 \Rightarrow \underline{q} = 640 \underline{i} + 640 \underline{j}$$

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2.

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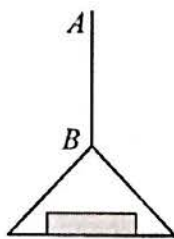
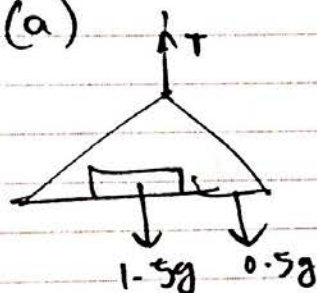


Figure 1

A vertical rope  $AB$  has its end  $B$  attached to the top of a scale pan. The scale pan has mass  $0.5 \text{ kg}$  and carries a brick of mass  $1.5 \text{ kg}$ , as shown in Figure 1. The scale pan is raised vertically upwards with constant acceleration  $0.5 \text{ m s}^{-2}$  using the rope  $AB$ . The rope is modelled as a light inextensible string.

- (a) Find the tension in the rope  $AB$ . (3)
- (b) Find the magnitude of the force exerted on the scale pan by the brick. (3)

2(a)



$$\uparrow 0.5 \text{ m s}^{-2}$$

∴



$$\uparrow 0.5 \text{ m s}^{-2}$$

$$\uparrow: F = ma$$

$$\Rightarrow T - 2g = 2 \times 0.5$$

$$\Rightarrow T = 1 + 2g \text{ N}$$

$$T = 20.6 \text{ N} \quad (2 \text{ sf})$$

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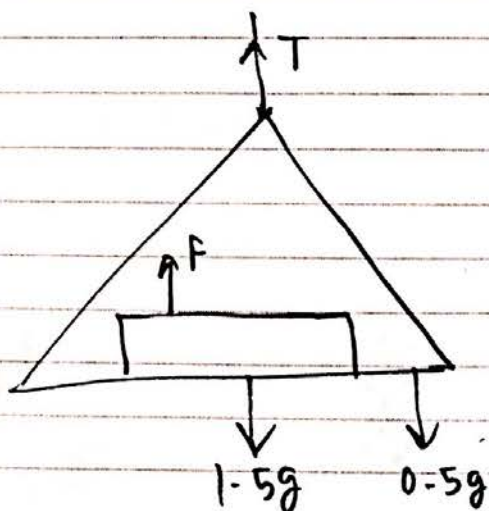
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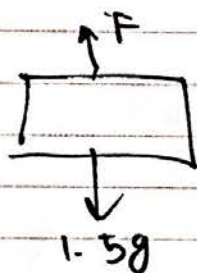
## Question 2 continued

(b)



$F$  is the contact force.

Consider just the brick:



$$a = 0.5 \text{ m s}^{-2}$$

$$\uparrow: F - 1.5g = 1.5 \times 0.5$$

$$\therefore F = \frac{3}{4} + 1.5g$$

$$\Rightarrow F = 15.5 \text{ N (3sf)}$$

(Total 6 marks)

Q2



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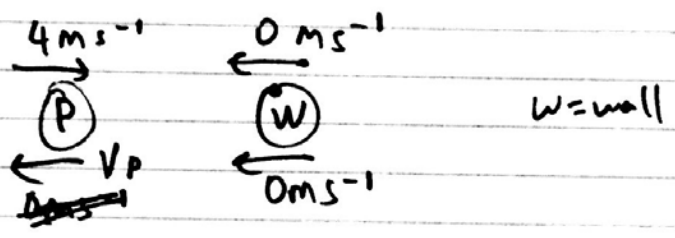
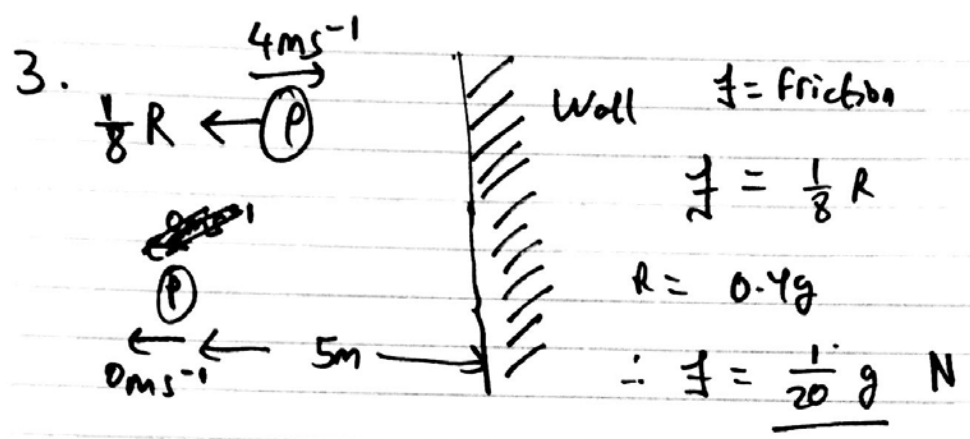
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3. A particle  $P$  of mass  $0.4 \text{ kg}$  is moving on rough horizontal ground when it hits a fixed vertical plane wall. Immediately before hitting the wall,  $P$  is moving with speed  $4 \text{ m s}^{-1}$  in a direction perpendicular to the wall. The particle rebounds from the wall and comes to rest at a distance of  $5 \text{ m}$  from the wall. The coefficient of friction between  $P$  and the ground is  $\frac{1}{8}$ .

Find the magnitude of the impulse exerted on  $P$  by the wall.

(7)



We need  $v_p$  to calculate Impulse.

$s = 5$   
 $u = v_p$   
 $v = 0$   
 $a = -\frac{g}{8}$   
 $t =$

$$+ \leftarrow F = ma$$

$$\Rightarrow -f = 0.4a$$

$$\Rightarrow -\frac{g}{20} = 0.4a$$



Question 3 continued

$$\therefore a = -\frac{1}{8}g \text{ ms}^{-2}$$

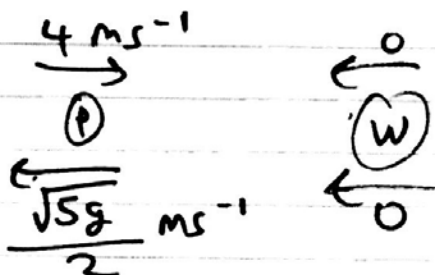
$$\therefore v^2 = u^2 + 2as$$

$$\Rightarrow 0 = (v_p)^2 - 2\left(\frac{g}{8}\right)(5)$$

$$\therefore 0 = (v_p)^2 - \frac{5}{4}g$$

$$(v_p)^2 = \frac{5}{4}g$$

$$\therefore v_p = \frac{\sqrt{5g}}{2} \text{ ms}^{-1}$$



$$+ \leftarrow I = m(v - u)$$

$$\therefore I = 0.4 \left( \frac{\sqrt{5g}}{2} + 4 \right)$$

$$\Rightarrow \underline{\underline{I = 3 \text{ N s}}}$$

(Total 7 marks)

Q3



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4. Two trains  $M$  and  $N$  are moving in the same direction along parallel straight horizontal tracks. At time  $t = 0$ ,  $M$  overtakes  $N$  whilst they are travelling with speeds  $40 \text{ m s}^{-1}$  and  $30 \text{ m s}^{-1}$  respectively. Train  $M$  overtakes train  $N$  as they pass a point  $X$  at the side of the tracks.

After overtaking  $N$ , train  $M$  maintains its speed of  $40 \text{ m s}^{-1}$  for  $T$  seconds and then decelerates uniformly, coming to rest next to a point  $Y$  at the side of the tracks.

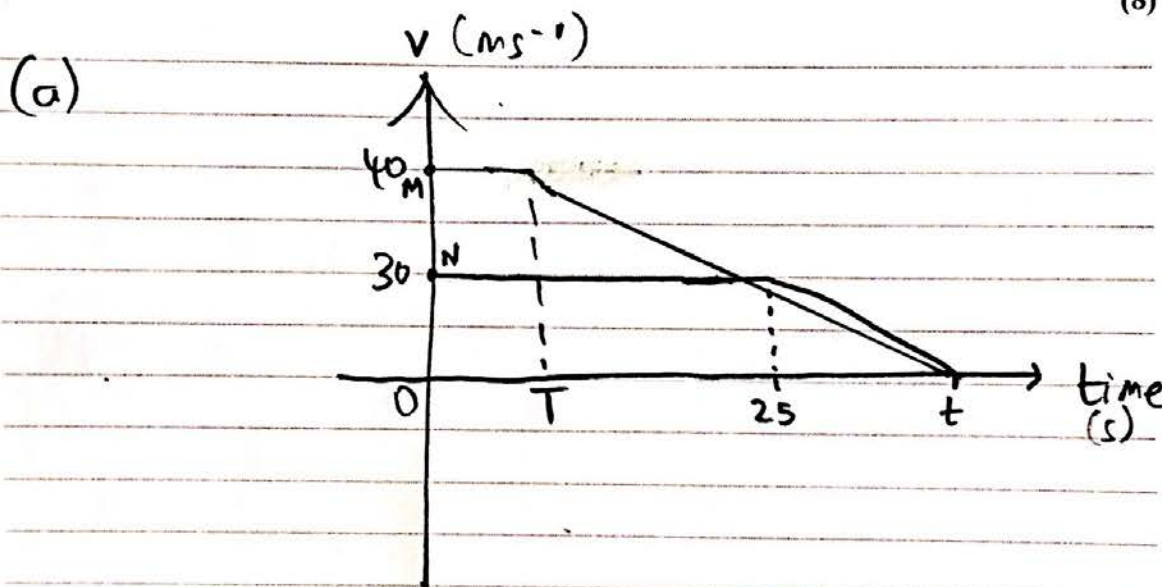
After being overtaken, train  $N$  maintains its speed of  $30 \text{ m s}^{-1}$  for  $25 \text{ s}$  and then decelerates uniformly, also coming to rest next to the point  $Y$ .

The times taken by the trains to travel between  $X$  and  $Y$  are the same.

(a) Sketch, on the same diagram, the speed-time graphs for the motions of the two trains between  $X$  and  $Y$ . (4)

Given that  $XY = 975 \text{ m}$ ,

(b) find the value of  $T$ . (8)



(b)  $XY = 975 \text{ m}$       let  $t = \Sigma \text{time}$

Consider Area train N:

$$\frac{30}{2} (t + 25) = 15(t + 25)$$

$$\therefore 15(t + 25) = 975 \Rightarrow t = 40$$

~~$t = 40$~~



Question 4 continued

Consider Area train M:

$$\frac{40}{2} (T + 40) = 20(T + 40)$$

$$\therefore 20(T + 40) = 975$$

$$\Rightarrow T = \underline{\underline{8.75 \text{ seconds}}}$$



5.

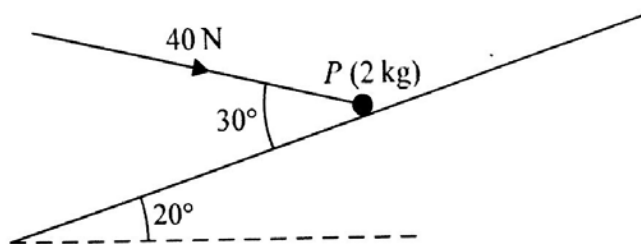
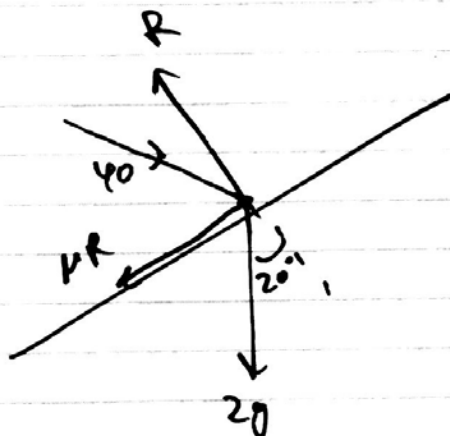


Figure 2

A particle  $P$  of mass  $2\text{ kg}$  is held at rest in equilibrium on a rough plane by a constant force of magnitude  $40\text{ N}$ . The direction of the force is inclined to the plane at an angle of  $30^\circ$ . The plane is inclined to the horizontal at an angle of  $20^\circ$ , as shown in Figure 2. The line of action of the force lies in the vertical plane containing  $P$  and a line of greatest slope of the plane. The coefficient of friction between  $P$  and the plane is  $\mu$ .

Given that  $P$  is on the point of sliding up the plane, find the value of  $\mu$ .

(10)



$$\rightarrow: 40 \cos 30 = \mu R + 2g \sin 20$$

$$\uparrow: R = 2g \cos 20 + 40 \sin 30 = 38.417\dots$$

$$\mu = \frac{40 \cos 30 - 2g \sin 20}{38.417\dots} = 0.7271\dots$$

$$\Rightarrow \mu = \underline{\underline{0.727}} \text{ (3sf)}$$



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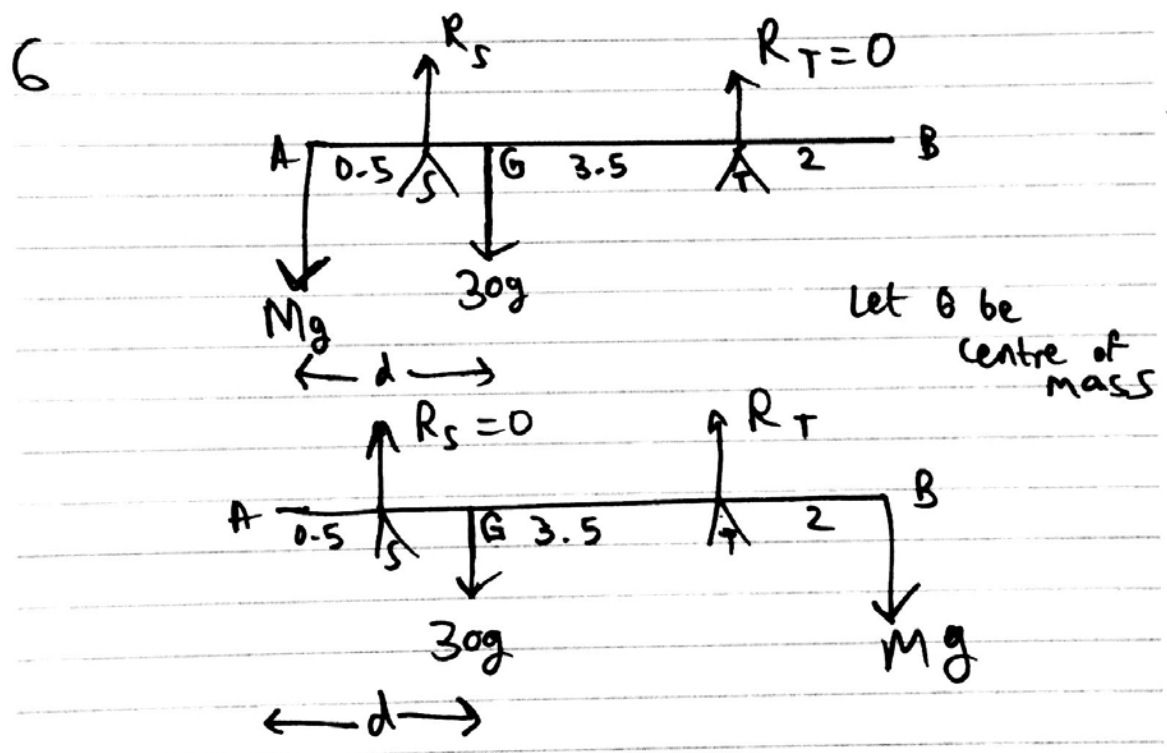
6. A non-uniform plank  $AB$  has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points  $S$  and  $T$  of the plank where  $AS = 0.5$  m and  $TB = 2$  m.

When a block of mass  $M$  kg is placed on the plank at  $A$ , the plank remains horizontal and in equilibrium and the plank is on the point of tilting about  $S$ .

When the block is moved to  $B$ , the plank remains horizontal and in equilibrium and the plank is on the point of tilting about  $T$ .

The distance of the centre of mass of the plank from  $A$  is  $d$  metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

- (i) the value of  $d$ ,
  - (ii) the value of  $M$ .
- (7)



Consider when block is placed at A:

$$M(A): 30g(d) = 0.5 R_S \quad (1)$$



## Question 6 continued

Consider when block is placed at B:

$$M(B): 30g(6-d) = 2R_T$$

$$\Rightarrow 180g - 30gd = 2R_T \quad (2)$$

Sub m ① in ②:

$$180g - \frac{1}{2}R_S = 2R_T$$

$$\text{Also: } (M+30)g = R_S = R_T$$

$$\therefore 180g - \frac{1}{2}(M+30)g = 2(M+30)g$$

$$\therefore 180g - \frac{g}{2}M - 15g = 2gM + 60g$$

$$\therefore \cancel{120g} = 105g = \left(2g + \frac{g}{2}\right)M$$

$$\therefore 105 = \left(2 + \frac{1}{2}\right)M = \frac{5}{2}M$$

$$\Rightarrow M = 42 \text{ kg}$$

$$M=42 \Rightarrow R_S = 72g$$



## Question 6 continued

Sub in ①

$$30g(d) = 0.5 \times 72g$$

$$30g d = 36g$$

$$30d = 36 \Rightarrow d = 1.2$$

$$\therefore (i) \quad d = \underline{\underline{1.2 \text{ m}}}$$

$$(ii) \quad M = \underline{\underline{42 \text{ kg}}}$$

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7. Two forces  $F_1$  and  $F_2$  act on a particle  $P$ .

The force  $F_1$  is given by  $F_1 = (-i + 2j)$  N and  $F_2$  acts in the direction of the vector  $(i + j)$ .

Given that the resultant of  $F_1$  and  $F_2$  acts in the direction of the vector  $(i + 3j)$ ,

(a) find  $F_2$  (7)

The acceleration of  $P$  is  $(3i + 9j) \text{ m s}^{-2}$ . At time  $t = 0$ , the velocity of  $P$  is  $(3i - 22j) \text{ m s}^{-1}$

(b) Find the speed of  $P$  when  $t = 3$  seconds. (4)

$$7. (a) \quad \underline{F_1} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \underline{F_2} = k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{Let } \underline{R} = \text{resultant}, \quad \underline{R} = \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\underline{R} = \underline{F_1} + \underline{F_2} \Rightarrow \underline{R} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \underline{F_2} = \underline{R} - \underline{F_1}$$

$$\therefore \underline{R} = \begin{pmatrix} k-1 \\ 2+k \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} k-1 &= \lambda & \Rightarrow k &= \lambda+1 \\ \& \quad k+2 &= 3\lambda \end{aligned}$$

$$\Rightarrow \lambda+1+2=3\lambda$$

$$\therefore 2\lambda=3 \Rightarrow \lambda = \frac{3}{2}$$

$$\therefore k = \frac{3}{2} + 1 = \frac{5}{2}$$



Question 7 continued

$$\therefore \underline{\underline{F_2}} = \frac{5}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \underline{\underline{F_2}} = \frac{5}{2} \underline{\underline{i}} + \frac{5}{2} \underline{\underline{j}}$$

$$(b) \quad \underline{\underline{a}} = \begin{pmatrix} 3 \\ 9 \end{pmatrix} \quad \underline{\underline{u}} = \begin{pmatrix} 3 \\ -22 \end{pmatrix} \quad t = 3$$

$$\underline{\underline{v}} = \underline{\underline{u}} + t\underline{\underline{a}}$$

$$\therefore \underline{\underline{v}} = \begin{pmatrix} 3 \\ -22 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$\therefore \underline{\underline{v}} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$\therefore |\underline{\underline{v}}| = \sqrt{12^2 + 5^2} = 13 \text{ m s}^{-1}$$

$$\underline{\underline{\text{Speed}}} = 13 \text{ m s}^{-1}$$



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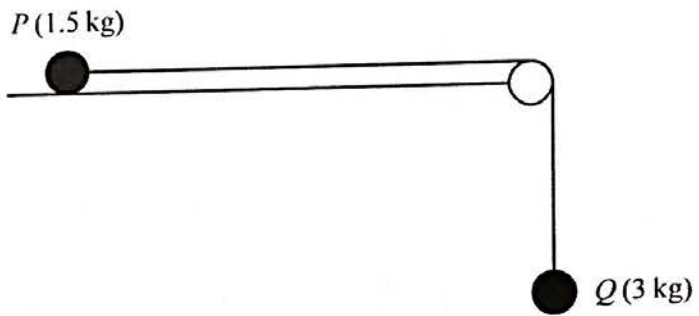


Figure 3

Two particles  $P$  and  $Q$  have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle  $P$  is held at rest on a fixed rough horizontal table. The coefficient of friction between  $P$  and the table is  $\frac{1}{5}$ . The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle  $Q$  hangs freely at rest vertically below the pulley, as shown in Figure 3. Particle  $P$  is released from rest with the string taut and slides along the table.

Assuming that  $P$  has not reached the pulley, find

- (a) the tension in the string during the motion, (8)
- (b) the magnitude and direction of the resultant force exerted on the pulley by the string. (4)

8 (a) Consider P:

$\rightarrow \quad R = 1.5g \quad \quad \quad \rightarrow \quad F = ma$

$\frac{1}{5}R = \frac{3}{10}g \quad \leftarrow \quad \text{P} \quad \rightarrow \quad T$

$\downarrow 1.5g$

$\therefore T - \frac{3}{10}g = 1.5a$

Consider Q:

$\uparrow T \quad \quad \quad \downarrow F = ma$

$\text{Q} \quad \downarrow$

$\downarrow 3g$

$3g - T = 3a$

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Question 8 continued

$$T - \frac{3}{10}g = \frac{3}{2}a \Rightarrow 2T - \frac{3}{5}g = 3a$$

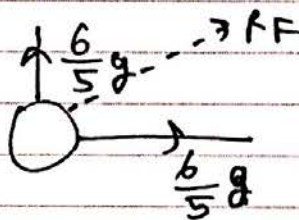
$$\& 3g - T = 3a$$

$$\therefore 3g - T = 2T - \frac{3}{5}g$$

$$\therefore 3T = 3g + \frac{3}{5}g = \frac{18}{5}g$$

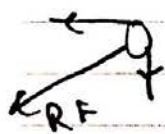
$$\Rightarrow T = \frac{6}{5}g \text{ N}$$

(b)



$$RF = \sqrt{\left(\frac{6}{5}g\right)^2 + \left(\frac{6}{5}g\right)^2} = 16.63\dots$$

$$\therefore RF = \underline{\underline{16.6 \text{ N} (3 \text{ sf})}}$$



Direction: 45° below pulley (south west)

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END