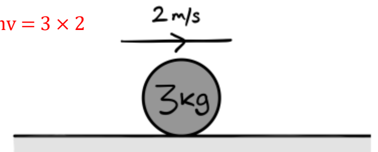
Momentum and impulse Cheat Sheet

Edexcel FM1

Momentum in one dimension

The units of momentum are $kgms^{-1}$, or Ns.

$Momentum = mv = 3 \times 2$
$= 6\ kgms^{-1}$



- The momentum of a body with mass m moving with velocity v is mv.

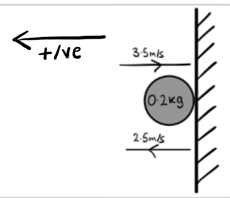
If a constant force F acts for a time t, then the impulse is defined to be $F \times t$. The units of impulse are therefore Ns (Newton seconds). A more helpful formulation of the impulse is:

- $Impulse = mv - mu = change\ in\ momentum,$ where v is the velocity after and u the velocity before.

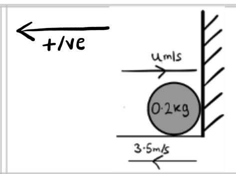
This is known as the impulse-momentum principle.

Remember that impulse is a vector quantity. An example of an impulse could be a baseball being hit by a bat, or a collision between two snooker balls.

Example 1: A ball of mass $0.2kg$ hits a fixed vertical wall at right angles with speed $3.5ms^{-1}$. The ball rebounds with speed $2.5ms^{-1}$. Find the magnitude of the impulse exerted on the wall by the ball.

We begin by drawing a diagram:	
We take the left direction to be positive and note the initial and final velocities.	$u = -3.5, v = 2.5$
Using $I = mv$ - mu:	$I = 0.2(2.5 - -3.5) = 0.2(6)$ $= 1.2\ Ns$

Example 2: A ball of mass $0.2kg$, moving along a horizontal surface, hits a fixed vertical wall at right angles. The ball rebounds at right angles to the wall with speed $3.5ms^{-1}$. Given that the magnitude of the impulse exerted on the ball by the wall is $2\ Ns$, find the speed of the ball just before it hits the wall.

We begin by drawing a diagram:	
We take the left direction to be positive and note the impulse and final velocity:	$I = 2, \quad v = 3.5$ u = -u as it is opposite to the our positive direction
Using $I = mv = mu$:	2 = 0.2(3.5 – - u) = 0.7 + 2u 1.3 = 0.2u --> u = 6.5 m/s

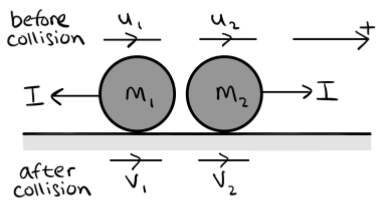
Conservation of momentum

You can use the principle of conservation of momentum to solve problems involving collisions between two bodies. The principle of conservation of momentum (commonly abbreviated as $P.C.L.M.$ or $C.L.M.$) states that:

- total momentum before impact = total momentum after impact

If we have two particles with masses m_1 and m_2 respectively moving with velocities, u_1 and u_2 respectively before the collision and velocities v_1 and v_2 respectively after the collision then we can rewrite this as:

- $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$



You should remember to perform the following steps whenever solving collision problems:

- Draw a collision diagram detailing the velocities before and after.
- Take one direction to be positive.
- Apply the $C.L.M.$ and/or the impulse-momentum principle, depending on what is required.

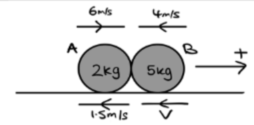
The following fact is important for questions involving impulses:

- When two bodies collide, they exert an impulse on each other of **equal magnitude** but in the opposite direction.

This is a consequence of Newton's 3rd law.

Example 3: Two particles A and B of masses $2kg$ and $5kg$ respectively are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide. Before the collision the speeds of A and B are $6ms^{-1}$ and $4ms^{-1}$ respectively. After the collision the direction of motion of A is reversed and its speed is $1.5ms^{-1}$. Find:

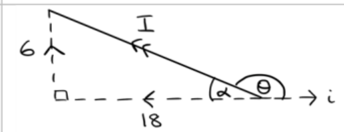
(a) the speed and direction of B after the collision.
(b) the magnitude of impulse given by A to B in the collision.

(a) We begin by drawing a diagram:	
Applying $C.L.M.$ and solving for v:	$2(6) + 5(-4) = 2(-1.5) + 5(\text{-}v)$ $-8 = -3 - 5v$ $\Rightarrow 5v = 5$ $\Rightarrow v = 1ms^{-1}$
Stating the direction:	We assumed B to be moving in the same direction, as A and v turned out to be positive so B is in fact moving in the same direction as A
(b) Using $I = m(v-u)$. We could consider A or B here, since they both experience an equal impulse. It is safer to use A though since we were given the initial/final speeds.	$I = m(v-u) = 2(-1.5-6) = -15\ Ns$ $\therefore \lvert I \rvert = 15\ Ns$

Momentum as a vector

You also need to be able to use the impulse-momentum principle and the principle of conservation of momentum to solve problems where velocities or impulses are given in vector form. The same procedure applies, but now with vectors.

Example 4: A cricket ball of mass $0.5kg$ is hit by a bat. Immediately before being hit the velocity of the ball is $(20i - 4j)ms^{-1}$ and immediately afterwards it is $(-16i + 8j)ms^{-1}$. Find the magnitude of the impulse exerted om the ball by the bat, and the angle between this impulse and $\boldsymbol{i}$.

We begin by applying the impulse-momentum principle:	$I = m(v-u) = 0.5\big(-16i + 8j - (20i - 4j)\big)$ $= 0.5(-36i + 12j) = -18i + 6j$
Applying $C.L.M.$ and solving for v:	$\therefore \lvert I \rvert = \sqrt{(-18)^2 + (6)^2} = 6\sqrt{10}\ Ns$
Drawing a diagram showing the impulse vector: The angle we want to find is θ.	
Working out α:	$tan\alpha = \frac{6}{18} = \frac{1}{3}$ $\therefore \alpha = tan^{-1}\left(\frac{1}{3}\right) = 18.4°$
Subtract α from 180 to find θ:	$\theta = 180 - \alpha = 180 - 18.4 = 162°\ \ (3\ s.f.)$

Example 5: A particle P of mass $0.5kg$ is moving with velocity $(4i + j)ms^{-1}$ when it receives an impulse $(2i - j)Ns$. Show that the kinetic energy gained by P as a result of the impulse is $12J$.

(a) We begin by applying the impulse-momentum principle:	$I = m(v-u)$ $2i - j = 0.5(v - 4i - j)$
Solving for v:	$4i - 2j = v - 4i - j$ $v = 8i - j$
Finding the magnitude of the final speed:	$\lvert v \rvert = \sqrt{(8)^2 + (-1)^2} = \sqrt{65}\ ms^{-1}$
Finding the magnitude of the initial speed:	$\lvert u \rvert = \sqrt{(4)^2 + (1)^2} = \sqrt{17}\ ms^{-1}$
Finding the K.E. before and after the impulse:	$K.E._{initial} = \frac{1}{2}(0.5)\left(\sqrt{17}\right)^2 = \frac{17}{4}\ J$ $K.E._{final} = \frac{1}{2}(0.5)\left(\sqrt{65}\right)^2 = \frac{65}{4}\ J$
$Gain\ in\ K.E. = K.E._{final} - K.E._{initial}$	$\therefore K.E.\ gained = \frac{65}{4} - \frac{17}{4} = 12J$