OCR A Further Maths A-level

Mechanics

Formula Sheet

Provided in formula book	Not provided in formula book

Work, Energy and Power

Work Done

Unit	$1\text{J} = 1\text{Nm}$
Work done by a force acting in the direction of motion	$\text{work done (J)} = \text{force (N)} \times \text{distance(m)}$ $= Fd$
Work done by or against gravity	$\text{work done (J)} = \text{weight (N)} \times \text{height(m)}$ $= mgh$

Energy

Kinetic Energy	$\text{KE} = \frac{1}{2}mv^2$
Work-Energy Principle	$\text{net work done} = \text{final KE} - \text{initial KE}$ $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
Gravitational Potential Energy	$\text{GPE} = mgh$
Principle of Conservation of Mechanical Energy	$\text{GPE} + \text{KE} = mgh + \frac{1}{2}mv^2 = \text{constant}$
$\text{GPE}_1 + \text{KE}_1 + \text{work done by driving forces} - \text{work done against resistive forces} = \text{GPE}_2 + \text{KE}_2$	

Work Done by Force at an Angle

For force acting at an angle θ to the direction of movement
$\text{work done} = \text{force} \times \cos\theta \times \text{distance}$

Work Done by a Variable Force

Work done by a variable force $f(x)$ dependent on displacement, x, in moving an object in a straight line from x_1 to x_2
$\text{work done} = \int_{x_1}^{x_2} f(x)\, dx$

Hooke's Law

Hooke's Law for Elastic String or Spring	$T = \frac{\lambda x}{l}$
Work done extending an elastic string or spring from extension x_1 to x_2	$\text{work done} = \frac{\lambda}{2l}(x_2^2 - x_1^2)$
Elastic Potential Energy	$\text{EPE} = \frac{\lambda x^2}{2l}$
Principle of Conservation of Energy	$\text{GPE} + \text{EPE} + \text{KE} = \text{constant}$

Power

Unit	$1\text{W} = 1\text{Js}^{-1}$
Average Power	$\text{Average power} = \frac{\text{work done}}{\text{time taken}} = \frac{Fd}{t}$
Power	$\text{Power} = \text{tractive force} \times \text{speed}$

Vectors in Work Done, Kinetic Energy and Power

Work done	work done $= \text{scalar product of force and displacement vectors}$ $= \mathbf{F} \cdot \mathbf{x} = (\lvert\mathbf{F}\rvert \cos\theta)(\lvert\text{displacement}\rvert)$
Kinetic energy	$\text{KE} = \frac{1}{2}m(\mathbf{v} \cdot \mathbf{v})$ $\mathbf{v} \cdot \mathbf{v} = \text{scalar product of velocity with itself}$
Equation of motion with constant acceleration	$\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{s}$ Where $\mathbf{v}$, $\mathbf{u}$, $\mathbf{a}$ and $\mathbf{s}$ are in vector form
Power	$\text{power} = \text{scalar product of force and velocity vectors}$ $= \mathbf{F} \cdot \mathbf{v}$

Impulse and Momentum

Momentum	$\text{momentum} = \text{mass} \times \text{velocity}$ $= mv$
Impulse of a constant force	$I = \text{force} \times \text{time} = Ft$ $= \text{change in momentum} = mv - mu$
Impulse of a variable force F acting for a time $t_1 \leq t \leq t_2$	$I = \int_{t_1}^{t_2} F\,dt$

Collisions

Conservation of Momentum	If there are no external impulses: total momentum before collision $=$ total momentum after collision $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Newton's Experimental Law for Two Smooth Spheres	$\frac{\text{speed of separation}}{\text{speed of approach}} = -e$ $\frac{v_1 - v_2}{u_1 - u_2} = -e$ For $0 \leq e \leq 1$ where e is the coefficient of restitution
Newton's Experimental Law for a Smooth Sphere and a Fixed Plane Surface	$v = -eu$

Vectors in Impulse and Momentum

Conservation of Momentum	$m_1\mathbf{u_1} + m_2\mathbf{u_2} = m_1\mathbf{v_1} + m_2\mathbf{v_2}$
Impulse	$\mathbf{I} = \mathbf{F}t = m\mathbf{v} - m\mathbf{u}$ where $\mathbf{I}$, $\mathbf{F}$, $\mathbf{v}$ **and** $\mathbf{u}$ are vectors

Centre of Mass

Centre of Mass of a System of Point Masses

Centre of mass, $\bar{x}$, for n particles with masses $m_1, m_2 \ldots m_n$ arranged in a straight line with positions $x_1, x_2 \ldots x_n$	$M = m_1 + m_2 + \cdots + m_n$ $M\bar{x} = m_1x_1 + m_2x_2 + \cdots + m_nx_n$
Centre of mass, $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$, for n particles with masses $m_1, m_2 \ldots m_n$ with position vectors $\begin{pmatrix}x_1\\ y_1\end{pmatrix}, \begin{pmatrix}x_2\\ y_2\end{pmatrix} \ldots \begin{pmatrix}x_n\\ y_n\end{pmatrix}$	$M = m_1 + m_2 + \cdots + m_n$ $M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + m_2\begin{pmatrix}x_2\\ y_2\end{pmatrix} + \cdots + m_n\begin{pmatrix}x_n\\ y_n\end{pmatrix}$

Centre of Mass of Standard Shapes

Triangular lamina	$\frac{2}{3}$ along the median from the vertex, or the point of intersection of the medians $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
Solid hemisphere with radius r	$\frac{3r}{8}$ from centre
Hemispherical shell with radius r	$\frac{r}{2}$ from centre
Circular arc with radius r and angle at centre 2α	$\frac{r\sin\alpha}{\alpha}$ from centre
Sector of circle with radius r and angle at centre 2α	$\frac{2r\sin\alpha}{3\alpha}$ from centre
Solid cone or pyramid with height h	$\frac{h}{4}$ from the base on the line between centre of base and vertex
Conical shell with height h	$\frac{h}{3}$ from the base on the line between centre of base and vertex

Centre of Mass of a Composite Body

$$M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = m_1\begin{pmatrix}\bar{x}_1\\ \bar{y}_1\end{pmatrix} + m_2\begin{pmatrix}\bar{x}_2\\ \bar{y}_2\end{pmatrix} + \cdots + m_n\begin{pmatrix}\bar{x}_n\\ \bar{y}_n\end{pmatrix}$$
$$M = m_1 + m_2 + \cdots + m_n$$

Centre of Mass by Integration

For a rod of length a metres with variable density function $f(x)$	$\bar{x} = \dfrac{\int_0^a xf(x)dx}{\int_0^a f(x)dx}$
For a uniform lamina defined by $f(x)$ and the lines of $x = a$, $x = 0$ and $y = 0$	$\bar{x} = \dfrac{\int_0^a xf(x)dx}{\int_0^a f(x)dx}$ $\bar{y} = \dfrac{\frac{1}{2}\int_0^a (f(x))^2 dx}{\int_0^a f(x)dx}$ $\text{Area} = \int_0^a f(x)dx$

Centre of Mass of a Uniform Solid of Revolution

For a uniform solid of revolution with radius $f(x)$:
$\bar{x} = \dfrac{\int_0^a \pi x y^2 \, dx}{\int_0^a \pi y^2 \, dx}$
$\text{Volume} = \int_0^a \pi y^2 \, dx$

Motion in a Circle

Constant Acceleration

For a particle moving in a horizontal circular path of radius r m with constant angular speed $\dot{\theta}$ rad s^{-1}	
Linear (Tangential) Speed	$v = r\dot{\theta}$ $\dot{\theta} = \frac{d\theta}{dt}$
Radial Acceleration	$a = r\dot{\theta}^2 = v\dot{\theta} = \frac{v^2}{r}$ Towards the centre of circular motion
Tangential Acceleration	$a = \frac{dv}{dt} = r\ddot{\theta}$ $\ddot{\theta} = \frac{d^2\theta}{dt^2}$

Further Dynamics and Kinematics

Linear Motion Under Variable Force

If velocity is given as a function of displacement	
Acceleration	$a = \frac{dv}{dt} = v\frac{dv}{dx}$
Time	$t = \int \frac{1}{v(x)}\,dx$

If acceleration is given as a function of displacement
$\frac{1}{2}v^2 = \int a(x)\,dx$

If acceleration is given as a function of velocity
$t = \int \frac{1}{a(v)}\,dv$
$x = \int \frac{v}{a(v)}\,dv$

Variable Force

If force is given as a function of time
Using $F = ma$ and $a = \frac{dv}{dt}$: $v = \frac{1}{m}\int F(t)\,dt$

If force is given as a function of displacement
$\frac{1}{2}mv^2 = \int F(x)\,dx$

If force is given as a function of velocity
$t = \int \frac{m}{F(v)}\,dv$
$x = \int \frac{mv}{F(v)}\,dv$