

M1 MAY 2012

PMT

1. Two particles  $A$  and  $B$ , of mass  $5m$  kg and  $2m$  kg respectively, are moving in opposite directions along the same straight horizontal line. The particles collide directly. Immediately before the collision, the speeds of  $A$  and  $B$  are  $3 \text{ m s}^{-1}$  and  $4 \text{ m s}^{-1}$  respectively. The direction of motion of  $A$  is unchanged by the collision. Immediately after the collision, the speed of  $A$  is  $0.8 \text{ m s}^{-1}$ .

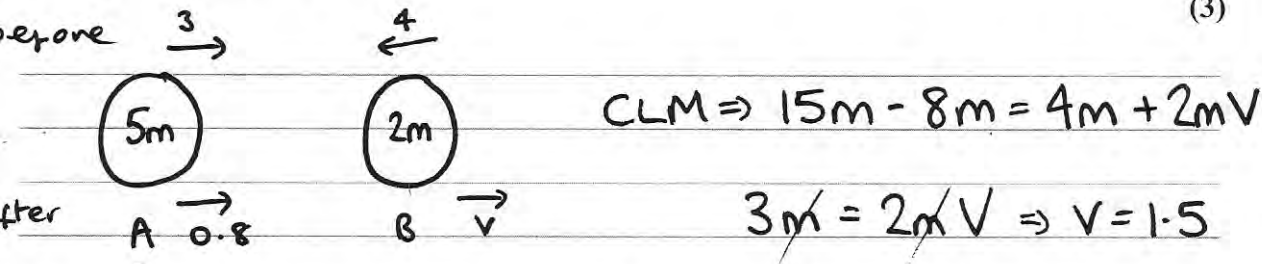
(a) Find the speed of  $B$  immediately after the collision.

(3)

In the collision, the magnitude of the impulse exerted on  $A$  by  $B$  is  $3.3 \text{ N s}$ .

(b) Find the value of  $m$ .

(3)



b) Mom A before =  $15m$   $\therefore$  Impulse =  $11m$   
 Mom A after =  $4m$

$11m = 3.3 \Rightarrow m = 0.3$

2.

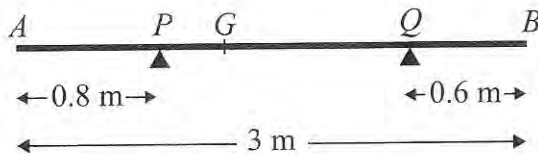
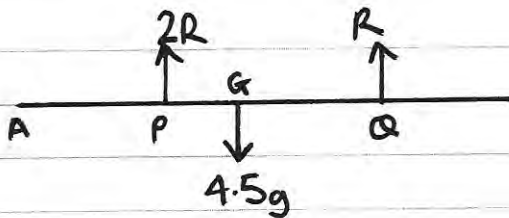


Figure 1

A non-uniform rod  $AB$  has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at  $P$  and at  $Q$ , where  $AP = 0.8$  m and  $QB = 0.6$  m, as shown in Figure 1. The centre of mass of the rod is at  $G$ . Given that the magnitude of the reaction of the support at  $P$  on the rod is twice the magnitude of the reaction of the support at  $Q$  on the rod, find

(a) the magnitude of the reaction of the support at  $Q$  on the rod, (3)

(b) the distance  $AG$ . (4)



$$R \uparrow = \downarrow \quad 3R = 4.5g$$

$$\therefore R = \underline{1.5g \text{ N}}$$

$$\text{b) } A \curvearrowright \quad 4.5g \times AG = 3g \times 0.8 + 1.5g \times 2.4$$

$$4.5g \times AG = 6g \quad \Rightarrow \quad AG = \frac{4}{3}$$

3.

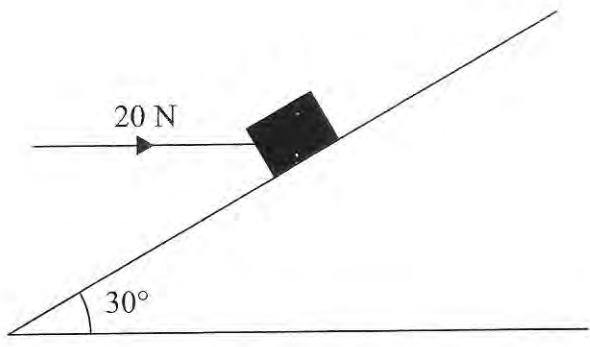
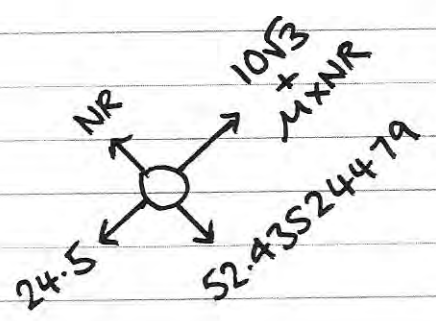
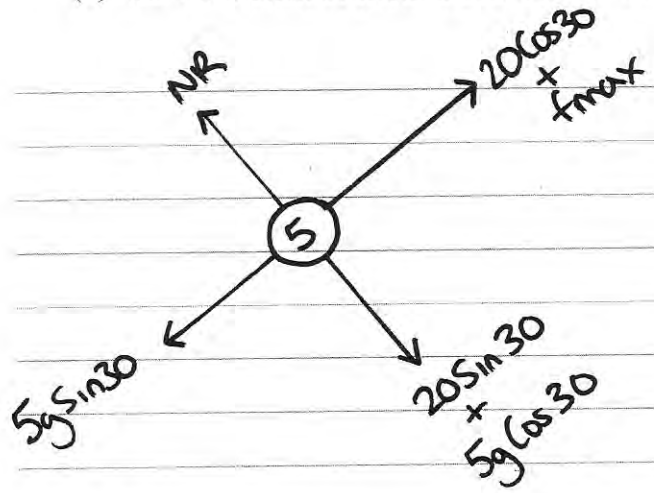


Figure 2

A box of mass 5 kg lies on a rough plane inclined at  $30^\circ$  to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

- (a) the magnitude of the normal reaction of the plane on the box, (4)
- (b) the coefficient of friction between the box and the plane. (5)



$$R_{f\uparrow} = 0 \Rightarrow NR = 52.4 \text{ (3sf)}$$

$$R_{f\nearrow} = 0 \quad 10\sqrt{3} + \mu(52.43\dots) = 24.5$$

$$\mu = 0.137 \text{ (3sf)}$$

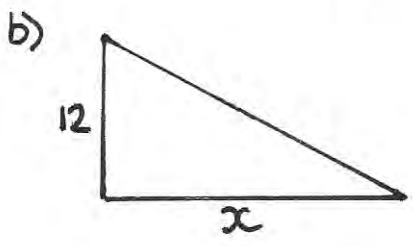
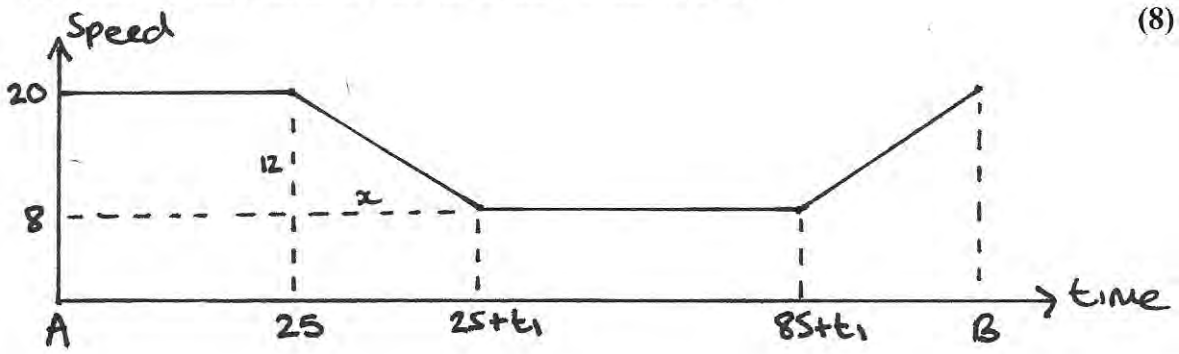
4. A car is moving on a straight horizontal road. At time  $t = 0$ , the car is moving with speed  $20 \text{ m s}^{-1}$  and is at the point  $A$ . The car maintains the speed of  $20 \text{ m s}^{-1}$  for  $25 \text{ s}$ . The car then moves with constant deceleration  $0.4 \text{ m s}^{-2}$ , reducing its speed from  $20 \text{ m s}^{-1}$  to  $8 \text{ m s}^{-1}$ . The car then moves with constant speed  $8 \text{ m s}^{-1}$  for  $60 \text{ s}$ . The car then moves with constant acceleration until it is moving with speed  $20 \text{ m s}^{-1}$  at the point  $B$ .

(a) Sketch a speed-time graph to represent the motion of the car from  $A$  to  $B$ . (3)

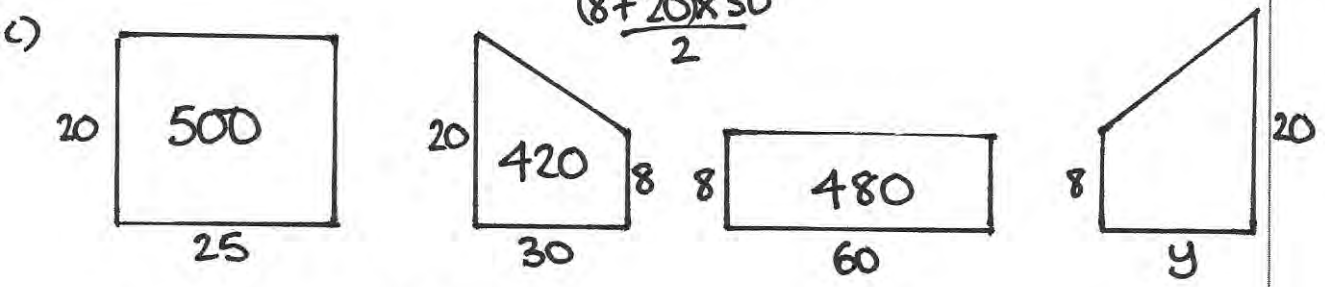
(b) Find the time for which the car is decelerating. (2)

Given that the distance from  $A$  to  $B$  is  $1960 \text{ m}$ ,

(c) find the time taken for the car to move from  $A$  to  $B$ . (8)



gradient =  $-0.4 = -\frac{12}{x} \Rightarrow x = \frac{12}{0.4}$   
 $x = 30 \text{ sec}$



$500 + 420 + 480 = 1400$

$1960 - 1400 = 560$

$\therefore \frac{(8+20) \times y}{2} = 560$

$28y = 1120$   
 $y = 40$

$\therefore \text{total time} = 25 + 30 + 60 + 40$   
 $= \underline{155 \text{ sec}}$

5. A particle  $P$  is projected vertically upwards from a point  $A$  with speed  $u \text{ m s}^{-1}$ . The point  $A$  is  $17.5 \text{ m}$  above horizontal ground. The particle  $P$  moves freely under gravity until it reaches the ground with speed  $28 \text{ m s}^{-1}$ .

(a) Show that  $u = 21$  (3)

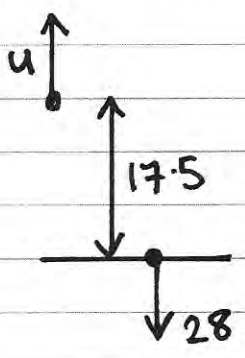
At time  $t$  seconds after projection,  $P$  is  $19 \text{ m}$  above  $A$ .

(b) Find the possible values of  $t$ . (5)

The ground is soft and, after  $P$  reaches the ground,  $P$  sinks vertically downwards into the ground before coming to rest. The mass of  $P$  is  $4 \text{ kg}$  and the ground is assumed to exert a constant resistive force of magnitude  $5000 \text{ N}$  on  $P$ .

(c) Find the vertical distance that  $P$  sinks into the ground before coming to rest. (4)

a)

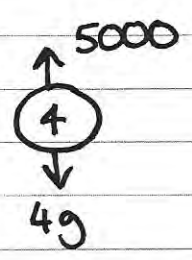


$$\begin{aligned}
 s &= -17.5 & v^2 &= u^2 + 2as \\
 u & & 784 &= u^2 + 343 \\
 v &= -28 & u^2 &= 441 \Rightarrow u = 21 \quad \# \\
 a &= -9.8 & & \\
 t & & &
 \end{aligned}$$

b)

$$\begin{aligned}
 s &= 19 & s &= ut + \frac{1}{2}at^2 & t &= \frac{21 \pm \sqrt{21^2 - 4(4.9)(19)}}{9.8} \\
 u &= 21 & 19 &= 21t - 4.9t^2 & t_1 &= 2.99 \text{ (3sf)} \\
 v & & 4.9t^2 - 21t + 19 &= 0 & t_2 &= 1.30 \text{ (3sf)} \\
 a &= -9.8 & & & & \\
 t & & & & &
 \end{aligned}$$

c)



$$\begin{aligned}
 Rf \downarrow &= ma \Rightarrow 39.2 - 5000 = 4a \\
 &\Rightarrow a = -1240.2
 \end{aligned}$$

$$u = 28, v = 0$$

$$\begin{aligned}
 v^2 &= u^2 + 2as \Rightarrow 0 = 784 - 2480 \cdot 4s \\
 &\Rightarrow s = 0.316 \text{ m (3sf)}
 \end{aligned}$$

6. [In this question  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

A ship  $S$  is moving with constant velocity  $(-12\mathbf{i} + 7.5\mathbf{j}) \text{ km h}^{-1}$ .

(a) Find the direction in which  $S$  is moving, giving your answer as a bearing. (3)

At time  $t$  hours after noon, the position vector of  $S$  is  $\mathbf{s}$  km. When  $t = 0$ ,  $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$ .

(b) Write down  $\mathbf{s}$  in terms of  $t$ . (2)

A fixed beacon  $B$  is at the point with position vector  $(7\mathbf{i} + 12.5\mathbf{j}) \text{ km}$ .

(c) Find the distance of  $S$  from  $B$  when  $t = 3$  (4)

(d) Find the distance of  $S$  from  $B$  when  $S$  is due north of  $B$ . (4)

a)

bearing =  $270 + A$   
 =  $270 + \tan^{-1}\left(\frac{7.5}{12}\right)$   
 = 302°

b) pos after  $t = \text{orig pos} + t(\text{vel})$

$$\mathbf{s} = \begin{pmatrix} 40 \\ -6 \end{pmatrix} + t \begin{pmatrix} -12 \\ 7.5 \end{pmatrix}$$

c)  $t = 3 \quad \mathbf{s} = \begin{pmatrix} 40 - 36 \\ -6 + 22.5 \end{pmatrix} \quad \mathbf{s} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix}$

$$\vec{BS} = \underline{\mathbf{s}} - \underline{\mathbf{B}} = \begin{pmatrix} 4 \\ 16.5 \end{pmatrix} - \begin{pmatrix} 7 \\ 12.5 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\text{dist} = |\vec{BS}| = \sqrt{3^2 + 4^2} = 5 \text{ km.}$$

d) due North  $\Rightarrow$  i component of  $\vec{BS} = 0$  (i part of  $\mathbf{s} = -7!$ )  
 $(40 - 12t) = 7 \quad 12t = 33 \quad t = \frac{11}{4}$

$$t = \frac{11}{4} \quad S = \begin{pmatrix} 40 - 12\left(\frac{11}{4}\right) \\ -6 + 7.5\left(\frac{11}{4}\right) \end{pmatrix} = \begin{pmatrix} 7 \\ \frac{117}{8} \end{pmatrix}$$

$$\vec{SB} = \frac{117}{8} - 12 \cdot S = \frac{17}{8} \text{ km}$$

7.



Figure 3

Two particles  $P$  and  $Q$ , of mass  $0.3\text{ kg}$  and  $0.5\text{ kg}$  respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time  $t = 0$ , a constant force  $F$  of magnitude  $4\text{ N}$  is applied to  $Q$  in the direction  $PQ$ , as shown in Figure 3. The system moves under the action of this force until  $t = 6\text{ s}$ . During the motion, the resistance to the motion of  $P$  has constant magnitude  $1\text{ N}$  and the resistance to the motion of  $Q$  has constant magnitude  $2\text{ N}$ .

Find

(a) the acceleration of the particles as the system moves under the action of  $F$ , (3)

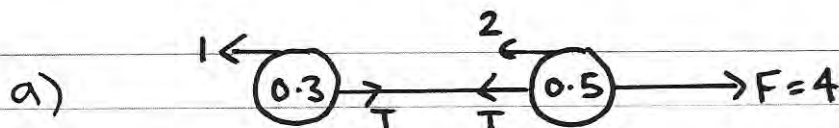
(b) the speed of the particles at  $t = 6\text{ s}$ , (2)

(c) the tension in the rod as the system moves under the action of  $F$ . (3)

At  $t = 6\text{ s}$ ,  $F$  is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

(d) the distance moved by  $P$  as the system decelerates, (4)

(e) the thrust in the rod as the system decelerates. (3)

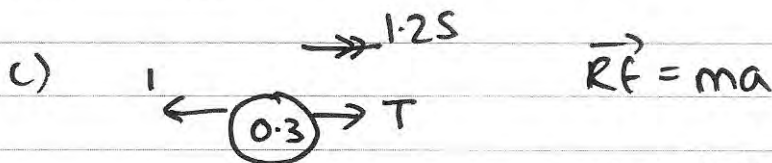


whole system  $Rf = ma$   $4 - 2 - 1 = 0.8a$   
 $\therefore a = \underline{1.25\text{ ms}^{-2}}$

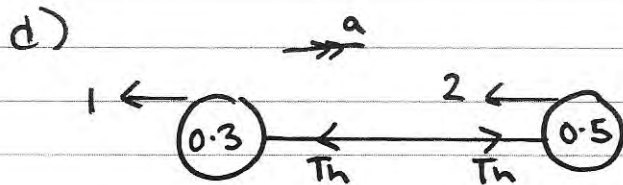
b)  $u = 0$   $t = 6$   $a = 1.25$   $v = u + at$

$v = 0 + 1.25 \times 6 \Rightarrow v = \underline{7.5\text{ ms}^{-1}}$





$$T - 1 = 0.3 \times 1.25 \Rightarrow T = \frac{11}{8} \text{ N}$$



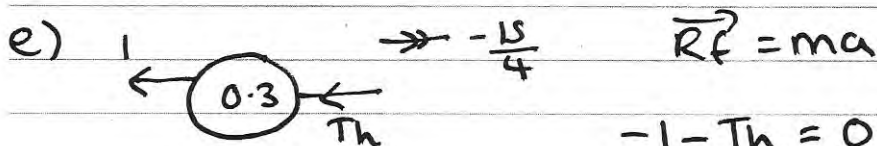
whole system  $\vec{R}F = ma$   $-1 - 2 = 0.8a$

$$\Rightarrow a = -\frac{15}{4}$$

$$u = 7.5 \quad v = 0 \quad a = -\frac{15}{4}$$

$$v^2 = u^2 + 2as \Rightarrow 0 = 86.25 - \frac{15}{2}s$$

$$\therefore s = 7.5 \text{ m}$$



$$-1 - T_h = 0.3 \left( -\frac{15}{4} \right)$$

$$-1 - T_h = -\frac{9}{8}$$

$$-T_h = -\frac{1}{8}$$

$$\therefore \text{Thrust} = \frac{1}{8} \text{ N}$$