Edexcel Further Maths A-level

Further Mechanics 2

Formula Sheet

Provided in formula book

Not provided in formula book

Motion in a Circle

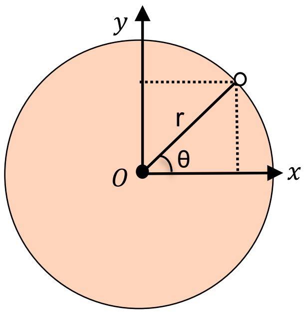
$r = radius$	$v = r\omega$
$v = linear\ speed$	
$\omega = \dot{\theta} = angular\ speed$	

Motion in a Horizontal Circle

$$Acceleration = r\omega^2 = \frac{v^2}{r}$$

(towards the centre of the circle)

Motion in a Vertical Circle

$\boldsymbol{r} = (r\cos\theta)\boldsymbol{i} + (r\sin\theta)\boldsymbol{j}$	
$\boldsymbol{r} = r\dot{\theta}(-\sin\theta\ \boldsymbol{i} + \cos\theta\ \boldsymbol{j})$	

Transverse velocity	$v = r\dot{\theta}$
Transverse acceleration	$v = r\ddot{\theta}$
Radial acceleration	$-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of Mass

Plane Figures

If a system consists of n particles with masses $m_1, m_2, \dots, m_n$ are positioned at $(x_1, 0), (x_2, 0), \dots, (x_n, 0)$ respectively, then $$\sum_{i=1}^{n} m_i x_i = \bar{x} \sum_{i=1}^{n} m_i$$ where $(\bar{x}, 0)$ is the position of the centre of mass of the system.
If a system consists of n particles with masses $m_1, m_2, \dots, m_n$ have position vectors $\boldsymbol{r_1}, \boldsymbol{r_2}, \dots \boldsymbol{r_n}$ then $$\sum_{i=1}^{n} m_i \boldsymbol{r_i} = \bar{\boldsymbol{r}} \sum_{i=1}^{n} m_i$$ where $\bar{\boldsymbol{r}}$ is the position vector of the centre of mass of the system.

For a rod with end positions (x_1, y_1) and (x_2, y_2) the centre of mass is the midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
For a uniform triangular lamina with coordinates $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3), the centre of mass has position $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Standard Results for Uniform Bodies

Triangular lamina	$\frac{2}{3}$ along median from vertex
Circular arc, radius r, angle at centre 2α	$\frac{r \sin \alpha}{\alpha}$ from the centre
Sector of circle, radius r, angle at centre 2α	$\frac{2r \sin \alpha}{3\alpha}$ from the centre
Semicircle, radius r	$\frac{4r}{3\pi}$ from the centre

Centre of Mass of Plane Figures Using Calculus

For a 2D model with equation $y = f(x)$:	
$\bar{x} = \dfrac{\int_a^b xy\,\mathrm{d}x}{\int_a^b y\,\mathrm{d}x}$	$\bar{y} = \dfrac{\int_a^b \frac{1}{2}y^2\mathrm{d}x}{\int_a^b y\,\mathrm{d}x}$
$M\bar{x} = \int_a^b \rho xy\,dx$	$M\bar{y} = \int_a^b \frac{1}{2}\rho y^2\,dx$
where $M = \int_a^b \rho y\ dx$ is the total mass of the lamina, and ρ is the mass per unit area of the lamina.	

Uniform Solid Bodies

For a uniform solid of revolution about the x-axis with mass M and density ρ, the centre of mass lies on the x-axis with position $\bar{x} = \frac{\int y^2 x dx}{\int y^2 dx}$ or $M\bar{x} = \int \rho\pi y^2 x\ dx$.

For a uniform solid of revolution about the y-axis with mass M and density ρ, the centre of mass lies on the y-axis with position $\bar{y} = \frac{\int x^2 y dy}{\int x^2 dy}$ or $M\bar{y} = \int \rho\pi x^2 y\ dy$.

Standard Results for Uniform Bodies

Solid hemisphere, radius r	$\frac{3}{8}r$ from the centre
Hemispherical shell, radius r	$\frac{1}{2}r$ from the centre
Solid cone or pyramid of height h	$\frac{1}{4}h$ above the base on the line from the centre of base to vertex
Conical shell of height h	$\frac{1}{3}h$ above the base on the line from the centre of base to vertex

Non-Uniform Solid Bodies

For a non-uniform rod with density variable $\rho = f(x)$ and length l, the distance of the centre of mass from the end of the rod is

$$\bar{x} = \frac{\int_0^l x\rho dx}{\int_0^l \rho dx} = \frac{\int_0^l xf(x)dx}{\int_0^l f(x)dx}.$$

Kinematics

Acceleration Varying with Time

$$a = f(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Acceleration Varying with Displacement

$$a = f(x) = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Acceleration Varying with Velocity

$$a = f(v) = \frac{dv}{dt}$$

Dynamics

Inverse Square Law

The force of attraction between two bodies of masses M_1 and M_2 is directly proportional to the product of their masses and inversely proportional to the square of the distance between them (G is the gravitational constant):	$F = \frac{GM_1M_2}{d^2}$

Simple Harmonic Motion

For a particle P moving in simple harmonic motion with amplitude a, defined by equation $\ddot{x} = -\omega^2 x$:
$v^2 = \omega^2(a^2 - x^2)$
$x = a\sin(\omega t + \alpha), a > 0$
$T = \frac{2\pi}{\omega}$