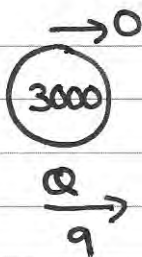
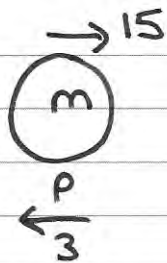


1. A railway truck P , of mass m kg, is moving along a straight horizontal track with speed 15 m s^{-1} . Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 m s^{-1} and the speed of Q is 9 m s^{-1} . The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

- (a) the magnitude of the impulse exerted by P on Q , (2)
- (b) the value of m . (3)



Mom Q before = 0

Mom Q after = 27000

\Rightarrow Impulse = 27000 Ns

b) Conservation of momentum \Rightarrow

$$15m = -3m + 27000 \Rightarrow 18m = 27000$$

$$\Rightarrow m = \underline{1500 \text{ kg}}$$

2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

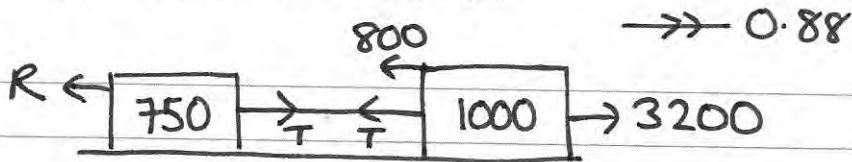
Given that the acceleration of the car and the caravan is 0.88 ms^{-2} ,

- (a) show that $R = 860$,

(3)

- (b) find the tension in the tow-bar.

(3)



a) whole system $\vec{R}F = ma$

$$3200 + T - T - 800 - R = (750 + 1000) \times 0.88$$

$$\Rightarrow 2400 - R = 1540 \Rightarrow R = 2400 - 1540 = \underline{860 \text{ N}}$$

b) Caravan $\vec{R}F = ma$

$$T - 860 = 750 \times 0.88 \Rightarrow T = \underline{1520 \text{ N}}$$

3. Three forces F_1 , F_2 and F_3 acting on a particle P are given by

$$F_1 = (7\mathbf{i} - 9\mathbf{j}) \text{ N}$$

$$F_2 = (5\mathbf{i} + 6\mathbf{j}) \text{ N}$$

$$F_3 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q .

(3)

The force F_3 is now removed. The resultant of F_1 and F_2 is \mathbf{R} . Find

(b) the magnitude of \mathbf{R} ,

(2)

(c) the angle, to the nearest degree, that the direction of \mathbf{R} makes with \mathbf{j} .

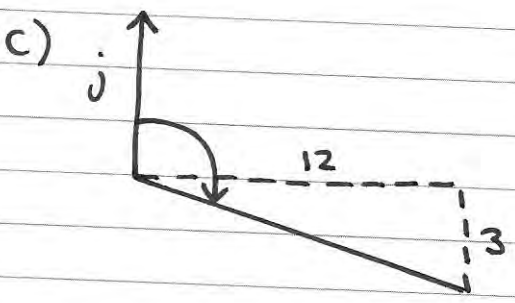
(3)

$$a) F_1 + F_2 + F_3 = 0 \Rightarrow \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow p = -12, q = 3$$

$$b) R_f = \begin{pmatrix} 7 \\ -9 \end{pmatrix} + \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix} \quad |R| = \sqrt{12^2 + 3^2}$$

$$|R| = 12.4 \text{ N (3sf)}$$



$$\text{angle} = 90 + \tan^{-1}\left(\frac{3}{12}\right)$$

$$= \underline{104^\circ} \text{ (n.d.)}$$

4.

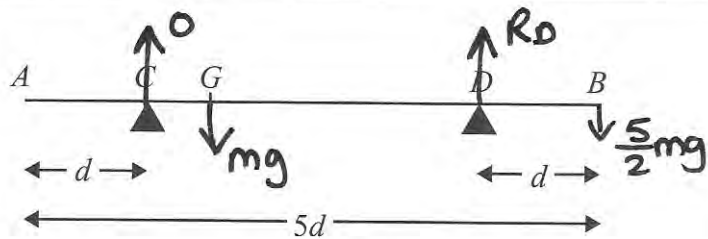


Figure 1

A non-uniform rod AB , of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D , where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G . A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D .

(a) Show that $GD = \frac{5}{2}d$. $\Rightarrow R_C = 0$

?(4)

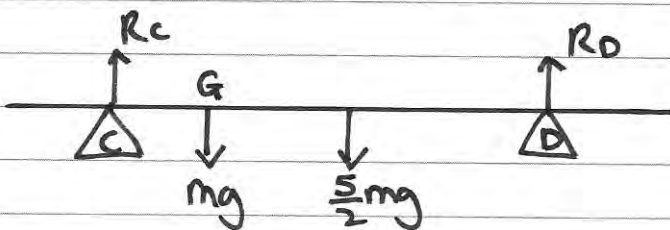
The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)?

$$a) \quad \curvearrowright \quad \frac{5}{2}mg \times d = mg \times GD \quad \Rightarrow \quad GD = \frac{5}{2}d$$

$$b) \quad R_C \uparrow = 0 \quad \Rightarrow \quad R_D = mg + \frac{5}{2}mg = \frac{7}{2}mg \quad \text{read the question}$$



$$CG = 3d - \frac{5}{2}d = \frac{1}{2}d$$

$$\curvearrowright \quad mg \times \frac{1}{2}d + \frac{5}{2}mg \times \frac{3}{2}d = R_D \times 3d$$

$$\frac{1}{2}mgd + \frac{15}{4}mgd = R_D \times 3d$$

$$\frac{17}{4}mgd = R_D \times 3d \quad \Rightarrow \quad R_D = \frac{17}{12}mg$$

5. A stone is projected vertically upwards from a point A with speed $u \text{ m s}^{-1}$. After projection the stone moves freely under gravity until it returns to A . The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$, (3)

(b) find the greatest height above A reached by the stone, (2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A . (6)

a) $s = 0$ $s = ut + \frac{1}{2}at^2$

u

v $0 = 3\frac{4}{7}u - 4.9(3\frac{4}{7})^2$

$a = -9.8$ ~~$3\frac{4}{7}u = 4.9(3\frac{4}{7})^2$~~

$t = 3\frac{4}{7}$ $u = 4.9 \times 3\frac{4}{7} = 17.5 \#$

b) s $v^2 = u^2 + 2as$

$u = 17.5$ $0 = 17.5^2 - 19.6s$

$v = 0$ $\Rightarrow s = 15.625 \approx 15.6 \text{ m (3sf)}$

$a = -9.8$

t

c) $s = 6.6$ $s = ut + \frac{1}{2}at^2$

$u = 17.5$ $6.6 = 17.5t - 4.9t^2$

v $4.9t^2 - 17.5t + 6.6 = 0$

$a = -9.8$ $49t^2 - 175t + 66 = 0$

t $(7t - 3)(7t - 22) = 0$

$t_1 = \frac{3}{7}$ $t_2 = \frac{22}{7}$

time above = $\frac{19}{7}$ sec

6. A car moves along a straight horizontal road from a point A to a point B , where $AB = 885$ m. The car accelerates from rest at A to a speed of 15 ms^{-1} at a constant rate $a \text{ ms}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of 15 ms^{-1} for T seconds. The car then decelerates at a constant rate of 2.5 ms^{-2} stopping at B .

(a) Find the time for which the car decelerates.

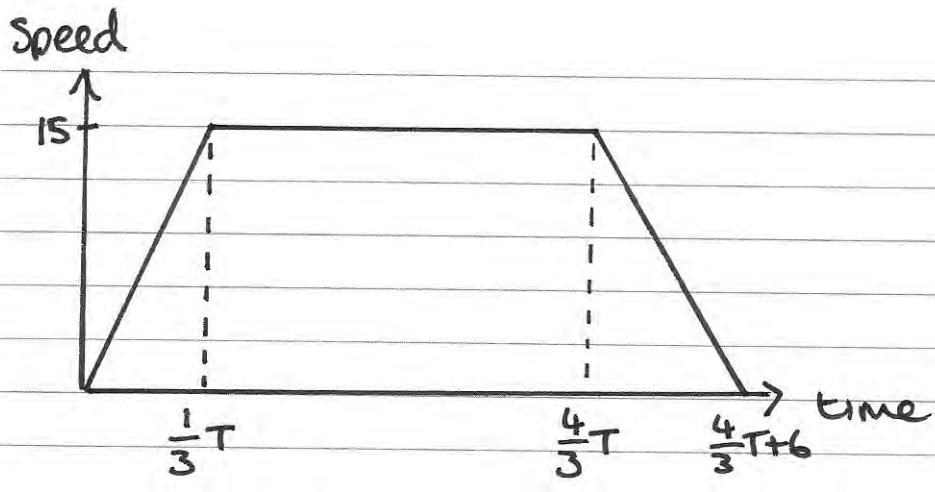
$$\frac{15}{2.5} = 6 \text{ sec} \quad (2)$$

(b) Sketch a speed-time graph for the motion of the car. (2)

(c) Find the value of T . (4)

(d) Find the value of a . (2)

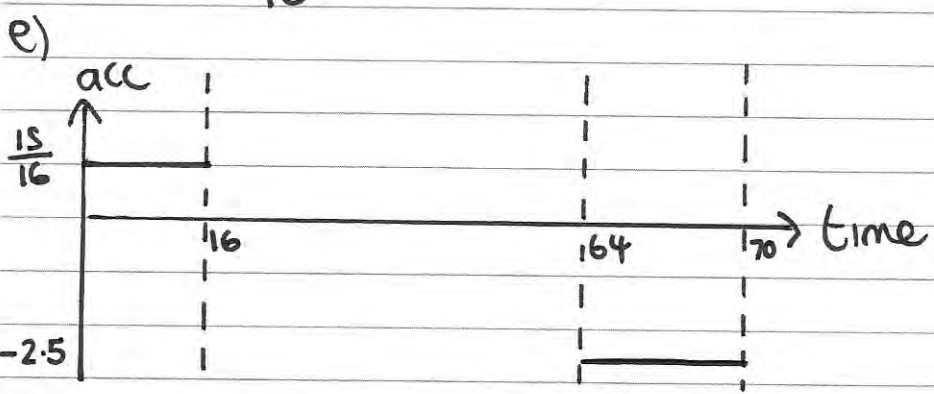
(e) Sketch an acceleration-time graph for the motion of the car. (3)



$$c) \left(\frac{1}{6}T \times 15\right) + (T \times 15) + (3 \times 15) = 885$$

$$17\frac{1}{2}T = 840 \Rightarrow T = \underline{48 \text{ sec}}$$

$$d) a = \frac{15}{16} \text{ ms}^{-2}$$



7. [In this question, the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively. Position vectors are relative to a fixed origin O .]

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$.

(a) Calculate the speed of P . $\sqrt{4^2 + 8^2} = \underline{8.94 \text{ km/h (3sf)}} \quad (2)$

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j}) \text{ km}$. At time t hours, the position vector of P is \mathbf{p} km.

(b) Write down \mathbf{p} in terms of t . $\mathbf{p} = \begin{pmatrix} 2 \\ -8 \end{pmatrix} + t \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 - 4t \\ 8t - 8 \end{pmatrix} \quad (1)$

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is \mathbf{q} km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j}) = \begin{pmatrix} 18 - 6t \\ 12 - 8t \end{pmatrix}$$

Find

- (c) the value of t when P is due west of Q , (3)
- (d) the distance between P and Q when P is due west of Q . (3)

c) due west \Rightarrow j component equal

$$8t - 8 = 12 - 8t \Rightarrow 16t = 20 \quad t = \frac{5}{4}$$

d) when $t = \frac{5}{4}$ i component of $\mathbf{p} = 2 - 4\left(\frac{5}{4}\right) = -3$

j component of $\mathbf{q} = 12 - 8\left(\frac{5}{4}\right) = 10 - 5 = 5$

\therefore distance between P and $Q = \underline{13.5 \text{ km}}$

8.

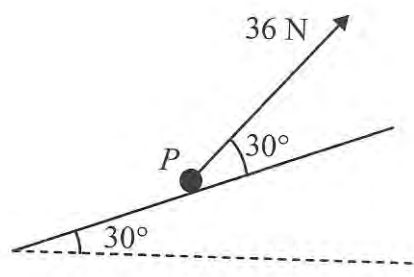


Figure 2

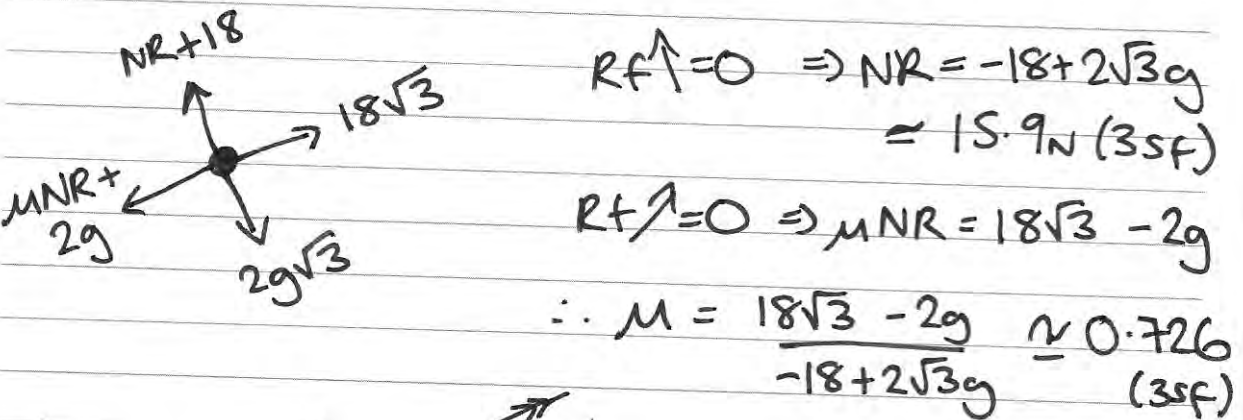
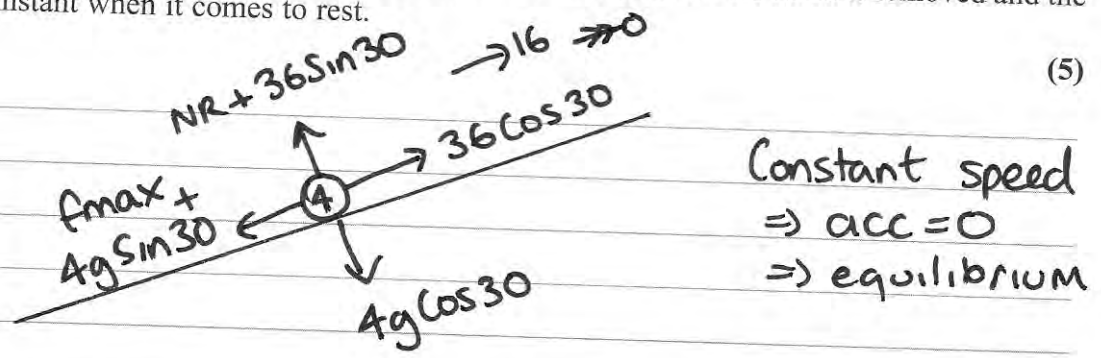
A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of 16 m s^{-1} under the action of a force of magnitude 36 N . The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P , and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ . Find

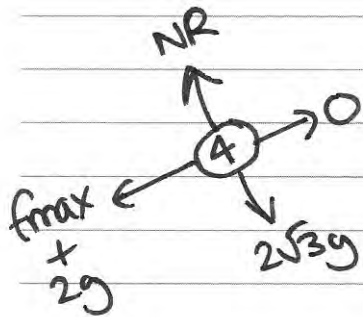
(a) the magnitude of the normal reaction between P and the plane, (4)

(b) the value of μ . (5)

The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest. (5)





$$NR = 2\sqrt{3}g$$

$$f_{max} = \mu \times 2\sqrt{3}g = 24.6432\dots$$

$$R_f = ma$$

$$-24.6432\dots - 2g = 4a$$

$$\Rightarrow a = -11.0608\dots$$

s

$$u = 16$$

$$v = 0$$

$$a = -11.0608\dots$$

t

$$v^2 = u^2 + 2as$$

$$0 = 256 - 22 \cdot 1216\dots s$$

$$\therefore s \approx \underline{\underline{11.6 \text{ m}}} \quad (3 \text{sf})$$