Work, Energy and Power I Cheat Sheet

Definition of Work

Force Acting Along the Line of Motion

The definition of work done is given by the product of force (F) and the distance moved in the direction of force (d).

$$\text{work done} = Fd$$

The unit of work done is in Joules (J), which is equivalent to Newton metres (Nm).

Example 1: A van is driven along a straight horizontal road. Given that the driving force of the engine is 500N and the van travelled 800m, find the work done by the engine.

Calculate the work done using the definition, work done $= Fd$.	Work done by engine $= 500\text{N} \times 800\text{m}$ $= 400000\text{Nm}$
The final answer can be written in terms of Joules (J) or kilojoules (kJ).	$= 400000\text{J}$ $= 400\text{kJ}$

When an object is raised, work is done against gravity. Similarly, work is done by gravity when an object descends. The work done against or by gravity for an object of mass m being raised or lowered by a height of h can be found by:

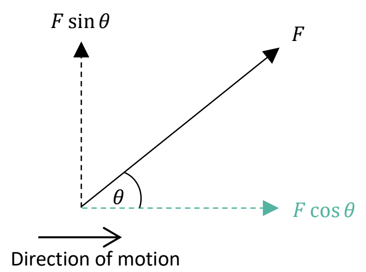
$$\text{work done} = mgh$$

Example 2: A box with a mass of 36kg is lifted from the floor using a pulley system. The box reached a height of 20m above the floor. Calculate the work done against gravity.

Calculate the work done against gravity using work done $= mgh$.	Work done against gravity $= 36 \times 9.8 \times 20$ $= 7056\text{J}$
Since the value of gravity used is to 2 significant figures, the answer should also be given to 2 significant figures.	$7056\text{J} \approx 7100\text{J}$ (2 s.f.)

Force at an Angle to the Direction of Motion (A Level Only)

When the force F is at an angle θ to the direction of motion for a distance (d), it can be resolved in the direction of the motion and perpendicular to the direction of motion.



$F \sin\theta$, which is perpendicular to the direction of motion, does no work so it is not included when calculating the work done. This gives the formula:

$$\text{work done} = F\cos\theta \times d$$

Example 3: A box with a mass of 7kg slides down a smooth slope which is inclined 30° to the horizontal. Given that the box travelled 10m, calculate the work done by its weight.

The direction of force here is vertically downwards, so we need to find the angle between the slope and the vertical.	$30°$ to the horizontal $= 60°$ to the vertical
Use the formula to calculate work done by force at an angle to the direction of motion.	work done $= 7\cos 60° \times 10$ $= 35\text{J}$

Gravitational Potential Energy

The gravitational potential energy of an object with mass m at a height h above ground can be calculated by:

$$GPE = mgh$$

Note that this is equivalent to the formula to calculate work done by or against gravity. When an object is travelling diagonally, only the vertical component causes a change in GPE and the horizontal component is not accounted for.

Kinetic Energy

The kinetic energy measured in Joules (J), of an object of mass m kg and speed v ms^{-1} is given by:

$$KE = \frac{1}{2}mv^2$$

The principle of conservation of mechanical energy states that when there is no external force acting on an object, and the only force acting on it is its weight, the sum of *KE* and *GPE* are constant. When ascending, *GPE* increases as h increases, and v decreases because *KE* decreases. When descending, *GPE* is lost and *KE* is gained.

Example 4: A ball of mass 250g is thrown upwards from the ground with a speed of 15ms^{-1}. Assuming that there are no external forces acting on the ball, find a) the kinetic energy of the ball at the start and b) the maximum height reached by the ball.

a) Calculate the starting kinetic energy using $KE = \frac{1}{2}mv^2$, remembering to change the unit of mass to kg.	$250\text{g} = 0.25\text{kg}$ $KE = \frac{1}{2}(0.25)(15)^2$ $= 28.125\text{J}$
b) Find the sum of mechanical energy at the start.	At the start, $h = 0$: Starting GPE $= 0.25(9.8)(0) = 0$ $28.125 + 0 = 28.125\text{J}$
Using the principal of conservation of mechanical energy, find h when $v = 0$. This is the maximum height reached by the ball.	$28.125 = mgh + \frac{1}{2}mv^2$ $28.125 = (0.25)(9.8)h + \frac{1}{2}(0.25)(0)$ $h = 11\text{m}$ (2 s.f.)

The work-energy principle states that the net work done on an object changes its kinetic energy. While work done by propulsive forces, such as driving force of an engine, increases kinetic energy, work done against resistive forces, such as forces from the brakes, will cause kinetic energy to be lost. Considering the work-energy principle along with the conservation of energy:

$$GPE_1 + KE_1 + \text{work done by propulsive forces} - \text{work done against resistive forces} = GPE_2 + KE_2$$

Example 5: A car is driven up a hill from rest. The driving force of the engine is 800N and the resistance experienced is 500N. The car travelled a distance of 1500m and gained a height of 20m. Given that the combined mass of the car and the driver is 1250kg, find a) the speed of the car at the end of the journey and b) the gain in gravitational potential energy.

a) Find the initial KE and GPE.	$v = 0 \Rightarrow KE = 0$ $h = 0 \Rightarrow GPE = 0$
Find the final KE using the work energy principle and conservation of energy principle.	work done by engine $-$ work done against resistance $= GPE_2 + KE_2$ $800(1500) - 500(1500) = 1250(9.8)(20) + KE$ $KE = 205000\text{J}$
Find the speed using the formula $KE = \frac{1}{2}mv^2$.	$205000 = \frac{1}{2}(1250)v^2$ $v^2 = 328$ $v = 18\text{ms}^{-1}$ (2 s.f.)
b) Calculate the change in GPE.	$\Delta GPE =$ final GPE $-$ initial GPE $= 1250(9.8)(20) - 0$ $= 245000\text{J}$ $= 245\text{kJ}$

Power

Power is defined by the rate of doing work. When the force applied is constant, average power can be calculated by:

$$\text{average power} = \frac{\text{work done}}{\text{time taken}} = \frac{Fd}{t}$$

When the force or speed is not constant, the power at a specific point can be found by:

$$\text{power} = \text{tractive force} \times \text{speed}$$

The unit of power is Watts (W), which is equivalent to Joule per second (Js^{-1}).

Example 6: The engine of a car has a driving force of 1400N. Given that the car travelled 375m in 15 seconds, find the average power.

Use the formula average power $= \frac{Fd}{t}$.	power $= \frac{1400 \times 375}{15}$ $= 35000\text{W}$

Example 7: A driver is driving a car on a straight horizontal road. The combined mass of the car and the driver is 1300kg. At a power of 20kW, the car is travelling at a constant speed of 72km/h. a) Find the resistance to motion. b) Assuming that the resistance to motion is constant, find the acceleration when the driver increases the power rating to 25kW.

a) At a constant speed, the tractive force is equivalent to the resistive force. Find the tractive force using power (in Watts) and speed (in ms^{-1}).	Resistance $=$ tractive force $= \frac{\text{power}}{\text{speed}}$ $20\text{kW} = 20000\text{W}$ $72\text{km h}^{-1} = (72 \div 3600 \times 1000)\text{m s}^{-1}$ $= 20\text{m s}^{-1}$ Resistance $= \frac{20000}{20}$ $= 1000\text{N}$
b) Find the tractive force at 25kW.	Tractive force $= \frac{25000}{20}$ $= 1250\text{N}$
Find the resultant force.	$F =$ tractive force $-$ resistance $= 1250 - 1000$ $= 250\text{N}$
Find the acceleration using $F = ma$.	$250 = 1300a$ $a = \frac{250}{1300}$ $= 0.192\text{m s}^{-2}$