Edexcel Further Maths AS-level

Core Pure

Formula Sheet

Provided in formula book

Not provided in formula book

Complex Numbers

Rectangular coordinate form	$z = x + i\,y$
Exponential form	$z = re^{i\theta}$ $r = \lvert z\rvert = \sqrt{x^2 + y^2}$ $\theta = \arg(z) = \tan^{-1}\frac{y}{x}$

Operations and Identities

For any two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:	$z_1 \cdot z_2 = (r_1 \cdot r_2)e^{i(\theta_1+\theta_2)}$
	$\frac{z_1}{z_2} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$

$\lvert z_1\rvert\lvert z_2\rvert = \lvert z_1 z_2\rvert$
$\left\lvert\frac{z_1}{z_2}\right\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$
$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
$\arg(\frac{z_1}{z_2}) = \arg(z_1) - \arg(z_2)$

Matrices

Elementary Operations

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{pmatrix}$$

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Matrix Multiplication

Matrices can be multiplied together if the number of columns in the first matrix is equal to the number of rows in the second matrix.

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

Determinant

For a 2×2 matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:	$\det M = ad - bc$
For a 3×3 matrix $M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$:	$\det(M) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

Singular matrix	$\det M = 0$
Non-singular matrix	$\det M \neq 0$

Inverse Matrices

$$M^{-1}M = MM^{-1} = I$$

If $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $M^{-1} = \frac{1}{\det M}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

For non-singular matrices A and B:	$(AB)^{-1} = B^{-1}A^{-1}$

Summations and Series

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^r}{r!}f^{(r)}(0) + \cdots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \cdots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r+1}\frac{x^r}{r} + \cdots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \cdots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r \frac{x^{2r}}{(2r)!} + \cdots \text{ for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^r \frac{x^{2r+1}}{2r+1} + \cdots \quad (-1 \leq x \leq 1)$$

Calculus

Differentiation

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\text{sech}^2\, x$
$\text{arsinh}\, x$	$\frac{1}{\sqrt{1+x^2}}$
$\text{arcosh}\, x$	$\frac{1}{\sqrt{x^2-1}}$
$\text{artanh}\, x$	$\frac{1}{1-x^2}$

Integration ($+$ *constant*, $a > 0$ **where appropriate)**

$f(x)$	$\int f(x)\ dx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\text{arcosh}\left(\frac{x}{a}\right), \quad \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\text{arsinh}\left(\frac{x}{a}\right), \quad \ln\{x + \sqrt{x^2 + a^2}\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a + x}{a - x}\right\rvert = \frac{1}{a}\text{artanh}\left(\frac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x - a}{x + a}\right\rvert$

Vectors

Vector product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \sin\theta\, \hat{\boldsymbol{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$
Scalar product	$\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Cartesian and Vector Equations

If A is the point with position vector $\boldsymbol{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\boldsymbol{b}$ is given by $\boldsymbol{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\boldsymbol{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \; (= \lambda)$$

The plane through A with normal vector $\boldsymbol{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation $n_1x + n_2y + n_3z + d = 0$ where $d = -\boldsymbol{a}.\boldsymbol{n}$

The plane through non-collinear points A, B and C has vector equation $\boldsymbol{r} = \boldsymbol{a} + \lambda(\boldsymbol{b} - \boldsymbol{a}) + \mu(\boldsymbol{c} - \boldsymbol{a}) = (1 - \lambda - \mu)\boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$

The plane through the point with position vector $\boldsymbol{a}$ and parallel to $\boldsymbol{b}$ and $\boldsymbol{c}$ has equation $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b} + t\boldsymbol{c}$

Calculating Distances

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$	$\dfrac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d\rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$