Momentum and Collisions II Cheat Sheet

Impulse

The impulse for a constant force is defined as the product of force and time, or a change in momentum. The unit for impulse is Newton second (Ns).

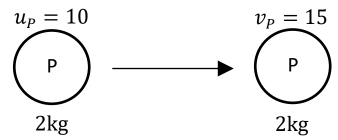
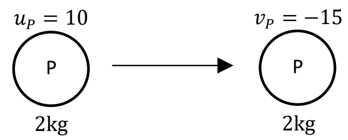
$$I = Ft = \Delta p = mv - mu$$

Impulse is a vector quantity, so it is important to keep in mind the direction when doing calculations.

Example 1: A particle P has mass $m$kg and is travelling along a smooth horizontal surface at a speed of $u\text{ms}^{-1}$. A constant force of magnitude F N is applied for t seconds in its direction of motion. Prove that $Ft = mv - mu$, where v is the speed of the particle after the force has been applied.

At constant mass and force, acceleration is also constant according to Newton's second law.	$F = ma$
Substitute $a = \frac{v-u}{t}$ into the equation.	$F = \frac{m(v-u)}{t}$ $Ft = m(v-u)$ $= mv - mu$

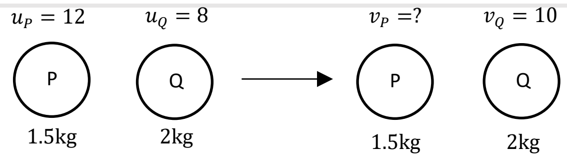
Example 2: A particle P of mass 2kg is travelling at a speed of 10ms^{-1}. An impulse is given to the particle and after the impulse P moves at a speed of 15ms^{-1}. Find the impulse when **a.)** the particle continues to move in the original direction and when **b.)** the direction of travel of the particle is reversed.

a.) Draw a simple diagram, taking the direction towards the right as the positive.	
Find the impulse using the change in momentum.	$I = mv - mu$ $= 2(15) - 2(10)$ $= 30 - 20$ $= 10\text{Ns}$
b.) Draw a simple diagram, taking the direction towards the right as the positive.	
Find the impulse using the change in momentum.	$I = mv - mu$ $= 2(-15) - 2(10)$ $= -30 - 20$ $= -50\text{Ns}$ Note that the impulse is negative, which means the force was applied in the opposite direction as the original motion of the particle.

Impulse in Collisions

When two particles collide, the same approach can be used to find the impulse. The impulse for each particle will be equal as there is no change in total momentum, but they will be in opposite direction as the forces exerted on each particle are in opposite direction.

Example 3: A particle P with a mass of 1.5kg is travelling along a smooth horizontal surface at a speed of 12ms^{-1}. It collides with particle Q, which has a mass of 2kg and is travelling in the same direction at 8ms^{-1}. Given that Q continues to travel in the same direction at 10ms^{-1} after the collision, find **a.)** the impulse on P and Q and **b.)** the velocity of P after the collision.

a.) Draw a simple diagram, taking the direction towards the right as the positive.	
Find the impulse on Q as the initial and final velocities are given	$I = mv - mu$ $= 2(10) - 2(8)$ $= 4\text{Ns}$
Find the impulse on P, which is equal and opposite from the impulse on Q.	$I = -4\text{Ns}$
b.) Find the final velocity of P using $I = mv - mu$.	$-4 = 1.5v - 1.5(12)$ $1.5v = -4 + 18$ $= 14$ $v = 9.33\text{ms}^{-1}(3\text{s.f.})$

Vector Notation

Since impulse is a vector, it can be written in the from $I = ai + bj$ or $I = \binom{a}{b}$. Force and velocity can also be written using the same forms.

Example 4: A particle P of mass 4kg is travelling along a smooth horizontal surface with velocity $(3i + 4j)\text{ms}^{-1}$ when a constant force of $F = (2i + 3j)$N is applied to the particle for 5 seconds. Find **a.)** the impulse on P and **b.)** the final velocity of P.

a.) Find the impulse using $I = Ft$.	$I = (2i + 3j)(5)$ $= (10i + 15j)\text{Ns}$
b.) Find the final velocity of P using $I = mv - mu$.	$10i + 15j = 4v - 4(3i + 4j)$ $= 4v - 12i - 16j$ $22i + 31j = 4v$ $v = \left(\frac{11}{2}i + \frac{31}{4}j\right)\text{ms}^{-1}$

Impulse for Variable Forces

When a variable force N is applied to an object from time t_1 to t_2, the impulse is given by:

$$I = \int_{t_1}^{t_2} F\,dt$$

In other words, the impulse is the area under graph in a force-time graph.

Example 5: A particle P of mass 3kg is travelling along a smooth horizontal surface with a speed of 8ms^{-1} when a variable force of $F = (3t^2 - 8t - 6)$N is applied in the direction of motion. Find the impulse for the first 10 seconds after the force is applied and the speed of the particle after 10 seconds.

Find I using $I = \int_{t_1}^{t_2} F\,dt$, taking $t_1 = 0$ and $t_2 = 10$ as the limits.	$I = \int_0^{10} 3t^2 - 8t - 6\,dt$ $= \left[\frac{3t^3}{3} - \frac{8t^2}{2} - 6t\right]_0^{10}$ $= [t^3 - 4t^2 - 6t]_0^{10}$ $= 10^3 - 4(10)^2 - 6(10) - 0$ $= 1000 - 400 - 60$ $= 540\text{Ns}$
Use $I = mv - mu$ to find v.	$540 = 3v - 3(8)$ $540 + 24 = 3v$ $3v = 564$ $v = 188\text{ms}^{-1}$