Circular Motion I Cheat Sheet

Circular Motion with a Constant Speed

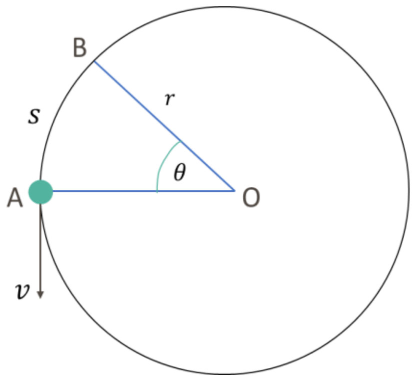
Velocity is a vector quantity so has magnitude and direction. The direction of velocity of a particle as it moves in a circle is constantly changing. To work out how the particle moves over time, consider how the angle θ at which the particle is at to some reference axis changes with time.

Consider a particle travelling in a circular path from A to B at a constant angular speed**,** ω. It is at an angle θ, which is measured in radians anticlockwise from the radius OB. The angular speed of the particle in a circular path, measured in radians per second, is defined as the rate of change of angle θ with respect to time, t:

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

Note that angular velocity can be written as ω or as a time derivative $\dot{\theta}$.

The distance the particle has travelled is equivalent to the arc length of the circle given by $s = r\theta$. The linear speed of the particle, v, is the rate of change of s with respect to time. This relation can be seen below.



$$v = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt}$$

$$v = = r\dot{\theta} = r\omega$$

Example 1: A conker is attached to a light, inextensible string of length 16cm and spun on a horizontal axis centred at an origin O. The frequency of the spins is constant at a rate of 90 revolutions per minute. Find:

- **a)** the number of revolutions that conker completes in 20 seconds.
- **b)** the linear speed of the particle.

a) Find out the number of revolutions per second by using 1 minute = 60 seconds. This can then be used to determine the answer.	$90 \div 60 = 1.5$ revolutions per second $0.5 \times 20 = 30$ The conker is spun 30 times in 20 seconds.
b) To find the linear speed, you should first find the angular speed which is determined from the angle the conker turns through per second. The result from part a) can be used to find how big θ after 1 second of travel.	$2\pi \times 1.5 = 3\pi$ radians are rotated in one second by the conker $\omega = 9.4 \text{ rads}^{-1}$
The relationship between the angular speed and linear speed is $v = r\,\omega$. You can substitute in the known values to find the linear speed. Make sure the values are in the correct units by converted centimetres to metres: $1\text{m} = 100\text{cm}$.	$v = r\omega$ $v = 0.16 \times 9.4 = 1.5 \text{ ms}^{-1}$

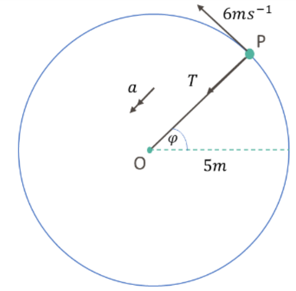
Acceleration and the Centripetal Force

For an object to move in a circular path, it must be subjected to a resultant force called the centripetal force that keeps it moving in a circle. The centripetal force is the one that acts towards the centre of the circle. This is due to Newton's second law of motion which states any object accelerating must be subjected to an external force.
As a particle on a circular path has a changing velocity, it therefore must be accelerating, and this is due to the centripetal force. If the resultant force did not exist, the particle would stop turning and move off at a tangent.

The acceleration is given by the formula $a = v\omega$ and can be rewritten as:

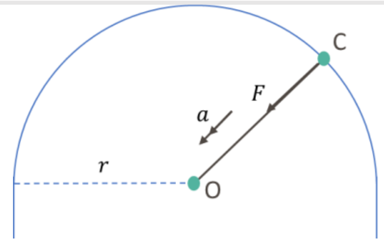
$$a = \omega^2 r = \frac{v^2}{r}$$

Example 2: On a theme park ride, a person sits in a seat attached to a uniform rod of length 4m. The other end is attached to a fixed-point O and the person and seat are rotated in a circular path with a linear speed of 6ms^{-1} . The total mass of the person and seat is 115kg. Calculate the tension in the rod. You may assume the person and seat can be modelled as a particle.

Sketch a diagram. The tension T in the string acts towards the centre of the circle. This is the resultant force in this question.	
Set up the equation for tension using Newton's Second law: $F = ma$. You can see there is an unknown value a.	$T = ma$
You can determine the acceleration a using the formula $a = \frac{v^2}{r}$. Then substitute the result back into the tension formula.	$a = \frac{v^2}{r} = \frac{(6)^2}{4} = 9 \text{ ms}^{-2}$ $m = 115\text{kg}$ $T = 115 \times 9 = 1035 \text{ N}$

Example 3: A car of mass 1500kg is racing on a horizontal racetrack and approaches a corner which bends as the arc of a circle with a radius 55m. Just before reaching the bend, the car was travelling at a maximum linear speed. Find the centripetal force when the car is travelling with:

- **a.)** a linear speed of 20 kmh^{-1}
- **b.)** an angular speed of $\frac{\pi}{25} \text{ rad s}^{-1}$

a.) Sketch a diagram and label it. In this diagram, F is the centripetal force, r is the radius, a is the acceleration. The position C is the location of the car and O is the centre of the circle. It is important to note that the centripetal force acts towards the centre of the circle and the radial acceleration also acts in the same direction.	
The unit of linear speed need to be converted into metres per second. This is done by using the relationships: $1\text{km} = 1000 \text{ m}$ and $1 \text{ hour} = 3600$ seconds. Set up the equation for tension using Newton's Second law: $F = ma$.	$20 \times \frac{1000}{3600} = 5.6\text{ms}^{-1}$ $a = \frac{v^2}{r} = \frac{(5.6)^2}{55} = 0.57\text{ms}^{-2}$ $F = ma = 1500 \times 0.57 = 860 \text{ N (2s.f.)}$
b.) This section of the question is answered in the same manner as part **a**, yet not you need to use the equation for a that includes ω.	$a = \omega^2 r = \left(\frac{\pi}{25}\right)^2 \times 55 = 0.87 \text{ ms}^{-2}$ $F = ma = 1500 \times 0.87 = 1300 \text{ N (2s.f.)}$