

M1 JUNE 2010

1. A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$. At time $t = 6 \text{ s}$ P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j}) \text{ m}$. Find the distance of P from the origin at time $t = 2 \text{ s}$.

(5)

$$\text{In 4 sec } P \text{ travels } 4(-3\mathbf{i} + 2\mathbf{j}) = -12\mathbf{i} + 8\mathbf{j}$$

$$\text{at } t=2 \text{ } P = (-4\mathbf{i} - 7\mathbf{j}) - (-12\mathbf{i} + 8\mathbf{j}) = 8\mathbf{j} - 15\mathbf{i}$$

$$\text{dist} = \sqrt{8^2 + 15^2} = \underline{17 \text{ m}}$$

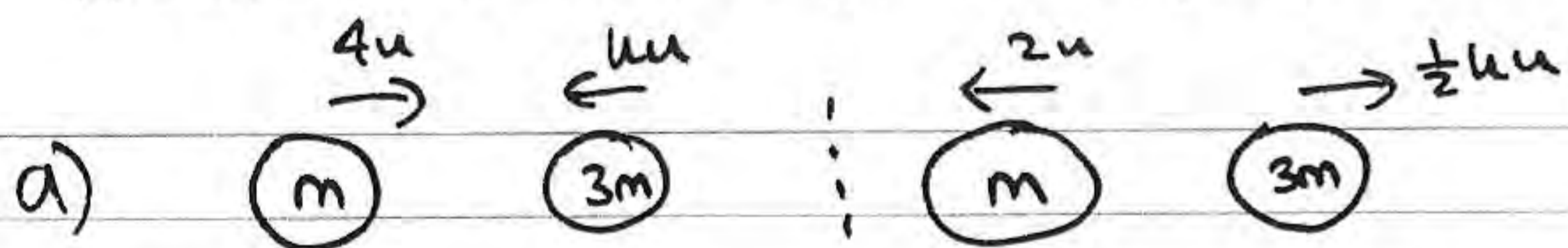
2. Particle P has mass $m \text{ kg}$ and particle Q has mass $3m \text{ kg}$. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision P has speed $4u \text{ m s}^{-1}$ and Q has speed $ku \text{ m s}^{-1}$, where k is a constant. As a result of the collision the direction of motion of each particle is reversed and the speed of each particle is halved.

- (a) Find the value of k .

(4)

- (b) Find, in terms of m and u , the magnitude of the impulse exerted on P by Q .

(3)



$$m \times 4u + 3m \times -ku = m \times -2u + 3m \left(\frac{1}{2}ku\right) \quad \text{Cons of Mom}$$

$$4mu - 3kmu = -2mu + \frac{3}{2}kmu$$

$$\Rightarrow 6mu = \frac{9}{2}kmu \quad k = \frac{12}{9} = \underline{\frac{4}{3}}$$

- b) Mom P before = $4mu$ } Impulse = change in
 Mom P after = $-2mu$ } Mom = $6mu \text{ N s}$

3.

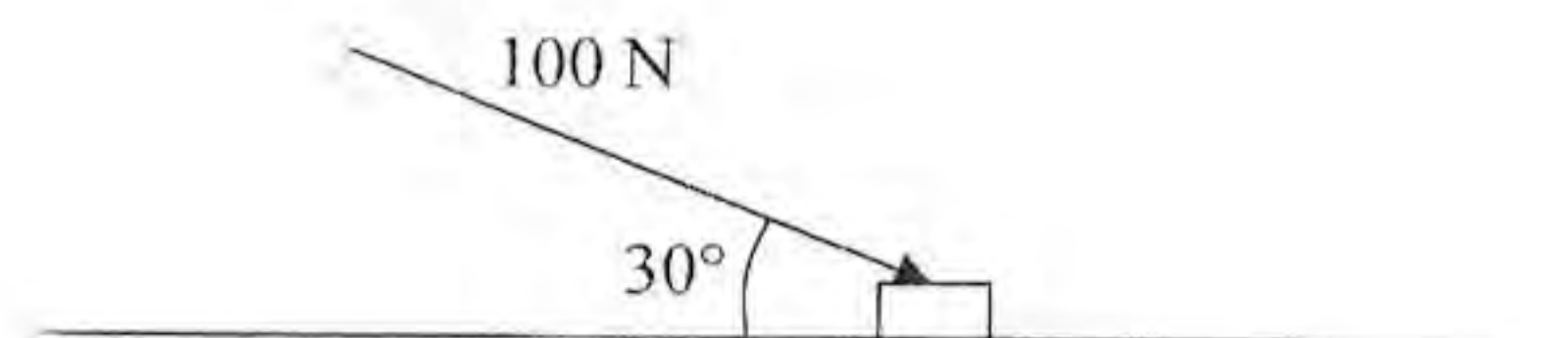
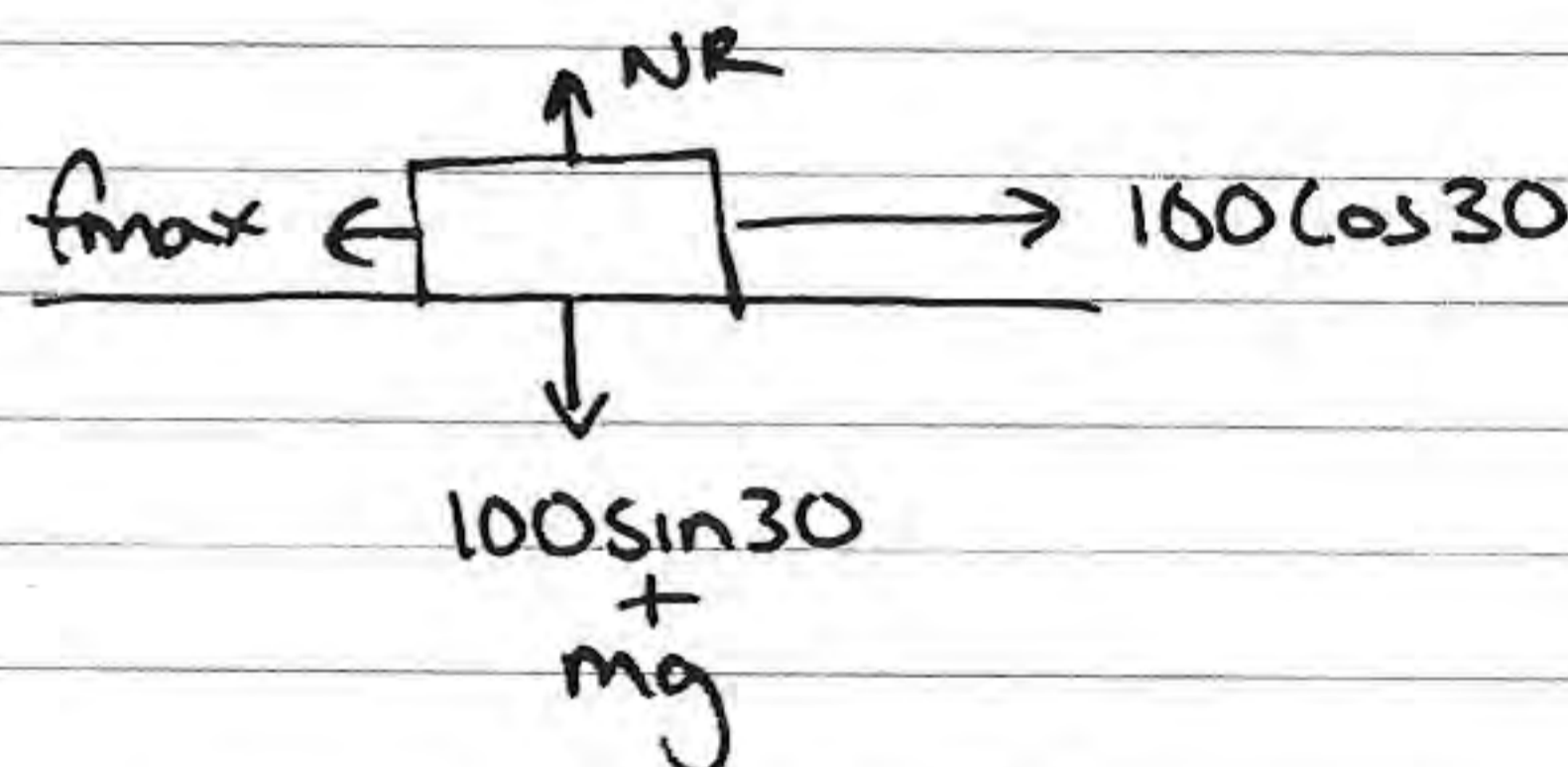


Figure 1

A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor, as shown in Figure 1.

Given that the box moves with constant speed, find the mass of the box.

$$\Rightarrow \text{acc} = 0 \Rightarrow \text{equilibrium} \quad (7)$$



$$Rf \uparrow = 0 \Rightarrow 50 + mg = NR$$

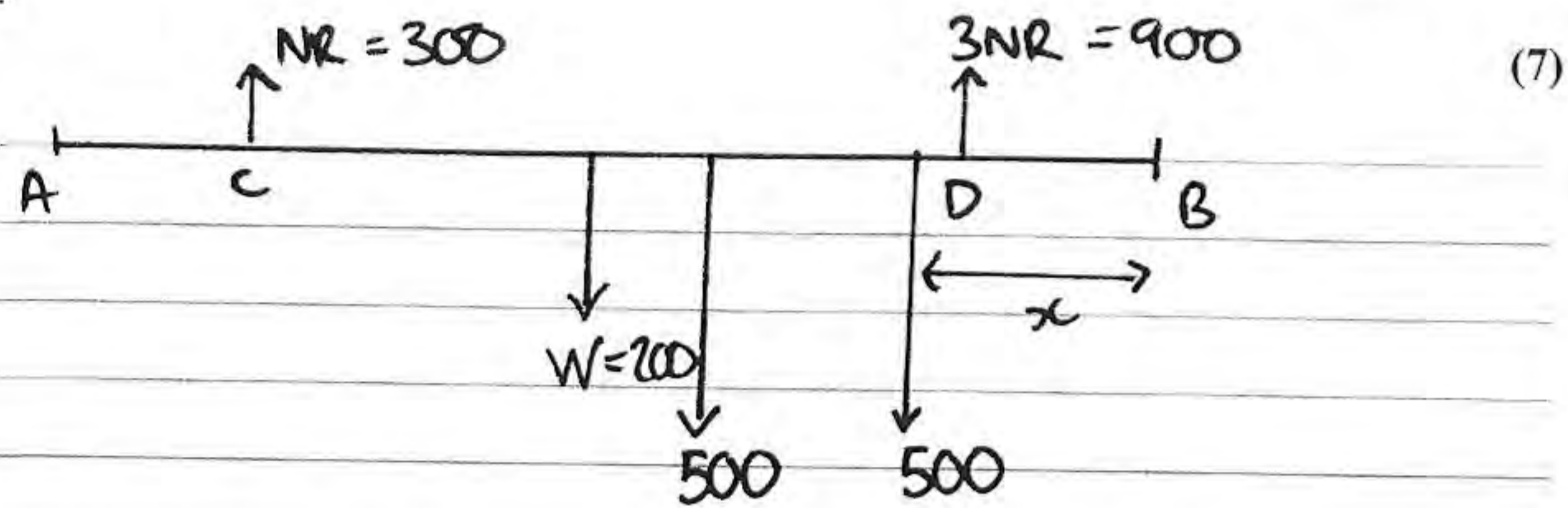
$$f_{\text{max}} = \mu NR = 25 + \frac{1}{2}mg$$

$$Rf = 0 \quad 25 + \frac{1}{2}mg = 100 \cos 30$$

$$\frac{1}{2}mg = 100 \cos 30 - 25$$

$$\Rightarrow m = \frac{200 \cos 30 - 50}{9} = \underline{12.6 \text{ kg}} \quad (3 \text{ sf})$$

4. A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D , where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C . By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B .



$$R \uparrow = 0 \Rightarrow 300 + 900 - 4NR = 1000 + W$$

$$4NR = 1200 \quad NR = 300 \text{ N}$$

$$NR_C = 300 \text{ N}$$

$$NR_D = 900 \text{ N}$$

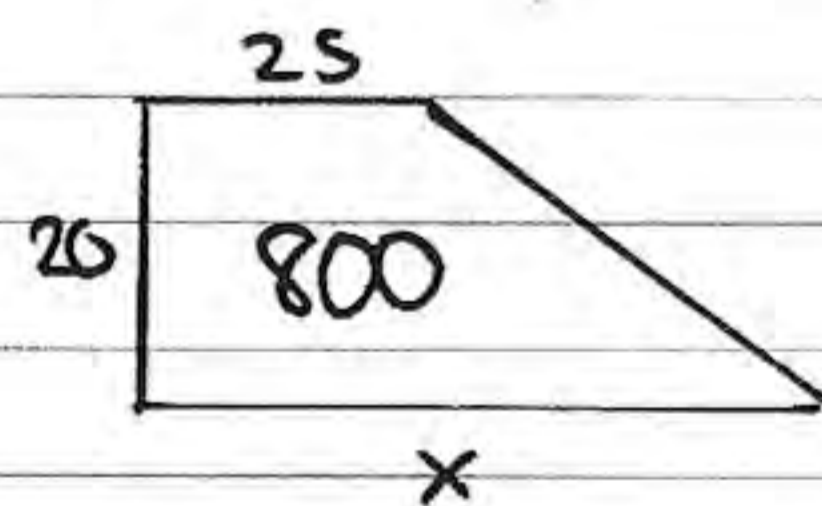
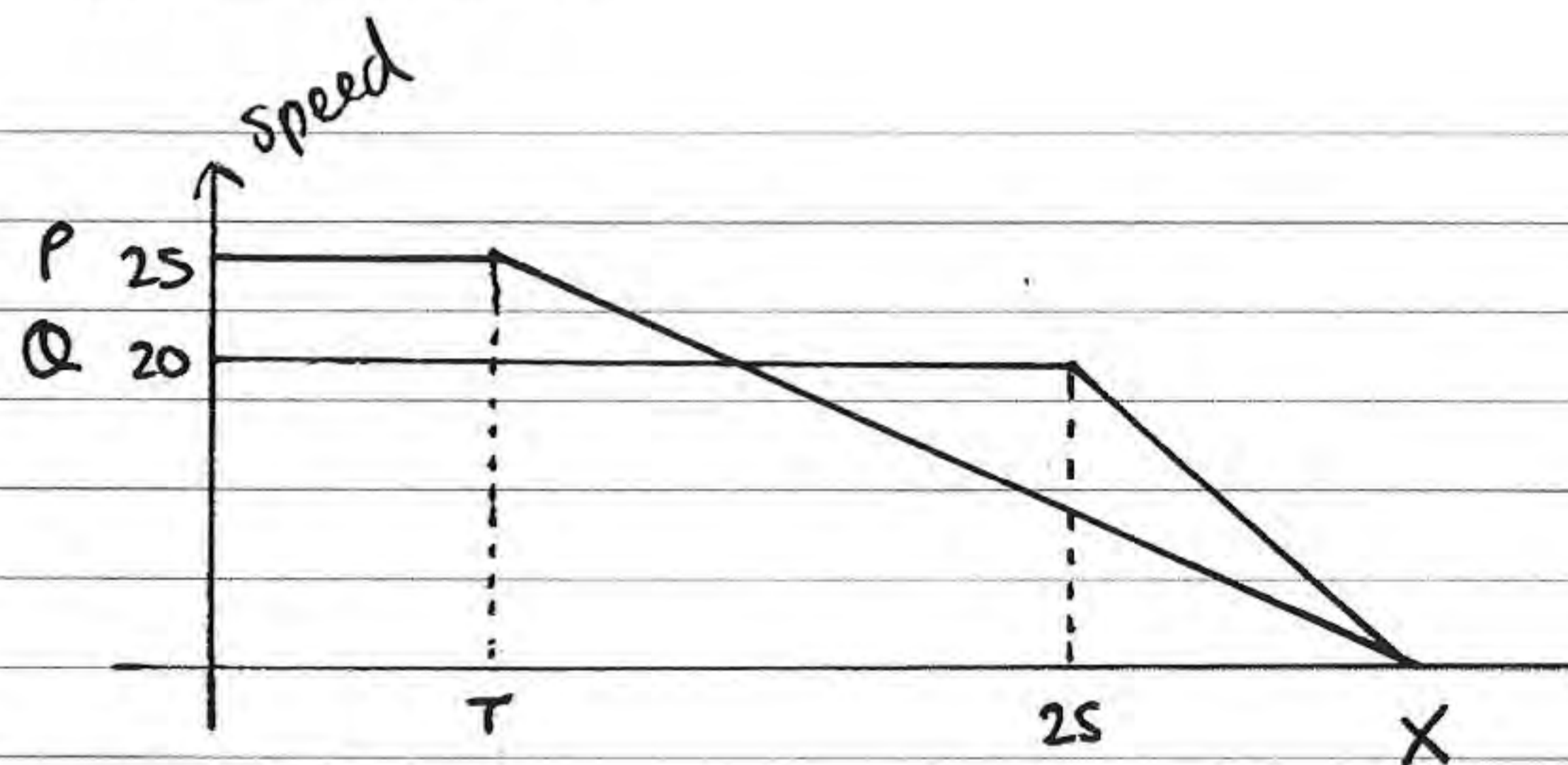
$$\text{Clockwise moments: } 500 \times x + 500 \times 2x + 200 \times 3 = 900 \times 1 + 300 \times 5$$

$$1500x = 1800 \quad \underline{x = 1.2 \text{ m}}$$

5. Two cars P and Q are moving in the same direction along the same straight horizontal road. Car P is moving with constant speed 25 m s^{-1} . At time $t = 0$, P overtakes Q which is moving with constant speed 20 m s^{-1} . From $t = T$ seconds, P decelerates uniformly, coming to rest at a point X which is 800 m from the point where P overtook Q . From $t = 25$ s, Q decelerates uniformly, coming to rest at the same point X at the same instant as P .

- (a) Sketch, on the same axes, the speed-time graphs of the two cars for the period from $t = 0$ to the time when they both come to rest at the point X . (4)

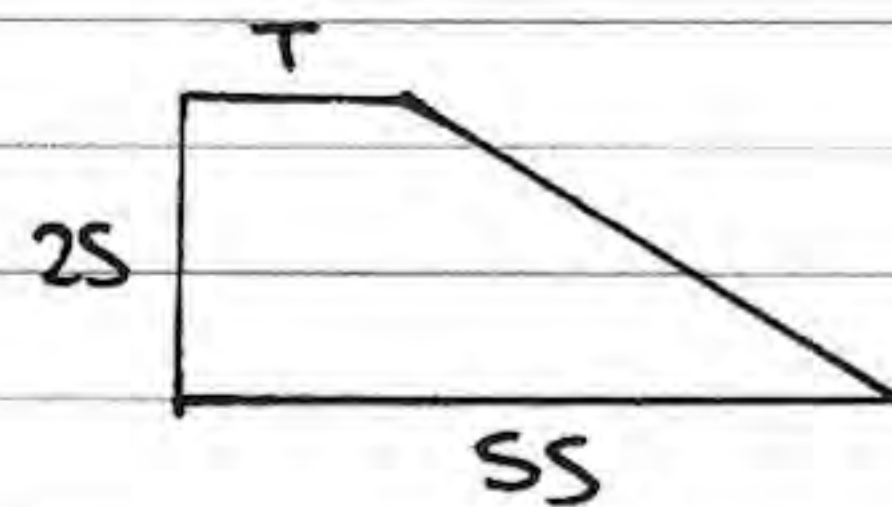
- (b) Find the value of T . (8)



$$\frac{1}{2}(20)(25 + x) = 800$$

$$25 + x = 80$$

$$x = 55 \text{ sec}$$



$$\frac{1}{2}(25)(55 + T) = 800$$

$$25(55 + T) = 1600$$

$$55 + T = 64$$

$$\underline{T = 9 \text{ sec}}$$

6. A ball is projected vertically upwards with a speed of 14.7 m s^{-1} from a point which is 49 m above horizontal ground. Modelling the ball as a particle moving freely under gravity, find

(a) the greatest height, above the ground, reached by the ball, (4)

(b) the speed with which the ball first strikes the ground, (3)

(c) the total time from when the ball is projected to when it first strikes the ground. (3)

$$a) \quad u \uparrow = 14.7 \quad v \uparrow = 0 \text{ (at gh)} \quad a \uparrow = -9.8$$

$$v^2 = u^2 + 2as \Rightarrow 0 = 14.7^2 - 19.6s$$

$$s = \frac{14.7^2}{19.6} = 11.025 \text{ m} \quad gh = 11.025 + 49 = 60.025 \text{ m}$$

$$\approx 60.0 \text{ (3sf)}$$

$$b) \quad u \uparrow = 14.7 \quad a \uparrow = -9.8 \quad s = -49$$

$$v^2 = u^2 + 2as \Rightarrow v^2 = 14.7^2 - 19.6(-49) \\ \Rightarrow v = \pm 34.3 \text{ m s}^{-1} \quad \text{speed} = 34.3 \text{ m s}^{-1}$$

$$c) \quad s = ut + \frac{1}{2}at^2 \Rightarrow -49 = 14.7t - 4.9t^2$$

$$\Rightarrow 4.9t^2 - 14.7t - 49 = 0 \quad (\div 4.9) \quad t^2 - 3t - 10 = 0$$

$$(t-5)(t+2) = 0 \quad \Rightarrow t = 5 \text{ sec}$$

7.

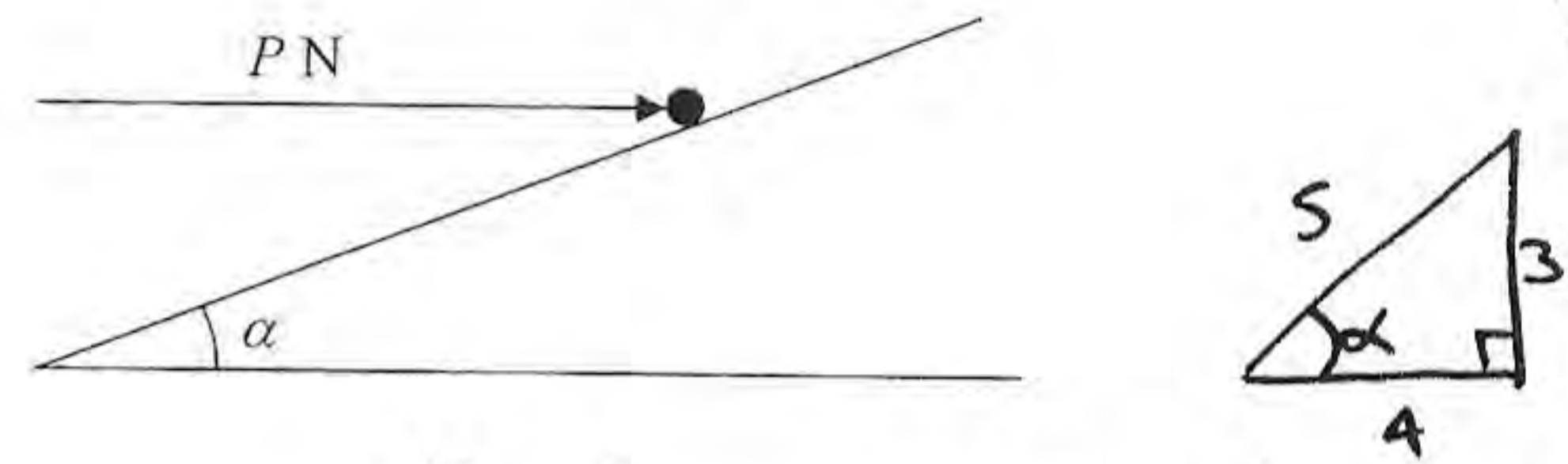


Figure 2

$$\cos \alpha = \frac{4}{5} \quad \sin \alpha = \frac{3}{5}$$

A particle of mass 0.4 kg is held at rest on a fixed rough plane by a horizontal force of magnitude P newtons. The force acts in the vertical plane containing the line of greatest slope of the inclined plane which passes through the particle. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 2.

The coefficient of friction between the particle and the plane is $\frac{1}{3}$.

Given that the particle is on the point of sliding up the plane, find

(a) the magnitude of the normal reaction between the particle and the plane, (5)

(b) the value of P . (5)



$$R_F \uparrow = 0 \Rightarrow NR = \frac{3}{5}P + 0.4g\left(\frac{4}{5}\right) = \frac{3}{5}P + \frac{8}{25}g$$

$$f_{\max} = \mu NR = \frac{1}{3}NR = \frac{1}{5}P + \frac{8}{75}g$$

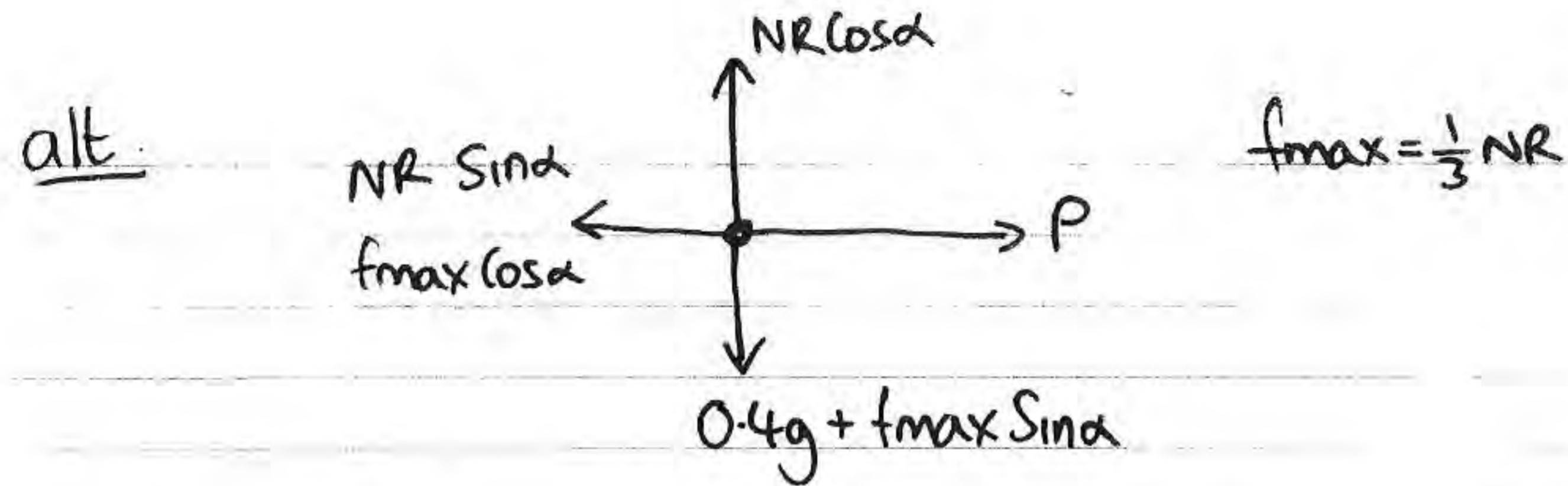
$$R_F \nearrow = 0 \quad \frac{4}{5}P = \left(\frac{1}{5}P + \frac{8}{75}g\right) + \frac{2}{5}g\left(\frac{3}{5}\right)$$

$$\Rightarrow \frac{3}{5}P = \frac{8}{75}g + \frac{6}{25}g \Rightarrow \frac{3}{5}P = \frac{26}{75}g$$

7 cont)

$$P = \frac{26}{45}g \quad P = 5.66 \text{ N (3sf)}$$

$$NR = \frac{3}{5}(5.66...) + \frac{8}{25}g = \frac{2}{3}g = 6.53 \text{ N (3sf)}$$



$$Rf \uparrow = 0 \quad -\frac{3}{5}f_{max} + \frac{4}{5}NR = \frac{2}{5}g$$

$$\Rightarrow -\frac{3}{5}\left(\frac{1}{3}NR\right) + \frac{4}{5}NR = \frac{2}{5}g \Rightarrow NR = \frac{2}{5}g$$

$$\frac{3}{5}NR = \frac{2}{5}g \quad NR = \frac{2}{3}g \text{ N}$$

$$\vec{Rf} = 0 \quad P = \frac{3}{5}NR + \frac{4}{5}f_{max} = \frac{3}{5}\left(\frac{2}{3}g\right) + \frac{4}{5}\left(\frac{1}{3} \times \frac{2}{3}g\right)$$

$$P = \frac{2}{5}g + \frac{8}{45}g \Rightarrow P = \frac{26}{45}g \text{ N}$$

\Rightarrow 8) cont

$$s = ut + \frac{1}{2}at^2$$

$$-1.175 = 0.7t - 4.9t^2 \Rightarrow 4.9t^2 - 0.7t - 1.175 = 0$$

$$t = \frac{0.7 \pm \sqrt{0.7^2 - 4(4.9)(-1.175)}}{2(4.9)} = \frac{0.7 \pm \sqrt{23.52}}{9.8}$$

$$t = 0.566 \text{ sec (3sf)}$$

8.

(A) $0.4g - T = 0.4a$
 (B) $T - 0.3g = 0.3a$
 $0.1g = 0.7a$
 $a = \frac{1}{7}g \text{ ms}^{-2}$

$$T = 0.3a + 0.3g$$

$$T = \frac{3}{70}g + \frac{3}{10}g$$

$$T = \frac{24}{70}g \text{ N}$$

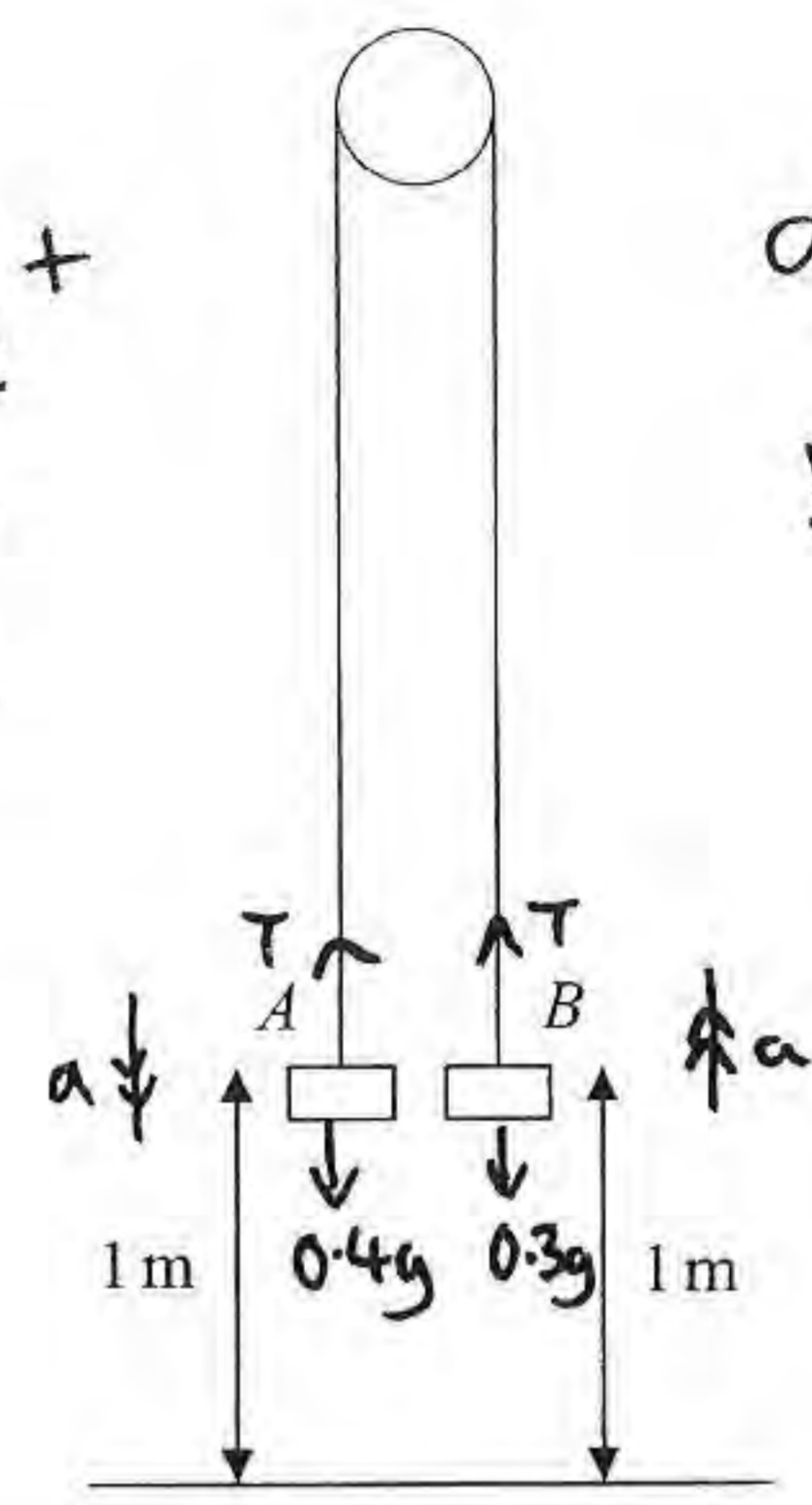


Figure 3

a) $T = 3.36 \text{ N}$
 b) $acc = 1.4 \text{ ms}^{-2}$

Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 3. The particles are released from rest and in the subsequent motion B does not reach the pulley.

- (a) Find the tension in the string immediately after the particles are released. (6)
 (b) Find the acceleration of A immediately after the particles are released. (2)

When the particles have been moving for 0.5 s, the string breaks.

- (c) Find the further time that elapses until B hits the floor. (9)

$$u \downarrow = 0 \quad a \downarrow = 1.4 \quad t = 0.5 \quad v = u + at \Rightarrow v \downarrow = 0 + 1.4(0.5)$$

$$v \downarrow = 0.7 \text{ ms}^{-1}$$

$$s = ut + \frac{1}{2}at^2 \quad s = 0 + \frac{1}{2}(1.4)(0.5)^2$$

$$s = 0.175 \text{ m}$$

B is 1.175 m above ground with a speed of 0.7 ms^{-1} when string breaks.

$$u \uparrow = 0.7 \quad a \uparrow = -9.8 \quad s = -1.175$$