# Volumes of Revolution (CP2)

## Questions

**Q1.**

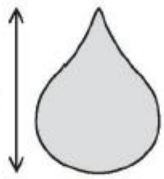


**Figure 2**

Figure 2 shows the image of a gold pendant which has height 2 cm. The pendant is modelled by a solid of revolution of a curve *C* about the *y*-axis. The curve *C* has parametric equations

$$x = \cos\theta + \frac{1}{2}\sin 2\theta, \quad y = -(1 + \sin\theta) \qquad 0 \leqslant \theta \leqslant 2\pi$$

(a) Show that a Cartesian equation of the curve *C* is

$$x^2 = -(y^4 + 2y^3)$$

**(4)**

(b) Hence, using the model, find, in $cm^3$, the volume of the pendant.

**(4)**

**(Total for question = 8 marks)**

**Q2.**

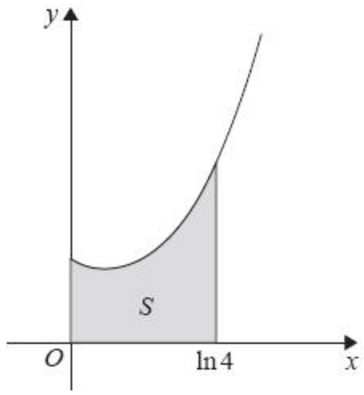


Diagram not drawn to scale

**Figure 2**

The finite region $S$, shown shaded in Figure 2, is bounded by the $y$-axis, the $x$-axis, the line with equation $x = \ln 4$ and the curve with equation

$$y = e^x + 2e^{-x}, \qquad x \geq 0$$

The region $S$ is rotated through $2\pi$ radians about the $x$-axis.

Use integration to find the exact value of the volume of the solid generated.
Give your answer in its simplest form.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

**(7)**

**(Total for question = 7 marks)**

**Q3.**

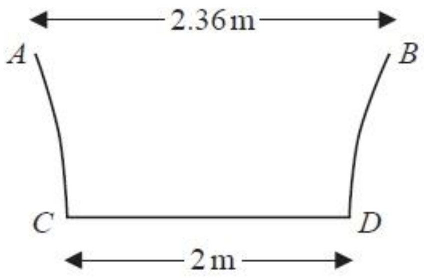


**Figure 1**

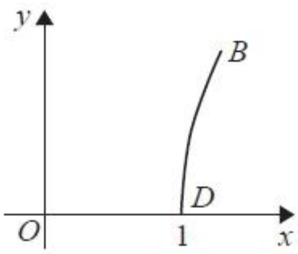


**Figure 2**

Figure 1 shows the central vertical cross section *ABCD* of a paddling pool that has a circular horizontal cross section. Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

Using these measurements, the curve *BD* is modelled by the equation

$$y = \ln(3.6x - k) \qquad 1 \leqslant x \leqslant 1.18$$

as shown in Figure 2.

(a) Find the value of *k*.

**(1)**

(b) Find the depth of the paddling pool according to this model.

**(2)**

The pool is being filled with water from a tap.

(c) Find, in terms of *h*, the volume of water in the pool when the pool is filled to a depth of *h* m.

**(5)**

Given that the pool is being filled at a constant rate of 15 litres every minute,

(d) find, in cm $h^{-1}$, the rate at which the water level is rising in the pool when the depth of the water is 0.2 m.

**(3)**

**(Total for question = 11 marks)**

**Q4.**

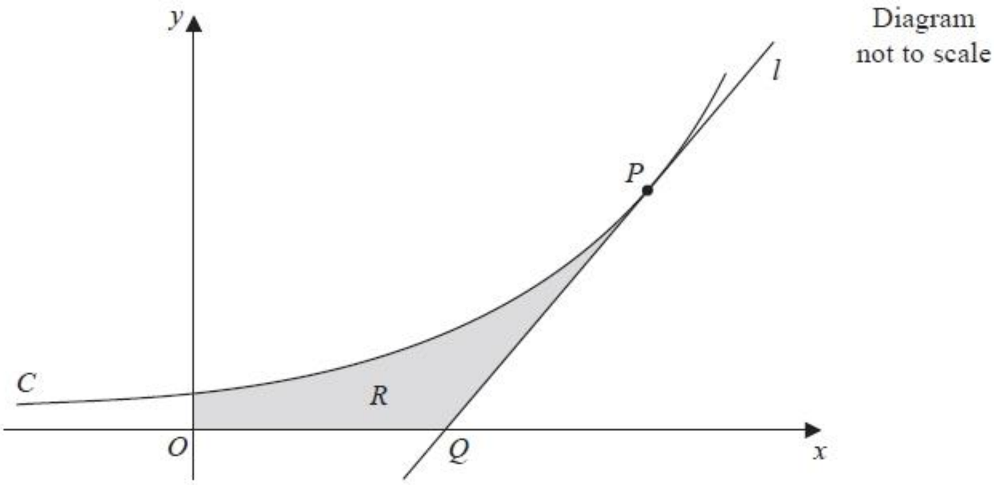


**Figure 3**

Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = 3^x$$

The point $P$ lies on $C$ and has coordinates (2, 9).

The line $l$ is a tangent to $C$ at $P$. The line $l$ cuts the $x$-axis at the point $Q$.

(a) Find the exact value of the $x$ coordinate of $Q$.

**(4)**

The finite region $R$, shown shaded in Figure 3, is bounded by the curve $C$, the $x$-axis, the $y$-axis and the line $l$. This region $R$ is rotated through 360° about the $x$-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$ where $p$ and $q$ are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

**(6)**

**(Total for question = 10 marks)**

# <u>Mark Scheme</u> – Volumes of Revolution (CP2)

**Q1.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $x = \cos\theta + \sin\theta\cos\theta = -y\cos\theta$ | M1 | 2.1 |
| | $\sin\theta = -y - 1$ | M1 | 2.1 |
| | $\left(\frac{x}{-y}\right)^2 = 1 - (-y-1)^2$ | M1 | 2.1 |
| | $x^2 = -(y^4 + 2y^3)$ * | A1* | 1.1b |
| | | **(4)** | |
| (b) | $V = \pi\int x^2\,dy = \pi\int -(y^4 + 2y^3)\,dy$ | M1 | 3.4 |
| | $= \pi\left[-\left(\frac{y^5}{5} + \frac{y^4}{2}\right)\right]$ | A1 | 1.1b |
| | $= -\pi\left[\left(\frac{(0)^5}{5} + \frac{(0)^4}{2}\right) - \left(\frac{(-2)^5}{5} + \frac{(-2)^4}{2}\right)\right]$ | M1 | 3.4 |
| | $= 1.6\pi\ \text{cm}^3$ or awrt $5.03\ \text{cm}^3$ | A1 | 1.1b |
| | | **(4)** | |
| | | **(8 marks)** | |

**Notes:**

(a)
**M1**: Obtains $x$ in terms of $y$ and $\cos\theta$
**M1**: Obtains an equation connecting $y$ and $\sin\theta$
**M1**: Uses Pythagoras to obtain an equation in $x$ and $y$ only
**A1***: Obtains printed answer

(b)
**M1**: Uses the correct volume of revolution formula with the given expression
**A1**: Correct integration
**M1**: Correct use of correct limits
**A1**: Correct volume

**Q2.**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | $y = e^x + 2e^{-x},\ x \geq 0$ | | |
| **Way 1** | $\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$ | For $\pi\int\left(e^x + 2e^{-x}\right)^2$. Ignore limits and $dx$. Can be implied. | B1 |
| | $= \{\pi\}\int_0^{\ln 4}\left(e^{2x} + 4e^{-2x} + 4\right)dx$ | Expands $\left(e^x + 2e^{-x}\right)^2 \to \pm\alpha e^{2x} \pm \beta e^{-2x} \pm \delta$ where $\alpha, \beta, \delta \neq 0$. Ignore $\pi$, integral sign, limits and $dx$. This can be implied by later work. | M1 |
| | $= \{\pi\}\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$ | Integrates at least one of either $\pm\alpha e^{2x}$ to give $\pm\frac{\alpha}{2}e^{2x}$ or $\pm\beta e^{-2x}$ to give $\pm\frac{\beta}{2}e^{-2x}$ $\alpha, \beta \neq 0$ | M1 |
| | | **dependent on the 2nd M mark** $e^{2x} + 4e^{-2x} \to \frac{1}{2}e^{2x} - 2e^{-2x}$, which can be simplified or un-simplified | A1 |
| | | $4 \to 4x$ or $4e^0x$ | B1 cao |
| | $= \{\pi\}\left(\left(\frac{1}{2}e^{2(\ln 4)} - 2e^{-2(\ln 4)} + 4(\ln 4)\right) - \left(\frac{1}{2}e^0 - 2e^0 + 4(0)\right)\right)$ | **dependent on the previous method mark.** Some evidence of applying limits of $\ln 4$ o.e. and 0 to a changed function in $x$ and subtracts the correct way round. **Note:** A proper consideration of the limit of 0 is required. | dM1 |
| | $= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$ | | |
| | $= \frac{75}{8}\pi + 4\pi\ln 4$ or $\frac{75}{8}\pi + 8\pi\ln 2$ or $\pi\left(\frac{75}{8} + 4\ln 4\right)$ or $\pi\left(\frac{75}{8} + 8\ln 2\right)$ or $\frac{75}{8}\pi + \ln 2^{8\pi}$ or $\frac{75}{8}\pi + \pi\ln 256$ or $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ or $\frac{1}{8}\pi(75 + 32\ln 4)$, etc | | A1 isw |
| | | | [7] |
| | | | 7 |

| | | Question Notes |
|---|---|---|
| | **Note** | $\pi$ is only required for the 1st B1 mark and the final A1 mark. |
| | **Note** | Give 1st B0 for writing $\pi\int y^2\,dx$ followed by $2\pi\int\left(e^x + 2e^{-x}\right)^2 dx$ |
| | **Note** | Give 1st M1 for $\left(e^x + 2e^{-x}\right)^2 \to e^{2x} + 4e^{-2x} + 2e^0 + 2e^0$ because $\delta = 2e^0 + 2e^0$ |
| | **Note** | A decimal answer of 46.8731... or $\pi(14.9201...)$ (without a correct **exact** answer) is A0 |
| | **Note** | $\pi\left[\frac{1}{2}e^{2x} - 2e^{-2x} + 4x\right]_0^{\ln 4}$ followed by awrt 46.9 (without a correct **exact** answer) is final dM1A0 |
| | **Note** | Allow exact equivalents which should be in the form $a\pi + b\pi\ln c$ or $\pi(a + b\ln c)$, where $a = \frac{75}{8}$ or $9\frac{3}{8}$ or 9.375. Do not allow $a = \frac{150}{16}$ or $9\frac{6}{16}$ |
| | **Note** | Give B1M0M1A1B0M1A0 for the common response $\pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx \to \pi\int_0^{\ln 4}\left(e^{2x} + 4e^{-2x}\right)dx = \pi\left[\frac{1}{2}e^{2x} - 2e^{-2x}\right]_0^{\ln 4} = \frac{75}{8}\pi$ |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **Way 2** | $y = e^x + 2e^{-x},\ x \geq 0$ | | |
| | $\{V =\}\ \pi\int_0^{\ln 4}\left(e^x + 2e^{-x}\right)^2 dx$ | For $\pi\int\left(e^x + 2e^{-x}\right)^2$ Ignore limits and $dx$. Can be implied. | B1 |
| | $u = e^x \Rightarrow \frac{du}{dx} = e^x = u$ and $x = \ln 4 \Rightarrow u = 4, x = 0 \Rightarrow u = e^0 = 1$ | | |
| | $V = \{\pi\}\int_1^4\left(u + \frac{2}{u}\right)^2 \frac{1}{u}du = \{\pi\}\int_1^4\left(u^2 + \frac{4}{u^2} + 4\right)\frac{1}{u}du$ | | |
| | $= \{\pi\}\int_1^4\left(u + \frac{4}{u^3} + \frac{4}{u}\right)du$ | $\left(e^x + 2e^{-x}\right)^2 \rightarrow \pm\alpha u \pm \beta u^{-3} \pm \delta u^{-1}$ where $u = e^x, \alpha, \beta, \delta \neq 0$. Ignore $\pi$, integral sign, limits and $du$. This can be implied by later work. | <u>M1</u> |
| | $= \{\pi\}\left[\frac{1}{2}u^2 - \frac{2}{u^2} + 4\ln u\right]_1^4$ | Integrates at least one of either $\pm\alpha u$ to give $\pm\frac{\alpha}{2}u^2$ or $\pm\beta u^{-3}$ to give $\pm\frac{\beta}{2}u^{-2}$ $\alpha, \beta \neq 0$, where $u = e^x$ | M1 |
| | | **dependent on the 2nd M mark** $u + 4u^{-3} \rightarrow \frac{1}{2}u^2 - 2u^{-2}$, simplified or un-simplified, where $u = e^x$ | A1 |
| | | $4u^{-1} \rightarrow 4\ln u$, where $u = e^x$ | B1 cao |
| | $= \{\pi\}\left(\left(\frac{1}{2}(4)^2 - \frac{2}{(4)^2} + 4\ln 4\right) - \left(\frac{1}{2}(1)^2 - \frac{2}{(1)^2} + 4\ln 1\right)\right)$ | **dependent on the previous method mark.** Some evidence of applying limits of 4 and 1 to a changed function in $u$ [or $\ln 4$ o.e. and 0 to an integrated function in $x$] and subtracts the correct way round. | dM1 |
| | $= \{\pi\}\left(\left(8 - \frac{1}{8} + 4\ln 4\right) - \left(\frac{1}{2} - 2\right)\right)$ | | |
| | $= \frac{75}{8}\pi + 4\pi\ln 4$ **or** $\frac{75}{8}\pi + 8\pi\ln 2$ **or** $\pi\left(\frac{75}{8} + 4\ln 4\right)$ **or** $\pi\left(\frac{75}{8} + 8\ln 2\right)$ **or** $\frac{75}{8}\pi + \ln 2^{8\pi}$ **or** $\frac{75}{8}\pi + \pi\ln 256$ **or** $\ln\left(2^{8\pi}e^{\frac{75}{8}\pi}\right)$ **or** $\frac{1}{8}\pi(75 + 32\ln 4)$, etc | | A1 isw |
| | | | [7] |

**Q3.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $k = 2.6$ | B1 | 3.4 |
| | | (1) | |
| (b) | $x = 1.18 \Rightarrow \ln(3.6 \times 1.18 - "2.6") = ...$ | M1 | 1.1b |
| | $h = 0.4995...$ m | A1 | 2.2b |
| | | (2) | |
| (c) | $y = \ln(3.6x - 2.6) \Rightarrow x = \frac{e^y + 2.6}{3.6}$ or $\frac{5e^y + 13}{18}$ | B1ft | 1.1a |
| | $V = \pi \int \left(\frac{e^y + 2.6}{3.6}\right)^2 dy = \frac{\pi}{3.6^2} \int (e^{2y} + 5.2e^y + 6.76) dy$ or $\frac{\pi}{324} \int (25e^{2y} + 130e^y + 169) dy$ | M1 | 3.3 |
| | $= \frac{\pi}{3.6^2}\left[\frac{1}{2}e^{2y} + 5.2e^y + 6.76y\right]$ $\left(\text{or } \frac{\pi}{324}\left[\frac{25}{2}e^{2y} + 130e^y + 169y\right]\right)$ | A1 | 1.1b |
| | $= \frac{\pi}{3.6^2}\left\{\left(\frac{1}{2}e^{2h} + 5.2e^h + 6.76h\right) - \left(\frac{1}{2}e^0 + 5.2e^0 + 6.76(0)\right)\right\}$ or e.g. $= \frac{\pi}{324}\left\{\left(\frac{25}{2}e^{2h} + 130e^h + 169h\right) - \left(\frac{25}{2}e^0 + 130e^0 + 6.76(0)\right)\right\}$ | M1 | 2.1 |
| | $= \frac{\pi}{3.6^2}\left(\frac{1}{2}e^{2h} + 5.2e^h + 6.76h - 5.7\right)$ | A1 | 1.1b |
| | | (5) | |
| (d) | $\frac{dV}{dh} = \frac{\pi}{3.6^2}(e^{2h} + 5.2e^h + 6.76) = \frac{\pi}{3.6^2}(e^{0.4} + 5.2e^{0.2} + 6.76)$ | M1 | 3.1a |
| | $\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt} = \frac{1}{3.539...} \times 0.015 \times 60$ | M1 | 1.1b |
| | $\frac{dh}{dt} = 25.4 \text{ cm h}^{-1}$ | A1 | 3.2a |
| | | (3) | |
| (d) Way 2 | $y = 0.2 \Rightarrow x = \frac{2.6 + e^{0.2}}{3.6} \Rightarrow A = \pi\left(\frac{2.6 + e^{0.2}}{3.6}\right)^2 (= 3.54)$ | M1 | 3.1a |
| | $\frac{dh}{dt} = \frac{0.015 \times 60}{3.54}$ | M1 | 1.1b |
| | $\frac{dh}{dt} = 25.4 \text{ cm h}^{-1}$ | A1 | 3.2a |
| | | | |
| | | | (11 marks) |

| Notes |
| --- |
| (a)<br>B1: Uses the model to obtain a correct value for $k$. Must be 2.6 not -2.6<br>(b)<br>M1: Substitutes their value of $k$ and $x = 1.18$ into the given model to find a value for $y$<br>A1: Infers that the depth of the pool could be awrt 0.5 m<br>(c)<br>B1ft: Uses the model to obtain $x$ correctly in terms of $y$ (follow through their $k$)<br>M1: Uses the model to obtain an expression for the volume of the pool using $\pi\int (\textit{their } f(y))^2 \, dy$ – must expand in order to reach an integrable form (allow poor squaring e.g. $(a + b)^2 = a^2 + b^2$. **Note that the $\pi$ may be recovered later.**<br>A1: Correct integration<br>M1: Selects limits appropriate to the model ($h$ and 0) substitutes and clearly shows the use of both limits (i.e. including zero)<br>A1: Correct expression **(allow unsimplified and isw if necessary)**<br>(d)<br>**Way 1**<br>M1: Recognises that $\frac{dV}{dh}$ is required and attempts to find $\frac{dV}{dh}$ or $\frac{dh}{dV}$ from their integration or using the earlier result (before integrating). Must clearly be identified as $\frac{dV}{dh}$ or $\frac{dh}{dV}$ unless this implied by subsequent work.<br>M1: Evidence of the correct use of the chain rule (ignore any confusion with units). Look for an attempt to divide 15 or their converted 15 by their $\frac{dV}{dh}$ or to multiply 15 or their converted 15 by $\frac{dh}{dV}$ **but must reach a <u>value</u> for** $\frac{dh}{dt}$ **<u>but you do not need to check their value.</u>**<br>A1: Interprets their solution correctly to obtain the correct answer (awrt 25.4) **with the correct units**<br>**Way 2**<br>M1: Uses $y = 0.2$ to find $x$ and the surface area of the water at that instant<br>M1: Attempts to divide the rate by their area (ignore any confusion with units)<br>A1: Interprets their solution correctly to obtain the correct answer (awrt 25.4) **with the correct units** |

**Q4.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $\left\{y=3^x \Rightarrow\right\} \frac{dy}{dx}=3^x\ln 3$ — $\frac{dy}{dx}=3^x\ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$ | B1 |
| | **Either T**: $y-9=3^2\ln 3(x-2)$ or **T**: $y=(3^2\ln 3)x+9-18\ln 3$, where $9=(3^2\ln 3)(2)+c$ — See notes | M1 |
| | {Cuts $x$-axis $\Rightarrow y=0\Rightarrow$} | |
| | $-9=9\ln 3(x-2)$ or $0=(3^2\ln 3)x+9-18\ln 3$, — Sets $y=0$ in their tangent equation and progresses to $x=\ldots$ | M1 |
| | So, $x=2-\frac{1}{\ln 3}$ — $2-\frac{1}{\ln 3}$ or $\frac{2\ln 3-1}{\ln 3}$ o.e. | A1 cso |
| | | [4] |
| (b) | $V=\pi\int\left(3^x\right)^2\{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$ — $V=\pi\int\left(3^x\right)^2$ with or without $dx$, which can be implied | B1 o.e. |
| | $=\{\pi\}\left(\frac{3^{2x}}{2\ln 3}\right)$ or $=\{\pi\}\left(\frac{9^x}{\ln 9}\right)$ — Eg: either $3^{2x}\to\frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x\to\frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$, $\alpha\in$ | M1 |
| | $3^{2x}\to\frac{3^{2x}}{2\ln 3}$ or $9^x\to\frac{9^x}{\ln 9}$ or $e^{2x\ln 3}\to\frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$ | A1 o.e. |
| | $\left\{V=\pi\int_0^2 3^{2x}\,dx=\{\pi\}\left[\frac{3^{2x}}{2\ln 3}\right]_0^2\right\}=\{\pi\}\left(\frac{3^4}{2\ln 3}-\frac{1}{2\ln 3}\right)\left\{=\frac{40\pi}{\ln 3}\right\}$ — **Dependent on the previous method mark.** Substitutes $x=2$ and $x=0$ and subtracts the correct way round. | dM1 |
| | $V_{cone}=\frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right)\left\{=\frac{27\pi}{\ln 3}\right\}$ — $V_{cone}=\frac{1}{3}\pi(9)^2(2-\text{their }(a))$. See notes. | B1ft |
| | $\left\{\text{Vol}(S)=\frac{40\pi}{\ln 3}-\frac{27\pi}{\ln 3}\right\}=\underline{\frac{13\pi}{\ln 3}}$ — $\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$ etc., isw | A1 o.e. |
| | $\{\text{Eg: } p=13\pi,\ q=\ln 3\}$ | [6] |
| | | 10 |
| (b) | **Alternative Method 1: Use of a substitution** | |
| | $V=\pi\int\left(3^x\right)^2\{dx\}$ | B1 o.e. |
| | $\left\{u=3^x\Rightarrow\frac{du}{dx}=3^x\ln 3=u\ln 3\right\}$ $V=\{\pi\}\int\frac{u^2}{u\ln 3}\{du\}=\{\pi\}\int\frac{u}{\ln 3}\{du\}$ | |
| | $=\{\pi\}\left(\frac{u^2}{2\ln 3}\right)$ — $\left(3^x\right)^2\to\frac{u^2}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)u^2$, where $u=3^x$ | M1 |
| | $\left(3^x\right)^2\to\frac{u^2}{2(\ln 3)}$, where $u=3^x$ | A1 |
| | $\left\{V=\pi\int_0^2\left(3^x\right)^2dx=\{\pi\}\left[\frac{u^2}{2\ln 3}\right]_1^9\right\}=\{\pi\}\left(\frac{9^2}{2\ln 3}-\frac{1}{2\ln 3}\right)\left\{=\frac{40\pi}{\ln 3}\right\}$ — Substitutes limits of 9 and 1 in $u$ (or 2 and 0 in $x$) and subtracts the correct way round. | dM1 |
| | *then apply the main scheme.* | |

| | | Question Notes |
|---|---|---|
| (a) | B1 | $\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$. Can be implied by later working. |
| | M1 | Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{dy}{dx}$ which is a function of $x$ or $y$ to find $m_T$ **and**<br>• **either** applies $y - 9 = \left(\text{their } m_T\right)\left(x - 2\right)$, where $m_T$ is a numerical value.<br>• **or** applies $y = \left(\text{their } m_T\right)x + \text{their } c$, where $m_T$ is a numerical value and $c$ is found by solving $9 = \left(\text{their } m_T\right)(2) + c$ |
| | Note | The first M1 mark can be implied from later working. |
| | M1 | Sets $y = 0$ in their *tangent* equation, where $m_T$ is a numerical value, (seen or implied) and progresses to $x = ...$ |
| | A1 | An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only. |
| | Note | Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$, where $\lambda$ is an integer, and ignore subsequent working. |
| | Note | Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$) is M0 M0 in part (a). |
| | Note | Candidates who invent a value for $m_T$ (which bears no resemblance to their gradient function) **cannot** gain the 1st M1 and 2nd M1 mark in part (a). |
| | Note | A decimal answer of 1.089760773… (without a correct **exact** answer) is A0. |
| (b) | B1 | A correct expression for the volume with or without $dx$ |
| | Note | **Eg:** Allow B1 for $\pi\int\left(3^x\right)^2\{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$ or $\pi\int\left(e^{x\ln 3}\right)^2\{dx\}$ or $\pi\int\left(e^{2x\ln 3}\right)\{dx\}$ or $\pi\int e^{x\ln 9}\{dx\}$ with or without $dx$ |
| | M1 | Either $3^{2x} \to \frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$<br>$e^{2x\ln 3} \to \frac{e^{2x\ln 3}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)e^{2x\ln 3}$ or $e^{x\ln 9} \to \frac{e^{x\ln 9}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)e^{x\ln 9}$, etc where $\alpha \in$ |
| | Note | $3^{2x} \to \frac{3^{2x+1}}{\pm\alpha(\ln 3)}$ or $9^x \to \frac{9^{x+1}}{\pm\alpha(\ln 3)}$ are allowed for M1 |
| | Note | $3^{2x} \to \frac{3^{2x+1}}{2x+1}$ or $9^x \to \frac{9^{x+1}}{x+1}$ are both M0 |
| | Note | M1 can be given for $9^{2x} \to \frac{9^{2x}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^{2x}$ |
| | A1 | Correct integration of $3^{2x}$. Eg: $3^{2x} \to \frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \to \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$ |
| | dM1 | **dependent on the previous method mark being awarded.**<br>Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round. |
| | Note | Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0. |

| | | |
|---|---|---|
| | dM1 | **dependent on the previous method mark being awarded.**<br>Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round. |
| | Note | Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0. |
| | B1ft | $V_{\text{cone}} = \frac{1}{3}\pi(9)^2\left(2 - \text{their answer to part }(a)\right)$.<br>Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark. |
| | Note | Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark. |
| | A1 | $\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$, etc. , where their answer is in the form $\frac{p}{q}$ |
| | Note | The $\pi$ in the volume formula is only needed for the 1st B1 mark and the final A1 mark. |
| | Note | A decimal answer of 37.17481128… (without a correct **exact** answer) is A0. |
| | Note | A candidate who applies $\int 3^x \mathrm{d}x$ will either get B0 M0 A0 M0 **B0** A0 or B0 M0 A0 M0 **B1** A0 |
| | Note | $\pi\int 3^{x^2}\,\mathrm{d}x$ unless recovered is B0. |
| | Note | **<u>Be careful! A correct answer may follow from incorrect working</u>**<br>$V = \pi\int_0^2 3^{x^2}\,\mathrm{d}x - \frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right) = \pi\left[\frac{3^{x^2}}{2\ln 3}\right]_0^2 - \frac{27\pi}{\ln 3} = \frac{\pi 3^4}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}$<br>would score B0 M0 A0 dM0 M1 A0. |
| (b) | **<u>2nd B1ft mark for finding the Volume of a Cone</u>** | |

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2}\left(9x\ln 3 - 18\ln 3 + 9\right)^2\mathrm{d}x$$

$$= \pi\left[\frac{\left(9x\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right]_{2-\frac{1}{\ln 3}\text{ or their part (a)answer}}^{2} \quad ****$$

Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

$$= \pi\left(\left(\frac{(18\ln 3 - 18\ln 3 + 9)^3}{27\ln 3}\right) - \left(\frac{\left(9\left(2 - \frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \pi\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{(18\ln 3 - 9 - 18\ln 3 + 9)^3}{27\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

(b)

**2nd B1ft mark for finding the Volume of a Cone**

**Alternative method 2:**

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2\,\mathrm{d}x$$

$$= \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(81x^2(\ln 3)^2 - 324x(\ln 3)^2 + 162x\ln 3 - 324\ln 3 + 324(\ln 3)^2 + 81\right)\mathrm{d}x$$

$$= \pi\left[27x^3(\ln 3)^2 - 162x^2(\ln 3)^2 + 81x^2\ln 3 - 324x\ln 3 + 324x(\ln 3)^2 + 81x\right]_{2-\frac{1}{\ln 3}}^{2}$$

Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

****

$$= \pi\left(\begin{array}{l}\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right)\\ \quad - \left(\begin{array}{l}27\left(2-\frac{1}{\ln 3}\right)^3(\ln 3)^2 - 162\left(2-\frac{1}{\ln 3}\right)^2(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)^2\ln 3\\ -324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\end{array}\right)\end{array}\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l}27\left(8 - \frac{12}{\ln 3} + \frac{6}{(\ln 3)^2} - \frac{1}{(\ln 3)^3}\right)(\ln 3)^2 - 162\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)(\ln 3)^2\\ + 81\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)\ln 3 - 324\left(2 - \frac{1}{\ln 3}\right)\ln 3\\ \quad + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2 - \frac{1}{\ln 3}\right)\end{array}\right)\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(\begin{array}{l}216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162\\ \quad +324\ln 3 - 324 + \frac{81}{\ln 3} - 648\ln 3 + 324\\ \quad + 648(\ln 3)^2 - 324\ln 3 + 162 - \frac{81}{\ln 3}\end{array}\right)\right)$$

$$= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$