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M1 June 03 Model Solutions

(1) $R_A = R_C = R$

a) $R + R = 80g + 40g \Rightarrow 2R = 120g \Rightarrow R = 60g$ N

b) M(A): $R \times 3 = 80g \times x + 40g \times 2$
$180g = 80g + 80g x$
$\frac{100}{80} = x \Rightarrow x = 1.25$ m

(2) BEFORE: A 0.12 kg, 3 ms^{-1} →; B 0.08 kg, 0 ms^{-1} →. AFTER: A, 1.2 ms^{-1} →; B, v ms^{-1} →.

a) $I = mv - mu$
$= 0.12 \times 3 - 0.12 \times 0$
$= 0.36$ Ns

b) $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_2$
$0.12 \times 3 + 0.08 \times 0 = 0.12 \times 1.2 + 0.08v$
$\Rightarrow 0.36 = 0.144 + 0.08v$
$\Rightarrow v = 2.7$ ms^{-1}

c) Take ← to be positive:
$I = mv - mu$
$= 1.2 \times 0.12 - 3 \times 0.12$
$= -0.216$ Ns
Impulse is 0.216 Ns ←

(3) +ve ↓, 8 ms^{-1}, 5 m, ↓ a

a) $u = -4$, $v = v$, $a = 9.8$, $s = +5$, $t =$ \
$v^2 = u^2 + 2as$
$= 16 + 2 \times 9.8 \times -5$
$= 114$
$\Rightarrow v = 10.7$ ms^{-1}

b) $s = ut + \frac{1}{2}at^2 \Rightarrow 5 = -4t + \frac{1}{2} \times 9.8t^2$
$\Rightarrow 4.9t^2 - 4t - 5 = 0$
$\Rightarrow t = \frac{4 \pm \sqrt{4^2 - 4 \times 4.9 \times -5}}{9.8} = \frac{4 \pm \sqrt{114}}{9.8} = 1.50$ s

c) We have ignored air resistance, the size of the diver, the horizontal component of velocity, spinning in the air, hitting the board on the way down.

(4) $\alpha = \tan^{-1}(\frac{1}{4}) \Rightarrow \cos\alpha = \frac{4}{\sqrt{17}}$, $\tan\alpha = \frac{1}{4}$
$F = \mu R$

$R = 5g\cos\alpha + 20\sin\alpha = 4g + 12$
$F = \mu R = \mu(4g + 12)$

$F + 20\cos\alpha = 5g\sin\alpha$
$\mu(4g + 12) + 20\cos\alpha = 5g\sin\alpha$
$\mu = \frac{5g\sin\alpha - 20\cos\alpha}{4g + 12} = \frac{3g - 16}{4g + 12} = \frac{67}{256} \approx 0.262$

(5)

a) $\underline{r} = 2\underline{i} - 3\underline{j}$, $t = 0$, $\underline{v} = -2\underline{i} + 7\underline{j}$
$\underline{v} = -2\underline{i} + 7\underline{j}$
$\underline{r} = \underline{v}$
$\underline{a} = 2\underline{i} - 3\underline{j}$
$s =$ \
$t = t$

$\underline{v} = \underline{u} + \underline{a}t$
$= -2\underline{i} + 7\underline{j} + (2\underline{i} - 3\underline{j})t$
$= (-2 + 2t)\underline{i} + (7 - 3t)\underline{j}$

Parallel to $\underline{i}$ means $\underline{j}$ component = 0:
$7 - 3t = 0$
$\Rightarrow t = 7/3$ s

b) $t = 3$, $\underline{v} = (-2 + 2 \times 3)\underline{i} + (7 - 3 \times 3)\underline{j}$
$= 4\underline{i} - 2\underline{j}$
Speed $= |\underline{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.47$ ms^{-1}

c) $t = 3$:
$\alpha = \tan^{-1}(\frac{2}{4})$
Angle $= 90 + \alpha = 116.6°$

(6) R, a, Fr, 30°, 3g cos 30, 3g sin 30

a) $\mu = 0.4$
$F_r = 0.4R$
$R = 3g\cos 30 \therefore F_r = 12g\cos 30 = 10.2$ N

b) $u = 6$, $v = 0$, $a = a$, $s = 3$, $t =$ \
To find a: $RF = ma$
$-F_r - 3g\sin 30 = 3a$
$\Rightarrow \frac{-12g\cos 30 - 3g\sin 30}{3} = a$
$\Rightarrow a = -8.29$ ms^{-2}

$v^2 = u^2 + 2as \Rightarrow 0 = 36 + 2s \times -8.29s$
$\Rightarrow 16.6s = 36$
$\Rightarrow s = 2.17$ m

(7) a) speed, 40, 30, 12, 40, 64, time; A, B

b) At T, both travelled same distance, ∴ area under graphs is equal.
$A = \frac{1}{2} \times 30 \times (T + (T - 12))$
$= 15(T + T - 12)$
$= 30T - 180$
$B = \frac{1}{2} \times 60 \times ((T - 40) + (T - 64))$
$= 30(2T - 104)$
$= 60T - 3120$

Since A = B, $30T - 180 = 60T - 3120$
$2940 = 30T$
$\Rightarrow T = 98$ s

(8) 400 N, 800 kg, T, T, 1200 kg, 2400 N, 600 N, a

a) $RF = ma$
Truck: $2400 - 600 - T = 1200a$
$1800 - T = 1200a$
$\Rightarrow T = 1800 - 1200a$ — (1)
Car: $T - 400 = 800a$ — (2)
Sub (1) into (2): $(1800 - 1200a) - 400 = 800a$
$1400 = 2000a$
$a = 0.7$ ms^{-2}

b) $T = 1800 - 1200a$
$= 1800 - 1200 \times 0.7 = 960$ N

c) 600 N, 1200 kg, 2400 N, a

New acceleration: $RF = ma$
$2400 - 600 = 1200a$
$1800 = 1200a$
$\Rightarrow a = 1.5$ ms^{-2}

To reach 28 ms^{-1}: $u = 20$, $v = 28$, $a = 1.5$, $s =$ \, $t = t$
$v = u + at \Rightarrow t = \frac{v - u}{a}$
$= \frac{28 - 20}{1.5}$
$= 5.33$ s.

If rope had not broken then time taken to reach 28 ms^{-1}:
$u = 20$, $v = 28$, $a = 0.7$, $s =$ \, $t = t$
$v = u + at \Rightarrow t = \frac{v - u}{a}$
$= \frac{28 - 20}{0.7}$
$= 11.43$ s.

Difference between times $= 11.43 - 5.33$
$= 6.1$
≈ 6 s QED