FP3 Differential Equations

1. June 2010 qu.4

(i) Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{dy}{dx} - y = x\cos\left(\frac{y}{x}\right),$$ giving your answer in a form without logarithms. [6]

(ii) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. [2]

2. June 2010 qu.6

(i) Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36$. [7]

(ii) Show that, when x is large and positive, the solution approximates to a linear function, and state its equation. [2]

3. Jan 2010 qu. 3

Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{-3x}$$

for which $y = 1$ when $x = 0$. Express your answer in the form $y = f(x)$. [6]

4. Jan 2010 qu. 6

The variables x and y satisfy the differential equation $\frac{d^2y}{dx^2} + 16y = 8\cos 4x$.

(i) Find the complementary function of the differential equation. [2]

(ii) Given that there is a particular integral of the form $y = px\sin 4x$, where p is a constant, find the general solution of the equation. [6]

(iii) Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. [4]

5. June 2009 qu.4

The differential equation $\frac{dy}{dx} + \frac{1}{1-x^2}y = (1-x)^{\frac{1}{2}}$, where $|x| < 1$,

can be solved by the integrating factor method.

(i) Use an appropriate result given in the List of Formulae (MF1) to show that the integrating factor can be written as $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$. [2]

(ii) Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = f(x)$. [6]

6. June 2009 qu.5

The variables x and y satisfy the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$.

(i) Find the complementary function. [3]

(ii) Explain briefly why there is no particular integral of either of the forms $y = ke^{3x}$ or $y = kxe^{3x}$. [1]

(iii) Given that there is a particular integral of the form $y = kx^2e^{3x}$, find the value of k. [5]

7. Jan 2009 qu. 4

Find the general solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65 \sin 2x$. [9]

8. Jan 2009 qu. 5

The variables x and y are related by the differential equation $x^3\frac{dy}{dx} = xy + x + 1$. (A)

(i) Use the substitution $y = u - \frac{1}{x}$, where u is a function of x, to show that the differential equation may be written as $x^2\frac{du}{dx} = u$. [4]

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y = f(x)$. [5]

9. June 2008 qu.3

(i) Use the substitution $z = x + y$ to show that the differential equation

$\frac{dy}{dx} = \frac{x+y+3}{x+y-1}$ (A) may be written in the form $\frac{dz}{dx} = \frac{2(z+1)}{z-1}$. [3]

(ii) Hence find the general solution of the differential equation (A). [4]

10. June 2008 qu.8

(i) Find the complementary function of the differential equation $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. [2]

(ii) It is given that $y = p(\ln \sin x) \sin x + qx \cos x$, where p and q are constants, is a particular integral of this differential equation.

(a) Show that $p - 2(p + q)\sin^2 x \equiv 1$. [6]

(b) Deduce the values of p and q. [2]

(iii) Write down the general solution of the differential equation. State the set of values of x, in the interval $0 \le x \le 2\pi$, for which the solution is valid, justifying your answer. [3]

11. Jan 2008 qu. 2

Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 4x$. [7]

12. Jan 2008 qu. 5

(i) Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin 2x$,

expressing y in terms of x in your answer. [6]

In a particular case, it is given that $y = \frac{2}{\pi}$ when $x = \frac{1}{4}\pi$.

(ii) Find the solution of the differential equation in this case. [2]

(iii) Write down a function to which y approximates when x is large and positive. [1]

13. June 2007 qu.3

Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^{3x}$. [6]

14. June 2007 qu.8

(i) Find the general solution of the differential equation $\frac{dy}{dx} + y\tan x = \cos^3 x$

expressing y in terms of x in your answer. [8]

(ii) Find the particular solution for which $y = 2$ when $x = \pi$. [2]

15. Jan 2007 qu. 4

The variables x and y are related by the differential equation $\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$. (A)

(i) Use the substitution $y = xz$, where z is a function of x, to obtain the differential equation

$$x\frac{dz}{dx} = \frac{1 - 2z^2}{z}.$$ [3]

(ii) Hence show by integration that the general solution of the differential equation (A) may be expressed in the form $x^2(x^2 - 2y^2) = k$, where k is a constant. [6]

16. Jan 2007 qu. 6

The variables x and y satisfy the differential equation $\frac{dy}{dx} + 3y = 2x + 1$ Find

(i) the complementary function, [1]

(ii) the general solution. [5]

In a particular case, it is given that $\frac{dy}{dx} = 0$ when $x = 0$.

(iii) Find the solution of the differential equation in this case. [3]

(iv) Write down the function to which y approximates when x is large and positive. [1]

17. June 2006 qu.4

Find the solution of the differential equation $\frac{dy}{dx} - \frac{x^2 y}{1 + x^3} = x^2$

for which $y = 1$ when $x = 0$, expressing your answer in the form $y = f(x)$. [8]

18. June 2006 qu.6

(i) Find the general solution of the differential equation $\frac{d^2 y}{dx^2} + 4y = \sin x$. [6]

(ii) Find the solution of the differential equation for which $y = 0$ and $\frac{dy}{dx} = \frac{4}{3}$ when $x = 0$. [4]

19. Jan 2006 qu. 3

(i) By using the substitution $y^3 = z$, find the general solution of the differential equation

$3y^2 \frac{dy}{dx} + 2xy^3 = e^{-x^2}$, giving y in terms of x in your answer. [6]

(ii) Describe the behaviour of y as $x \to \infty$. [1]

20. Jan 2006 qu. 8

(i) Find the general solution of the differential equation $\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + 4x = 0$,

where k is a real constant, in each of the following cases,

(a) $|k| > 2$

(b) $|k| < 2$

(c) $k = 2$ [8]

(ii) (a) In the case when $k = 1$, find the solution for which $x = 0$ and $\frac{dx}{dt} = 6$ when $t = 0$. [4]

(b) Describe what happens to x as $t \to \infty$ in this case, justifying your answer. [2]