MI JAN 2010

1. A particle A of mass 2 kg is moving along a straight horizontal line with speed $12\,\mathrm{m\,s^{-1}}$. Another particle B of mass m kg is moving along the same straight line, in the opposite direction to A, with speed $8\,\mathrm{m\,s^{-1}}$. The particles collide. The direction of motion of A is unchanged by the collision. Immediately after the collision, A is moving with speed $3\,\mathrm{m\,s^{-1}}$ and B is moving with speed $4\,\mathrm{m\,s^{-1}}$. Find

(a) the magnitude of the impulse exerted by B on A in the collision,

(2)

(b) the value of m.

(4)

(b) $2\times 12 + m\times -8 = 2\times 3 + 4\times m$

$24 - 8m = 6 + 4m$

$18 = 12m$ $\quad$ $\underline{m = 1.5\,\text{kg}}$

(a) Mom A before $= 2\times 12 = 24\,\text{Ns}$

Mom A after $= 2\times 3 = 6\,\text{Ns}$ $\Rightarrow$ Impulse $= \underline{18\,\text{Ns}}$

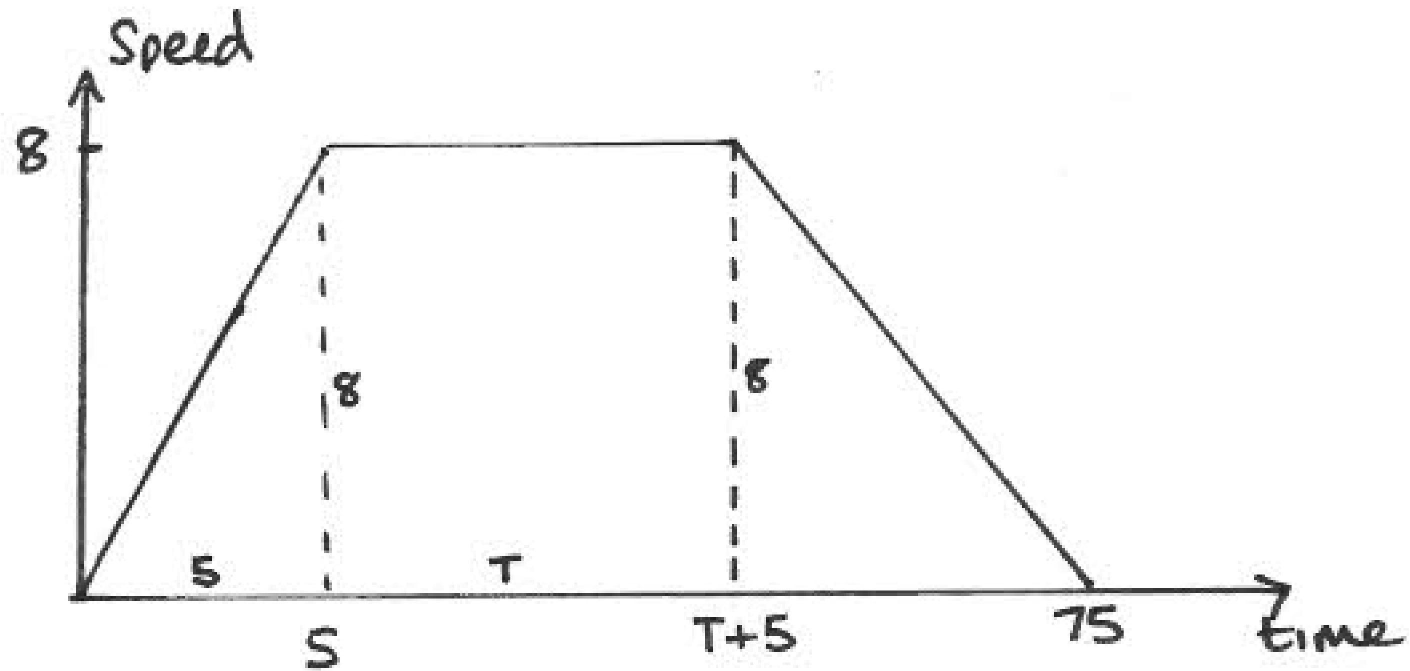
2. An athlete runs along a straight road. She starts from rest and moves with constant acceleration for 5 seconds, reaching a speed of $8\,\mathrm{m\,s^{-1}}$. This speed is then maintained for T seconds. She then decelerates at a constant rate until she stops. She has run a total of 500 m in 75 s.

(a) In the space below, sketch a speed-time graph to illustrate the motion of the athlete.

(3)

(b) Calculate the value of T.

(5)



(b)

$$\frac{1}{2} \times 8 \times (75 + T) = 500$$

$$75 + T = 125 \Rightarrow T = \underline{50 \text{ sec}}$$

3.

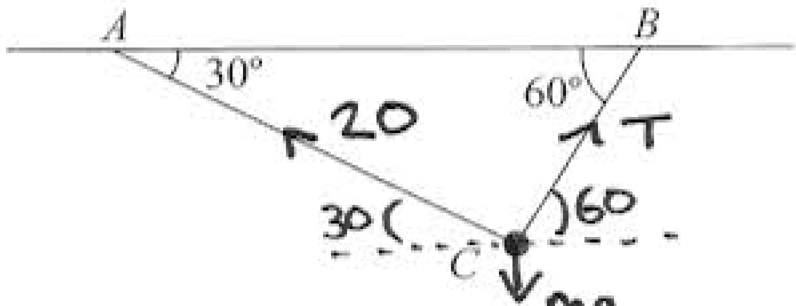


Figure 1

A particle of mass m kg is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 60° respectively, as shown in Figure 1.

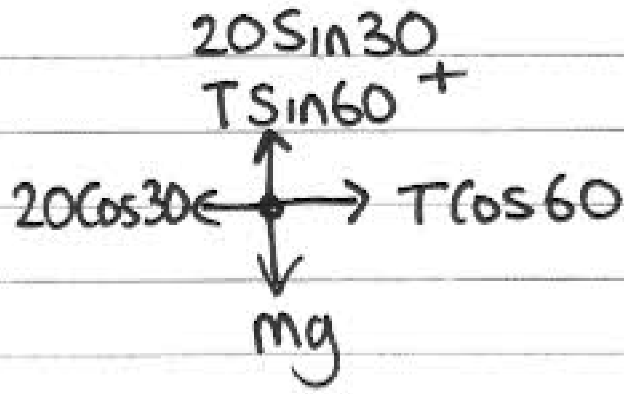
Given that the tension in AC is 20 N, find

(a) the tension in BC,

(4)

(b) the value of m.

(4)



$\overrightarrow{Rf} = 0 \Rightarrow T\cos 60 = 20\cos 30$

$T = \underline{34.6\text{N}}$

$Rf \uparrow = 0 \qquad 10 + T\sin 60 = mg \qquad mg = 40$

$\div$

$m = \underline{4.1\text{kg}}$

4.

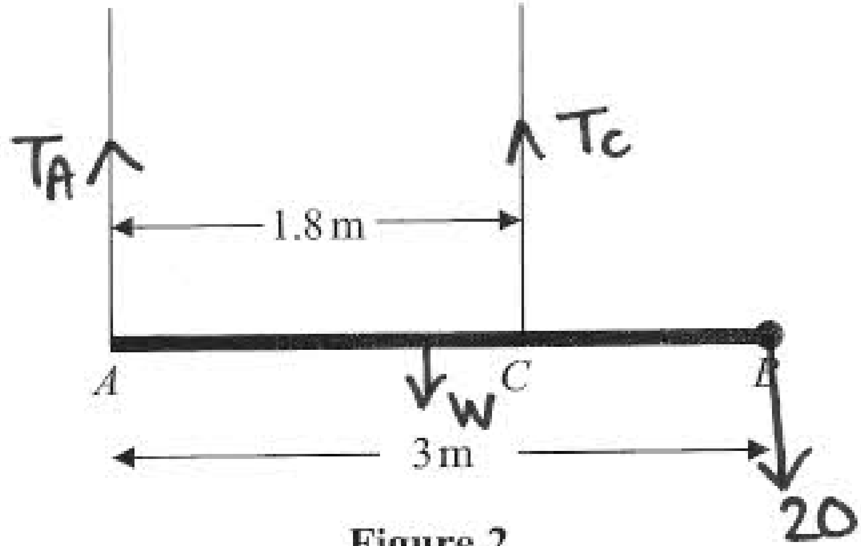


Figure 2

A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at C is $\left(\frac{5}{6}W + \frac{100}{3}\right)$ N.

(4)

(b) Find, in terms of W, the tension in the rope attached to the pole at A.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of W.

(3)

(a) $A\circlearrowright$ $\quad W \times 1.5 + 20 \times 3 = T_C \times 1.8$

$$T_C = \frac{1.5W}{1.8} + \frac{60}{1.8} = \frac{5}{6}W + \frac{100}{3} \checkmark$$

(b) $\uparrow = \downarrow$ $\quad T_A + T_C = W + 20 \Rightarrow T_A = W + 20 - \frac{5}{6}W - \frac{100}{3}$

$$T_A = \frac{1}{6}W - \frac{40}{3}$$

(c) $8 \times T_A = T_C \Rightarrow \frac{8}{6}W - \frac{320}{3} = \frac{5}{6}W + \frac{100}{3}$

$$\Rightarrow \frac{3}{6}W = \frac{420}{3} \Rightarrow W = 280\text{N}$$

5. A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,
(3)

(b) the coefficient of friction between the particle and the plane.
(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.

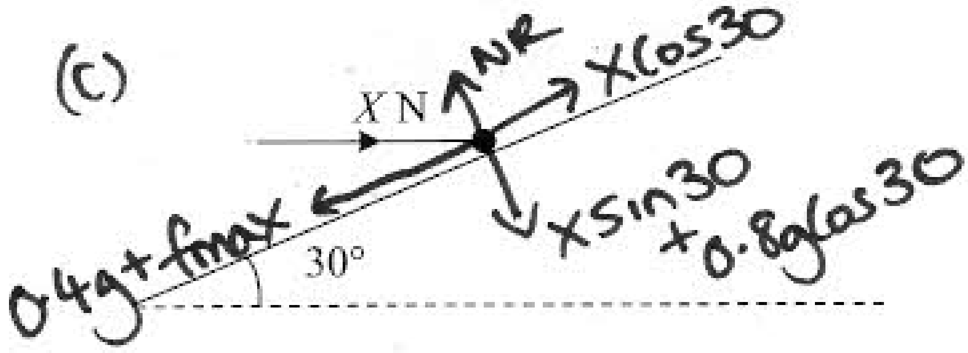


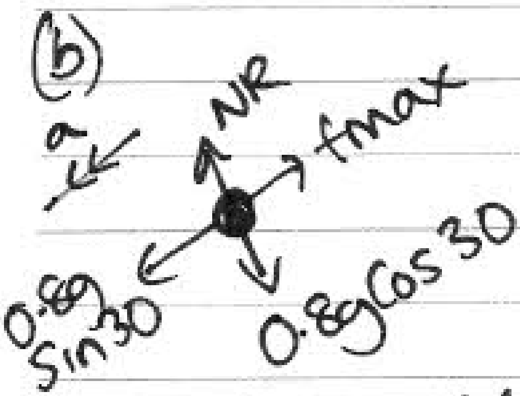
Figure 3

(c) Find the value of X.
(7)

(a) $u = 0 \quad s = 2.7 \quad t = 3$

$s = ut + \frac{1}{2}at^2 \Rightarrow 2.7 = 0 + \frac{1}{2} \times a \times 9 \Rightarrow a = \underline{0.6\ ms^{-2}}$

(b)



$R(\nwarrow) = 0 \Rightarrow NR = 0.8g\cos 30$
$NR = 6.789639$

$f_{max} = \mu NR = 6.789639\mu$.

$R(\swarrow) = ma \Rightarrow 0.4g - 6.789639\mu = 0.8 \times 0.6$

$3.44 = 6.789639\mu \quad \underline{\mu = 0.51}$

(c) $NR = \frac{1}{2}X + 6.789639 \Rightarrow f_{max} = \mu NR = 0.2533X + 3.44$

$R(\swarrow) = 0 \quad 0.4g + 0.2533X + 3.44 = 0.866X$
$7.36 = 0.6127X \quad \underline{X = 12N}$

6.

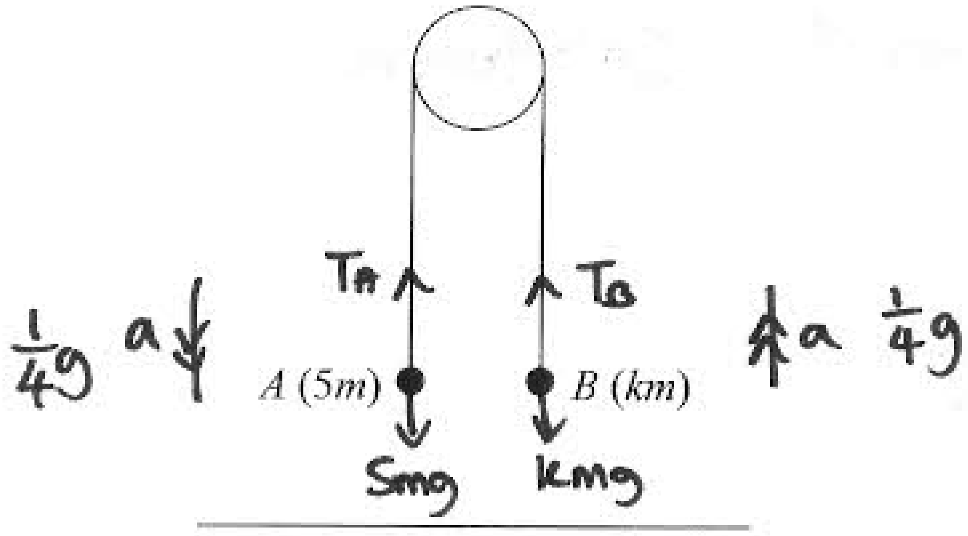


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(b) Find the value of k.

(3)

(c) State how you have used the information that the pulley is smooth.

(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.

(7)

(a) $R_f \downarrow = ma \Rightarrow 5mg - T_A = 5m \times \frac{1}{4}g$

$T_A = 5mg - \frac{5}{4}mg = \frac{15}{4}mg$

(b) $T_A = T_B \Rightarrow R_f \uparrow = ma \Rightarrow \frac{15}{4}mg - kmg = \frac{1}{4}kmg$

$\Rightarrow \frac{15}{\cancel{4}}\cancel{mg} = \frac{5}{\cancel{4}}k\cancel{mg} \Rightarrow 15 = 5k \Rightarrow \underline{k = 3}$

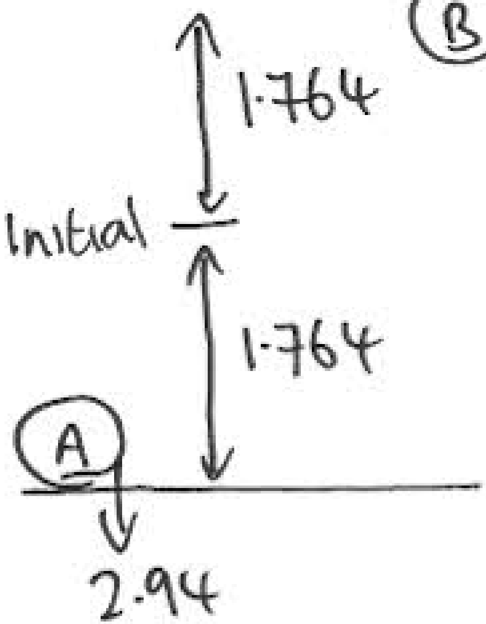
(c) Same tension either side of pulley.

(d) A $\downarrow$ $u=0$ $t=1.2$ $a=2.45$

$v\downarrow = u+at \Rightarrow v\downarrow = 2.94$

$s = ut+\frac{1}{2}at^2 \Rightarrow s = 1.764\text{m}$

B $\uparrow 2.94$ $u\uparrow = 2.94$ $a\uparrow = -9.8$ $v\uparrow = 0$



$v^2 = u^2+2as$

$0 = 8.6436 - 19.6s$

$s = 0.441\text{m}$

greatest height $= 0.441 + 2\times 1.764$

$= \underline{3.97\text{m}}$

7. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

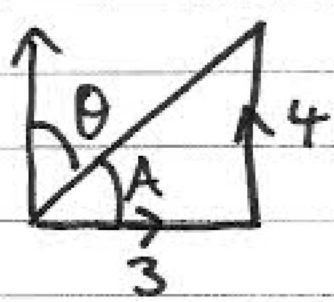
(d) Find the possible values of T.

(6)

(a) $\text{Vel} = \dfrac{(21i+10j)-(9i-6j)}{4} = \dfrac{12i+16j}{4} = 3i+4j \text{ kmh}^{-1}$

$\text{Speed} = \underline{5 \text{ km/h}}$

(b)



$\theta = 90 - \tan^{-1}\left(\frac{4}{3}\right) = 36.9°$

$\underline{\theta = 036.9°}$ $\quad (\underline{037°})$

(c) $S = (9i-6j)+t(3i+4j) = (9+3t)i+(-6+4t)j$

(d) $SL = (18-(9+3T))i+(6-(-6+4T))j$

$SL = (9-3T)i+(12-4T)j$

$SL^2 = (9-3T)^2+(12-4T)^2 \qquad SL^2 = 10^2 = 100$

$9T^2 - 54T + 81 + 16T^2 - 96T + 144 = 100$

$25T^2 - 150T + 125 = 0 \qquad \div 25$

$T^2 - 6T + 5 = 0$

$(T - 5)(T - 1) = 0$

$T = \underline{5\text{hrs}}$ $\quad$ $T = \underline{1\text{hr}}$