M1 MAY 2013 INTERNATIONAL

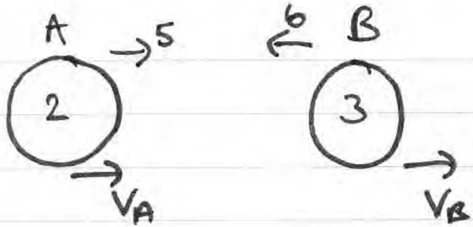
1. Two particles A and B, of mass 2 kg and 3 kg respectively, are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately before the collision the speed of A is 5 m s^{-1} and the speed of B is 6 m s^{-1}. The magnitude of the impulse exerted on B by A is 14 N s. Find

(a) the speed of A immediately after the collision,

(3)

(b) the speed of B immediately after the collision.

(3)



Mom A before $= 10$ $\Rightarrow$ Impulse $= 14$ $\therefore 2V_A = -4$

Mom A after $= 2V_A$ $\underline{V_A = -2}$

b) CLM $5\times2 + 3\times-6 = 2\times-2 + 3V_B$

$\Rightarrow -8 = -4 + 3V_B \Rightarrow 3V_B = -4 \therefore V_B = -\frac{4}{3}$

2.

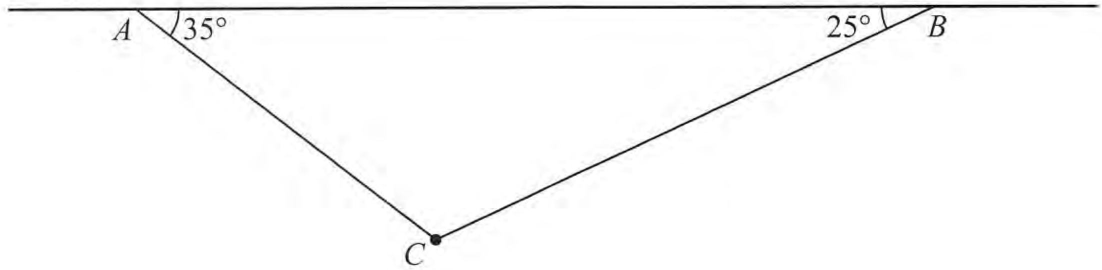


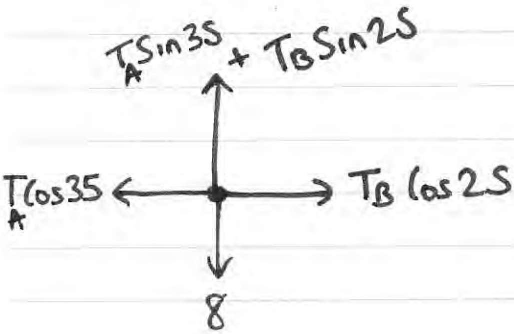
Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)



$\vec{Rf} = 0 \therefore T_A \cos 35 = T_B \cos 25$

$\Rightarrow T_B = \dfrac{T_A \cos 35}{\cos 25}$

$R\uparrow = 0 \Rightarrow T_A \sin 35 + T_B \sin 25 = 8$

$\Rightarrow T_A \sin 35 + \dfrac{T_A \cos 35 \sin 25}{\cos 25} = 8$

$\Rightarrow 0.955533 T_A = 8 \Rightarrow T_A = 8.37\text{N}$

$T_B = 7.57\text{N}$

3.

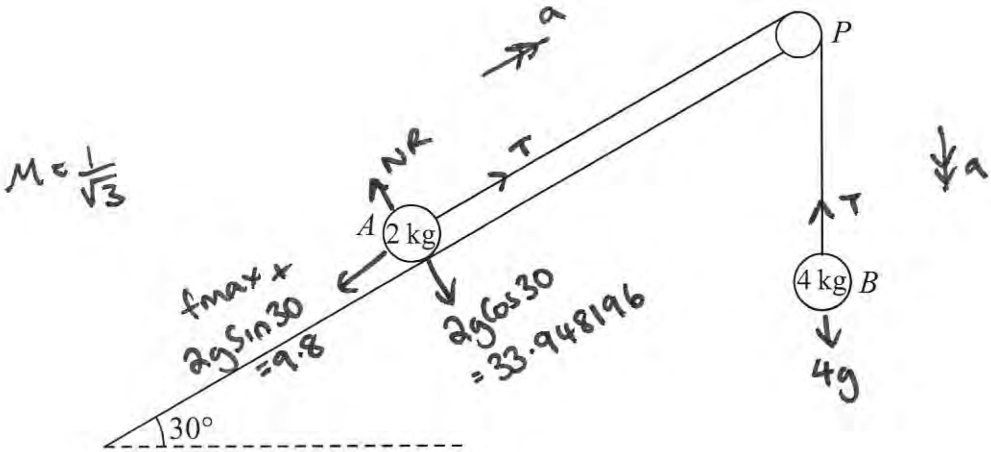


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

$NR = 33.948196 \Rightarrow f_{max} = \mu NR = \frac{1}{\sqrt{3}}(2g)\cos 30 = g$

A $\Rightarrow$ total force down the plane $= T - F_{max} - 2g \sin 30 = 2g$

B $4g - T = 4a$

A $T - 2g = 2a$

A & B $4g - T = 4a \Rightarrow 4g - T = 2T - 4g$

$\therefore 3T = 8g$

$\therefore T = \frac{8}{3}g$

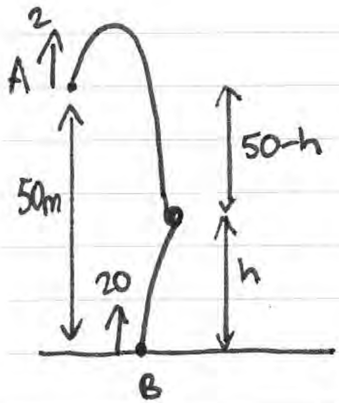
4. At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1}. At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T,

(5)

(b) the value of h.

(2)



(A) $s = -(50-h)$, $u = 2$, v, $a = -9.8$, $t = T$

(B) $s = h$, $u = 20$, v, $a = -9.8$, $t = T$

$s = ut + \frac{1}{2}at^2 \Rightarrow h - 50 = 2T - 4.9T^2$ (A)

$h = 20T - 4.9T^2$ (B) $4.9T^2 = 20T - h$

$\Rightarrow \cancel{h} - 50 = 2T + \cancel{h} - 20T \Rightarrow -50 = -18T \therefore T = \frac{25}{9}$ sec

b) $h = 20\left(\frac{25}{9}\right) - 4.9\left(\frac{25}{9}\right)^2 = \underline{17.7\text{m}}$

5.

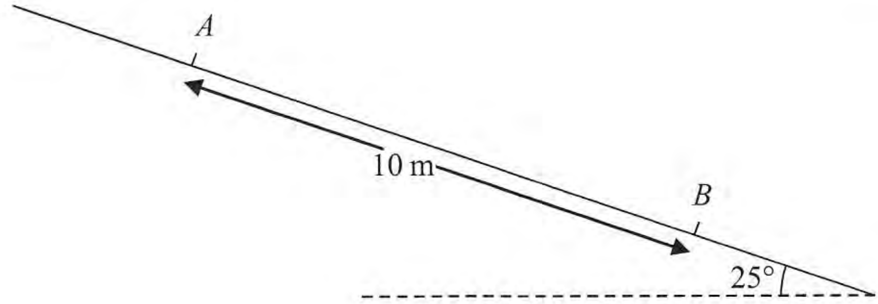


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where AB = 10 m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

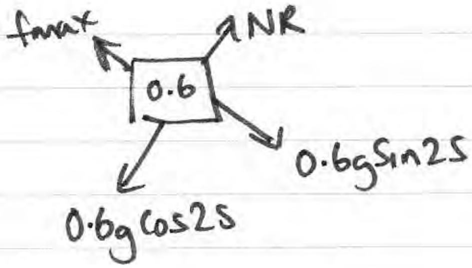
(3)

(b) the acceleration of P,

(2)

(c) the coefficient of friction between P and the plane.

(5)



$NR = 5.329089788$

$\therefore f_{max} = 5.3290898\mu$

$Rf \downarrow = ma$

$\Rightarrow 2.484995 - 5.3290898\mu = 0.6a$

$s = 10$
$u = 2$
v
a
$t = 3.5$

$s = ut + \frac{1}{2}at^2 \Rightarrow 10 = 7 + \frac{1}{2}a(3.5)^2$

b) $\therefore a = \frac{24}{49}$

a) $v = u + at \quad v = 2 + \left(\frac{24}{49}\right)\left(\frac{7}{2}\right) = \frac{26}{7}$

c) $2.484995 - 0.6\left(\frac{24}{49}\right) = 5.3290898\mu$

$\therefore \mu = 0.41$ (2sf)

PMT

6. *[In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.]*

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours. **(2)**

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n. **(5)**

(c) Find the distance OP. **(4)**

a) $v = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\quad S = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

b) $T = \begin{pmatrix} 6 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ n \end{pmatrix}$ $\quad 6 - 2t = -4 + 3t$

$10 = 5t \quad \underline{t = 2}$

$2 + 3t = 1 + tn \Rightarrow 2 + 6 = 1 + 2n \Rightarrow 2n = 7 \Rightarrow n = 3.5$

c) $P = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$ $\quad \vec{OP} = \sqrt{2^2 + 8^2}$

$= \underline{8.25 \text{ km } (3sf)}$

7.

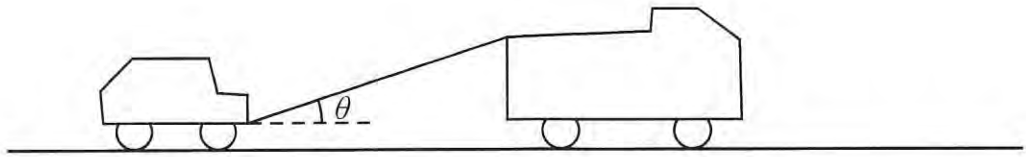


Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1}. The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

(a) Find the deceleration of the truck and the car. **(3)**

The constant braking force on the truck has magnitude R newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos\theta = 0.9$, find

(b) the force in the towbar, **(4)**

(c) the value of R. **(4)**

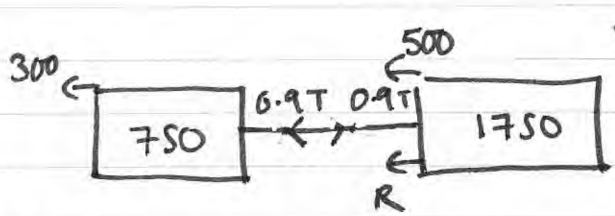
$s = 100$ $\quad v^2 = u^2 + 2as \quad 196 = 400 + 200a$

$u = 20$

$v = 14$ $\quad \Rightarrow 200a = -204 \Rightarrow a = -1.02$

a

t $\quad \therefore$ deceleration $= 1.02$



Note - when breaking tension is THRUST

c) whole system $\quad R + 500 + 300 + 0.9T - 0.9T = 2500 \times 1.02$

$\Rightarrow R + 800 = 2550 \quad \therefore R = 1750\text{N}$

b) Car $\quad 300 + 0.9T = \cancel{7}50(1.02) \Rightarrow$ Thrust $= 517\text{N}$

~~300~~ ~~7~~ (3sf)

8.

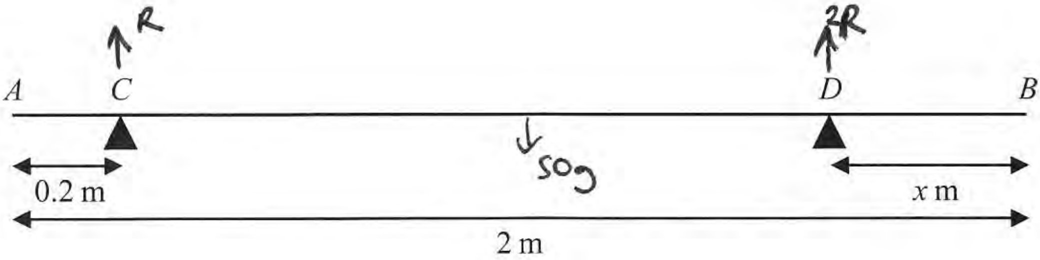


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C,

(a) find the value of x.

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B, and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C,

(b) find the value of m.

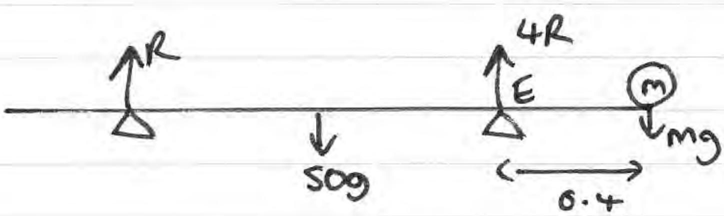
(7)

a) $\uparrow = \downarrow \Rightarrow 3R = 50g \quad R = \frac{50}{3}g \quad 2R = \frac{100}{3}g.$

$B \curvearrowleft \quad \frac{100}{3}g \times x + \frac{50}{3}g \times 1.8 = 50g \times 1$

$\Rightarrow \frac{100}{3}\cancel{g}x = 20\cancel{g} \quad \therefore x = 0.6\text{m}$

b)



$B \curvearrowleft \quad 4R \times 0.4 + R \times 1.8 = 50g \times 1 \quad \Rightarrow 3.4R = 50g$

$\therefore R = \frac{250}{17}g$

$\uparrow = \downarrow \Rightarrow 5R = 50g + mg \quad \Rightarrow \frac{1250}{17}\cancel{g} = \frac{850}{17}\cancel{g} + m\cancel{g}$

$\therefore m = \frac{400}{17} \quad 23.5\text{kg}$ (3sf)