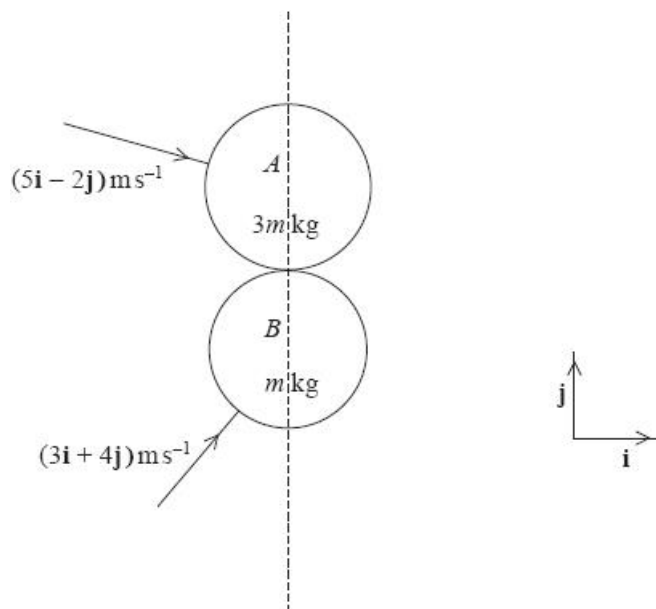# Elastic collisions in 2D

Questions

Q1.

Figure 1

Two smooth uniform spheres A and B have masses $3m$ kg and m kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(5\mathbf{i} - 2\mathbf{j})$ms^{-1} and sphere B has velocity $(3\mathbf{i} + 4\mathbf{j})$ms^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1.
The coefficient of restitution between the two spheres is e.
The kinetic energy of sphere B immediately after the collision is 85% of its kinetic energy immediately before the collision.

Find

(a)   the velocity of each sphere immediately after the collision,

(9)

(b)   the value of e.

(3)
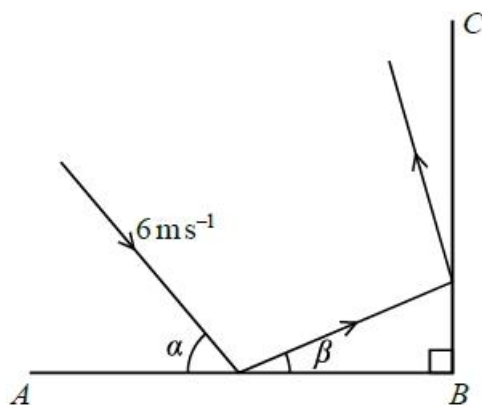
(Total for question = 12 marks)

Q2.

Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where *AB* and *BC* are fixed vertical walls with *AB* perpendicular to *BC*.

A small ball is projected along the floor towards *AB* with speed 6 m s⁻¹ on a path that

makes an angle *α* with *AB*, where $\tan \alpha = \frac{4}{3}$. The ball hits *AB* and then hits *BC*.

Immediately after hitting *AB*, the ball is moving at an angle *β* to *AB*, where $\tan \beta = \frac{1}{3}$

The coefficient of restitution between the ball and *AB* is *e*.

The coefficient of restitution between the ball and *BC* is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$

(5)

(b) find the speed of the ball immediately after it hits *BC*.

(4)

(c) Suggest two ways in which the model could be refined to make it more realistic.

(2)

(Total for question = 11 marks)

Q3.

[*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]
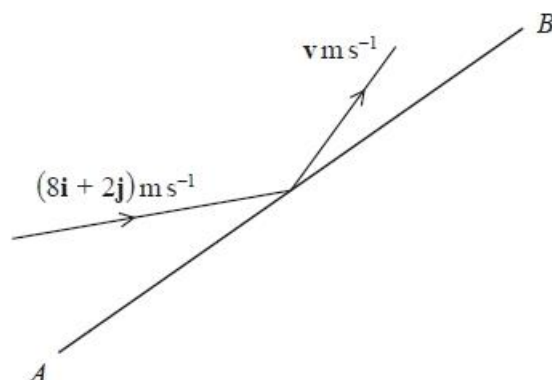

Figure 3

Figure 3 represents the plan view of part of a smooth horizontal floor, where *AB* is a fixed smooth vertical wall.

The direction of $\overrightarrow{AB}$ is in the direction of the vector (**i** + **j**)

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall *AB*.

Immediately before its impact with the wall *AB*, the velocity of the ball is (8**i** + 2**j**) m s⁻¹

Immediately after its impact with the wall *AB*, the velocity of the ball is **v** m s⁻¹

The coefficient of restitution between the ball and the wall is $\dfrac{1}{3}$

By modelling the ball as a particle,

(a) show that **v** = 4**i** + 6**j**

(6)

(b) Find the magnitude of the impulse received by the ball in the impact.

(3)
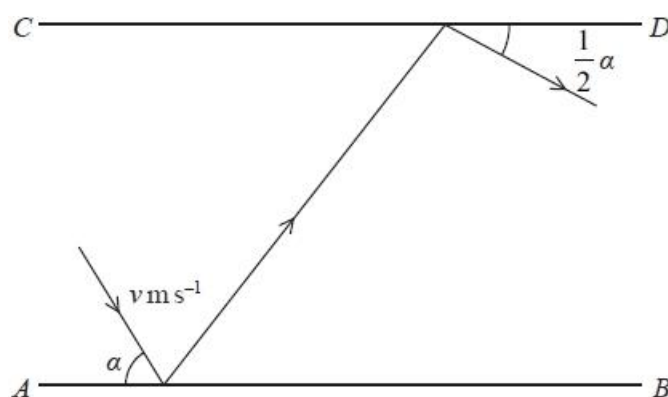
(Total for question = 9 marks)

Q4.

Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where *AB* and *CD* represent fixed vertical walls, with *AB* parallel to *CD*.

A small ball is projected along the floor towards wall *AB*. Immediately before hitting wall

AB, the ball is moving with speed v m s⁻¹ at an angle α to *AB*, where $0 < \alpha < \dfrac{\pi}{2}$

The ball hits wall *AB* and then hits wall *CD*.

After the impact with wall *CD*, the ball is moving at angle $\dfrac{1}{2}\alpha$ to *CD*.

The coefficient of restitution between the ball and wall *AB* is $\dfrac{2}{3}$

The coefficient of restitution between the ball and wall *CD* is also $\dfrac{2}{3}$

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\dfrac{1}{2}\alpha\right) = \dfrac{1}{3}$

(7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

(4)

(Total for question = 11 marks)

Q5.

Unless otherwise indicated, whenever a numerical value of *g* is required, take *g* = 9.8 ms^{-2} and give your answer to either 2 significant figures or 3 significant figures.

[In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere *A* has mass 2*m* kg and another smooth uniform sphere *B*, with the same radius as *A*, has mass 3*m* kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of *A* is (3**i** + 3**j**)m s^{-1} and the velocity of *B* is (–5**i** + 2**j**)m s^{-1}.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\dfrac{1}{4}$

(a)   Find the velocity of *B* immediately after the collision.

(7)

(b)   Find, to the nearest degree, the size of the angle through which the direction of motion of *B* is deflected as a result of the collision.

(2)
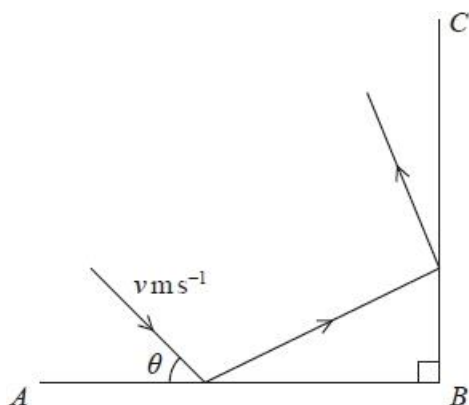
(Total for question = 9 marks)

Q6.

Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where *AB* and *BC* represent fixed vertical walls, with *AB* perpendicular to *BC*.

A small ball is projected along the floor towards the wall *AB*. Immediately before hitting the wall *AB* the ball is moving with speed v m s^{-1} at an angle θ to *AB*.

The ball hits the wall *AB* and then hits the wall *BC*.

The coefficient of restitution between the ball and the wall *AB* is $\dfrac{1}{3}$

The coefficient of restitution between the ball and the wall *BC* is e.

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall *AB*.

(a) Find the exact value of cos θ.

(5)

The ball loses half of its remaining kinetic energy in the impact with the wall *BC*.

(b) Find the exact value of *e*.

(5)

(Total for question = 10 marks)

Q7.

Unless otherwise indicated, whenever a numerical value of *g* is required, take *g* = 9.8 ms^{-2} and give your answer to either 2 significant figures or 3 significant figures.
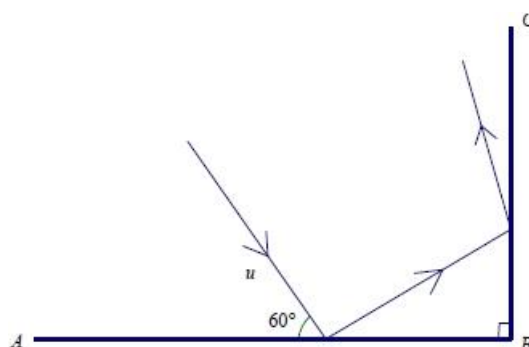
Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where *AB* and *BC* are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards *AB* with speed *u* m s^{-1} on a path at an angle of 60° to *AB*. The ball hits *AB* and then hits *BC*.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall *AB* is $\dfrac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall *BC* is $\sqrt{\dfrac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

(Total for question = 9 marks)

Q8.

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

A line of greatest slope of a fixed smooth plane is parallel to the vector $(-4\mathbf{i} - 3\mathbf{j})$. A particle P falls vertically and strikes the plane. Immediately before the impact, P has velocity $-7\mathbf{j}$ ms^{-1}. Immediately after the impact, P has velocity $(-a\mathbf{i} + \mathbf{j})$ ms^{-1}, where a is a positive constant.

(a)   Show that $a = 6$

(2)

(b)   Find the coefficient of restitution between P and the plane.

(6)

(Total for question = 8 marks)

Mark Schemeu – Elastic Collisions in2D

Q1.

a	For A, component perpendicular to loc = 5		B1	
	For B, component perpendicular to loc = 3		B1	
	$\frac{1}{2}m \times 25 \times \frac{85}{100} = \frac{1}{2}m\left(3^3 + v^2\right)$		M1	Equation for kinetic energy of B For their "3"
	$\frac{85}{4} = 9 + v^2, \ v^2 = \frac{49}{4}$		A1	
	$-6m + 4m = 3mw - mv$ $\qquad \left(= 3mw - 3.5m\right)$		M1	CLM parallel to loc. No missing/additional terms Condone sign error(s)
			A1ft	Correct unsimplified equation for CLM (with their values if substituted)
	$w = 0.5$			
	Select correct root and state velocities:		DM1	
	$\mathbf{v}_B = \left(3\mathbf{i} - 3.5\mathbf{j}\right)\left(\text{m s}^{-1}\right)$		A1	One correct
	$\mathbf{v}_A = \left(5\mathbf{i} + 0.5\mathbf{j}\right)\left(\text{m s}^{-1}\right)$		A1	Both correct
			(9)	
b	$v + w = e(2 + 4)$		M1	Impact law parallel to loc. Used the right way round. Condone sign error(s)
	$0.5 + 3.5 = 6e$		A1ft	Correct unsimplified or with their values
	$e = \frac{2}{3}$		A1	
			(3)	
			[12]	

Q2.

Question	Scheme	Marks	AOs	Notes
(a)	After hit AB: $\rightarrow 6\cos\alpha\,(=v\cos\beta)\quad(=3.6)$	B1	3.1b	Use model to find component parallel to the wall
	Use of impact law:	M1	3.4	Use model and impact law perpendicular to the wall
	$\uparrow 6e\sin\alpha$ $(=v\sin\beta)\left(=\dfrac{24e}{5}\right)(=4.8e)$	A1	1.1b	Correct perpendicular component
	$\tan\beta=\dfrac{1}{3}=\dfrac{6e\sin\alpha}{6\cos\alpha}\left(=\dfrac{24e}{5}\div\dfrac{18}{5}\right)$	M1	2.1	Use $\frac{1}{3}$ and their components to form equation in e $\left(v=\dfrac{6\sqrt{10}}{5}=3.79\right)$
	$e=\dfrac{18}{3\times24}=\dfrac{1}{4}$ *	A1*	2.2a	Correct answer from correct exact working
				If only see $e\tan\alpha=\tan\beta$ with no explanation of where it comes from then score 0/5
		(5)		

Question	Scheme	Marks	AOs	Notes
(b)	After hit BC: $\quad\uparrow\dfrac{6}{5}$	B1	1.1b	First component correct
	$\rightarrow\dfrac{1}{2}\times\dfrac{18}{5}\left(=\dfrac{9}{5}\right)$	B1	3.4	Second component correct
				Alternative: B1 for speed of impact with $BC=3.79.....$ B1 for path on leaving BC at $56.3...°$ to BC
	Speed $=\dfrac{3}{5}\sqrt{2^2+3^2}$	M1	1.1b	Use Pythagoras' theorem or trigonometry to find the speed
	$=\dfrac{3\sqrt{13}}{5}$ (m s^{-1})	A1	1.1b	Any equivalent form. 2.2 or better $(2.1633....)$
		(4)		

(c)	An appropriate refinement	B1	3.5c	Two independent refinements relating to the modelling e.g.
	A second independent appropriate refinement and no incorrect refinements	B1	3.5c	• Include friction between the floor and the ball • Include friction between the ball and the walls • Give the ball dimensions • Consider air resistance • Spin / rotation Do not accept comments about mass / gravity / levels / perpendicularity
		(2)		
		(11 marks)		

Q3.

Question	Scheme	Marks	AOs
(a)	Component parallel to the wall: $\left[\dfrac{1}{\sqrt{2}}(i+j).(8i+2j)\right]$	M1	2.1
	$= 5\sqrt{2}$	A1	1.1b
	Use of impact law perpendicular to wall:	M1	3.4
	Component perpendicular to wall after impact $\dfrac{1}{3}\left[\dfrac{1}{\sqrt{2}}(-i+j).(8i+2j)\right]=-\sqrt{2}$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow v=(5i+5j)+(-i+j)=(4i+6j)$ *	A1*	2.2a
		(6)	
(a) alt	If $v=ai+bj$ component parallel to the wall:	M1	2.1
	$(8i+2j).(i+j)=(ai+bj).(i+j)$ $\quad(a+b=10)$	A1	1.1b
	Use of impact law:	M1	3.4
	$-\dfrac{1}{3}(8i+2j).(-i+j)=(ai+bj).(-i+j)$ $\quad(2=-a+b)$	A1	1.1b
	For a complete method to find **v**	M1	1.1b
	$\Rightarrow v=(4i+6j)$ *	A1*	2.2a
		(6)	

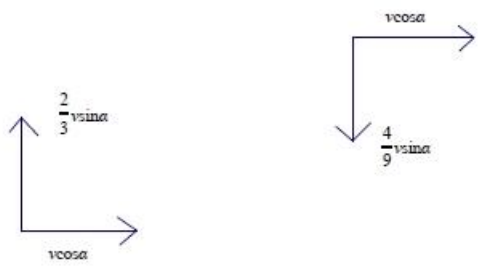| (a) alt 2	Angle to wall $= 31°$, component parallel to the wall:	M1	2.1
	$= \sqrt{68}\cos 31° = 7.07$	A1	1.1b
	Component perpendicular to the wall	M1	3.4
	$= \frac{1}{3}\sqrt{68}\sin 31° = 1.42$	A1	1.1b
	For a complete method to find v	M1	1.1b
	$\Rightarrow \mathbf{v} = \left(\sqrt{52}\cos 56.3°\mathbf{i} + \sqrt{52}\sin 56.3°\mathbf{j}\right) = (4\mathbf{i} + 6\mathbf{j})$	A1*	2.2a
		(6)	
(b)	$\mathbf{I} = 0.25(4\mathbf{i}+6\mathbf{j}) - 0.25(8\mathbf{i}+2\mathbf{j})$ $\left(\mathbf{I} = 0.25(-\mathbf{i}+\mathbf{j}) - 0.25(3\mathbf{i}-3\mathbf{j})\right)\;\;\left(\mathbf{I} = (-\mathbf{i}+\mathbf{j})\right)$	M1	3.1b

	Use of Pythagoras	M1	1.1b
	$\lvert\mathbf{I}\rvert = \sqrt{2}\;(\text{Ns})$	A1	1.1b
		(3)	
		(9 marks)	

Notes:

(a)M1	Use of scalar product or equivalent. Allow M1 if not using unit vector
A1	Correct unsimplified expression for component parallel to wall
M1	Correct use of impact law perpendicular to the wall. Condone sign error
A1	Correct unsimplified expression for component perpendicular to wall
M1	Complete method to solve for v
A1*	Obtain given result from correct working
(b) M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ with velocities or perpendicular components of velocities. Must be subtracting but allow subtraction in either order.
M1	Correct use of Pythagoras to find modulus
A1	Accept 1.4 Ns or better

Q4.

Question	Scheme	Marks	AOs
(a)	Use model to find components of velocity after the impacts:	B1	3.1b
		B1	3.4
		B1	3.1b
		B1	3.4
	$\tan\dfrac{\alpha}{2} = \dfrac{\frac{4}{9}v\sin\alpha}{v\cos\alpha}\left(=\dfrac{4}{9}\tan\alpha\right)$	M1	3.1b
	$t = \tan\dfrac{\alpha}{2} \Rightarrow t = \dfrac{4\times 2t}{9\left(1-t^2\right)}$	M1	1.1b
	$1-t^2 = \dfrac{8}{9}, \quad t = \dfrac{1}{3}$ *	A1*	2.2a
		(7)	
(b)	$\tan\alpha = \dfrac{\frac{2}{3}}{1-\frac{1}{9}} = \dfrac{3}{4}$	B1	1.1b
	change in KE $\quad \dfrac{1}{2}mv^2 - \dfrac{1}{2}m\left(v^2\cos^2\alpha + \left(\dfrac{4}{9}v\right)^2\sin^2\alpha\right)$	M1	3.1b
	% of KE lost $=100\left(1-\dfrac{\frac{1}{2}mv^2\left(\frac{16}{25}+\frac{16}{81}\times\frac{9}{25}\right)}{\frac{1}{2}mv^2}\right)$	M1	1.1b
	$= 28.888...(\%)$	A1	1.1b
		(4)	
		(11 marks)	

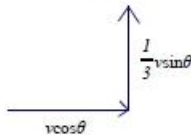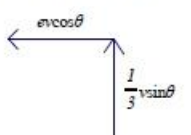| Notes:	
(a)B1 B1 B1 B1	One mark for each component correct.
M1	Form expression for $\tan\dfrac{\alpha}{2}$ in terms of $\tan\alpha$
M1	Form and solve equation in $\tan\dfrac{\alpha}{2}$
A1*	Obtain given answer from correct working
	NB: This is a "Show that .." question. A candidate who assumes, without proof, that $\tan\dfrac{\alpha}{2}=e^{2}\tan\alpha$ can only score the last two marks.
(b)B1	Correct use of $t=\dfrac{1}{3}$ Must be seen / used in part (b)
M1	Dimensionally correct expression for change in KE NB note that they may not show component parallel to the wall
M1	Dimensionally correct expression for the percentage of KE lost.
A1	Accept $29(\%)$ or better Accept $\dfrac{260}{9}$

Q5.

Question	Scheme	Marks	AOs
(a)			
	Overall strategy to find $\mathbf{v}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision $= 3\mathbf{j}$ (m s^{-1})	B1	3.4
	CLM parallel to loc.	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv \quad (-9 = 3w - 2v)$	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \dfrac{1}{4}(3+5) \ (=2)$	A1	1.1b
	Solve for w $\quad$ $\begin{aligned} 3w - 2v &= -9 \\ 2v + 2w &= 4 \end{aligned}$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s^{-1}),	A1ft	1.1b
		(7)	
(b)	$\cos\theta = \dfrac{(-5\mathbf{i} + 2\mathbf{j}).(-\mathbf{i} + 2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63...° = 42°$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1}\dfrac{2}{5} = 41.63...° = 42°$ (nearest degree)		
		(2)	
		(9 marks)	

Notes:

(a)

M1: Correct overall strategy to form sufficient equations and solve for $\mathbf{v}_A$

B1: Use the model to find the component of $\mathbf{v}_A$ perpendicular to the line of centres.

M1: Use CLM to form equation in v and w. Need all 4 terms, dimensionally correct

A1: Correct unsimplified

M1: Must be used the right way round

A1: Correct unsimplified

A1ft: v_B correct. Follow their $2\mathbf{j}$

(b)

M1: Complete method for finding the required angle. Follow their v_B

A1: cao

Q6.

Question	Scheme	Marks	AOs
(a)	Use the model to find components of velocity after first impact: $\frac{1}{3}v\sin\theta$ (upward), $v\cos\theta$ (rightward)	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{2}m\left(v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{2} = \frac{1}{9} + \frac{8}{9}\cos^2\theta$	M1	1.1b
	$\frac{7}{16} = \cos^2\theta, \quad \cos\theta = \frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	
(a) alt	Working with initial velocity $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$, after impact $\mathbf{v} = x\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4
	KE: $\frac{1}{2} \times \frac{1}{2}m\left(x^2 + y^2\right) = \frac{1}{2}m\left(x^2 + \frac{1}{9}y^2\right)$	M1	3.1b
	$y^2 = \frac{9}{7}x^2, \quad \frac{y}{x} = \tan\theta = \frac{3}{\sqrt{7}}$	M1	1.1b
	$\cos\theta = \frac{\sqrt{7}}{4}$	A1	1.1b
		(5)	

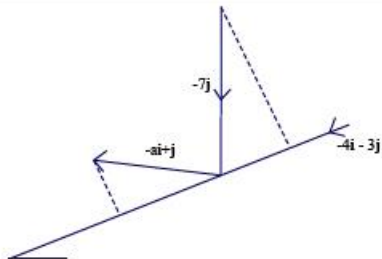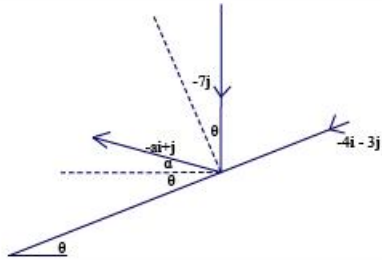| Question	Scheme	Marks	AOs
(b)	Use the model to find components of velocity after second impact: $ev\cos\theta$ (leftward), $\frac{1}{3}v\sin\theta$ (upward)	B1 B1	1.1b 3.4
	Kinetic energy: $\frac{1}{4} \times \frac{1}{2}mv^2 = \frac{1}{2}m\left(e^2v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right)$ or $\frac{1}{2} \times \frac{1}{2}m\left(v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right) = \frac{1}{2}m\left(e^2v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta\right)$	M1	3.1b
	$\frac{1}{4} = \frac{7}{16}e^2 + \frac{1}{9} \times \frac{9}{16}$	M1	1.1b
	$\Rightarrow e^2 = \frac{3}{7}, \quad e = \sqrt{\frac{3}{7}}$	A1	1.1b
		(5)	
(b) alt	After second impact $\mathbf{v} = -ex\mathbf{i} + \frac{1}{3}y\mathbf{j}$	B1 B1	1.1b 3.4

	KE: $\dfrac{1}{4} \times \dfrac{1}{2} m \left(x^2 + y^2 \right) = \dfrac{1}{2} m \left(e^2 x^2 + \dfrac{1}{9} y^2 \right)$	M1	3.1b
	$4e^2 x^2 + \dfrac{4}{9} y^2 = x^2 + y^2, \quad 4e^2 = 1 + \dfrac{5}{9} \left(\dfrac{y}{x} \right)^2$	M1	1.1b
	$\Rightarrow e^2 = \dfrac{3}{7}, \quad e = \sqrt{\dfrac{3}{7}}$	A1	1.1b
		(5)	

(10 marks)

Notes:

(a) B1	Parallel component correct
B1	Perpendicular component correct Check the diagram
M1	Equation for KE in v, θ . Dimensionally correct. Includes all components. Condone $\dfrac{1}{2}$ used on wrong side
M1	Form and solve equation in $\cos \theta$
A1	Or exact equivalent
(b) B1	Parallel component correct
B1	Perpendicular component correct
M1	Equation for KE in x, y. Dimensionally correct. Includes all components. Condone $\dfrac{1}{2}$ used on wrong side
M1	Use their $\cos \theta$ to form and solve equation in e
A1	Or exact equivalent

Q7.

Question	Scheme	Marks	AOs
(a)	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to *AB* after collision: $u\cos 60°$	M1	3.1b
	Perpendicular to *AB* after collision: $\dfrac{1}{\sqrt{3}}u\sin 60°$	M1	3.4
	Components of velocity after first impact: $\dfrac{u}{2},\ \dfrac{u}{2}$	A1	1.1b
	Parallel to *BC* after collision: $\dfrac{u}{2}\quad\left(u\times\dfrac{1}{\sqrt{3}}\sin 60°\right)$	M1	3.1b
	Perpendicular to *BC* after collision: $\sqrt{\dfrac{2}{5}}\times\dfrac{u}{2}\left(=\dfrac{1}{\sqrt{10}}u\right)$ $\left(\sqrt{\dfrac{2}{5}}\times u\cos 60°\right)$	M1	3.4
	Components of velocity after second impact: $\dfrac{u}{2},\ \dfrac{u}{\sqrt{10}}$	A1	1.1b
	Final KE $=\dfrac{1}{2}m\left(\dfrac{u^2}{4}+\dfrac{u^2}{10}\right)\quad\left(=\dfrac{mu^2}{2}\times\dfrac{7}{20}\right)$		
	Fraction of initial KE $=\dfrac{\dfrac{mu^2}{2}\times\dfrac{7}{20}}{\dfrac{mu^2}{2}}=\dfrac{7}{20}=35\%$ *	A1*	2.2a
		(8)	
(b)	The answer is too large - rough surface means resistance so final speed will be lower.	B1	3.5a
		(1)	
		(9 marks)	

Notes:

(a)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Correct expression for total KE using their components after 2nd collision.

A1*: Obtain **given** answer with sufficient working to justify it

(b)

Clear explanation of how the modelling assumption has affected the outcome

Q8.

Q	Scheme	Marks	Notes
a			
	Components parallel to the plane unchanged: $\left(\begin{pmatrix} 0 \\ -7 \end{pmatrix} \cdot \dfrac{1}{5}\begin{pmatrix} -4 \\ -3 \end{pmatrix} = \begin{pmatrix} -a \\ 1 \end{pmatrix} \cdot \dfrac{1}{5}\begin{pmatrix} -4 \\ -3 \end{pmatrix}\right)$ $\Rightarrow 21 = 4a - 3$	M1	Use of scalar product. Do not need to see $\dfrac{1}{5}$
	$a = 6$	A1	*Given Answer*
		(2)	
a alt			
	Components parallel to the plane: $7\sin\theta = v\cos(\theta + \alpha)$		$\theta + \alpha = 46.3...°$
	$7\sin\theta = v(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$, $\Rightarrow 7\tan\theta = a - \tan\theta$	M1	Equate components and form an equation in a and θ
	$8\tan\theta = a = 6$	A1	
		(2)	

b	Component of -7**j** parallel to the plane $= \dfrac{21}{|4\mathbf{i} + 3\mathbf{j}|}$	M1	Scalar product of -7**j** and unit vector parallel to plane
	$= 4.2$	A1	
	$\sqrt{49 - 4.2^2} = \sqrt{31.36}$	M1	Use Pythagoras to find components perpendicular to the plane
	$\sqrt{36 + 1 - 4.2^2} = \sqrt{19.36}$	A1	Both correct
	$\sqrt{19.36} = e \times \sqrt{31.36}$	DM1	Use if impact law Dependent on preceding M mark
	$e = 0.786$	A1	
		(6)	

Alt b	Component of -7**j** perpendicular to the plane $=\dfrac{1}{5}\begin{pmatrix}0\\-7\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Component of $-a\mathbf{i}+\mathbf{j}$ perpendicular to the plane $=\dfrac{1}{5}\begin{pmatrix}-6\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Impact law: $e=\dfrac{\frac{1}{5}\times22}{\frac{1}{5}\times28}=\dfrac{22}{28}=\dfrac{11}{14}(=0.786)$	DM1 A1	
		(6)	
Alt b	Components perpendicular to the plane: $e\times7\cos\theta=v\sin(\theta+\alpha)$	M1	
	$e\times7\cos\theta=v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	$\theta+\alpha=46.3...°,\ v=\sqrt{37}$
	Substitute for α: $7e=6\tan\theta+1$	M1A1	
	Solve for e: $7e=6\times\dfrac{3}{4}+1=\dfrac{11}{2},\ e=\dfrac{11}{14}$	DM1 A1	
		(6)	
Alt b	Components perpendicular to the plane: $e\times7\cos\theta=v\sin(\theta+\alpha)$	M1	
	$e\times7\cos\theta=v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	
	Divide and substitute for α: $e\cot\theta=\tan(\theta+\alpha)=\dfrac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$	M1	
	$=\dfrac{\frac{3}{4}+\frac{1}{6}}{1-\frac{3}{4\times6}}=\dfrac{3\times6+4}{4\times6-3}$	A1	
	Solve for e: $e=\dfrac{22}{21}\times\dfrac{3}{4}=\dfrac{11}{14}$	DM1 A1	
		(6)	

Q	Scheme	Marks	Notes
Alt b	Parallel: $7\sin\theta=\sqrt{37}\cos(\theta+\alpha)$ Perpendicular: $e7\cos\theta=\sqrt{37}\sin(\theta+\alpha)$	M1A1	Pair of equations
	$49\sin^2\theta=37\cos^2(\theta+\alpha)$ $\Rightarrow\sin^2(\theta+\alpha)=1-\dfrac{49}{37}\sin^2\theta$	M1	Square and substitute to eliminate $\theta+\alpha$
	$49e^2\cos^2\theta=37\sin^2(\theta+\alpha)=37-49\sin^2\theta$	A1	
	$e^2=\dfrac{37-49\times\frac{9}{25}}{49\times\frac{16}{25}}=\dfrac{121}{196},\ e=\dfrac{11}{14}$	M1A1	Substitute for θ to obtain e.
		(6)	
		[8]	