Edexcel Maths FP3

Topic Questions from Papers

Matrices

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3.

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(a) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(5)

(b) Find an eigenvector corresponding to the eigenvalue 7.

(4)

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blank

Question 3 continued

6.
$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$,

(a) find the eigenvalue of $\mathbf{M}$ corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$,

(2)

(b) show that $k = 3$,

(2)

(c) show that $\mathbf{M}$ has exactly two eigenvalues.

(4)

A transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by $\mathbf{M}$.

The transformation T maps the line l_1, with cartesian equations $\dfrac{x-2}{1} = \dfrac{y}{-3} = \dfrac{z+1}{4}$, onto the line l_2.

(d) Taking $k = 3$, find cartesian equations of l_2.

(5)

Leave blank

Question 6 continued

N 3 5 3 8 9 R A 0 1 7 2 8

7. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1$$

(a) Show that det $\mathbf{M} = 2 - 2k$.

(2)

(b) Find $\mathbf{M}^{-1}$, in terms of k.

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented

by the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$.

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(c) Find a vector equation for the line l_1.

(5)

Question 7 continued

P 3 5 4 1 4 A 0 2 1 2 8

8. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

(a) Show that 4 is an eigenvalue of **M**, and find the other two eigenvalues.

(5)

(b) For the eigenvalue 4, find a corresponding eigenvector.

(3)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **M**.

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(c) Find a vector equation for the line l_2.

(5)

Leave blank

Question 8 continued

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Q8

4. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},$$

where s and t are real parameters.

The plane Π_1 is transformed to the plane Π_2 by the transformation represented by the matrix $\mathbf{T}$, where

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find an equation of the plane Π_2 in the form $\mathbf{r} \cdot \mathbf{n} = p$

(9)

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blank

Question 4 continued

6. It is given that $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

and a and b are constants.

(a) Find the eigenvalue of $\mathbf{A}$ corresponding to the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

(3)

(b) Find the values of a and b.

(3)

(c) Find the other eigenvalues of $\mathbf{A}$.

(5)

Leave blank

Question 6 continued

5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & a \\ 2 & b & c \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}$$

(a) Given that **j** + **k** and **i** – **k** are two of the eigenvectors of **M**,

find

(i) the values of a, b and c,

(ii) the eigenvalues which correspond to the two given eigenvectors.

(8)

(b) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

Leave blank

Question 5 continued

P 4 3 1 4 3 A 0 1 3 2 8

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda:\mu$ is $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}=\begin{pmatrix} a_2b_3-a_3b_2 \\ a_3b_1-a_1b_3 \\ a_1b_2-a_2b_1 \end{pmatrix}$

$\mathbf{a.(b\times c)}=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=\mathbf{b.(c\times a)}=\mathbf{c.(a\times b)}$

If A is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\;(=\lambda)$$

The plane through A with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$$n_1x+n_2y+n_3z+d=0 \text{ where } d=-\mathbf{a.n}$$

The plane through non-collinear points A, B and C has vector equation

$$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation

$$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$$

The perpendicular distance of (α,β,γ) from $n_1x+n_2y+n_3z+d=0$ is $\dfrac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$.

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\text{arcosh } x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \qquad (x \geq 1)$$

$$\text{arsinh } x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\text{artanh } x = \tfrac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right) \qquad (|x| < 1)$$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \dfrac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\dfrac{a}{e}$	$x = -a$	$x = \pm\dfrac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\dfrac{x}{a} = \pm\dfrac{y}{b}$	$x = 0, y = 0$

Differentiation

f(*x*)	f′(*x*)
arcsin x	$\dfrac{1}{\sqrt{1-x^2}}$
arccos x	$-\dfrac{1}{\sqrt{1-x^2}}$
arctan x	$\dfrac{1}{1+x^2}$
sinh x	cosh x
cosh x	sinh x
tanh x	$\text{sech}^2\,x$
arsinh x	$\dfrac{1}{\sqrt{1+x^2}}$
arcosh x	$\dfrac{1}{\sqrt{x^2-1}}$
artanh x	$\dfrac{1}{1-x^2}$

Integration (+ *constant*; $a > 0$ *where relevant*)

f(*x*)	$\displaystyle\int$ f(*x*) d*x*
sinh x	cosh x
cosh x	sinh x
tanh x	ln cosh x
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\text{arcosh}\left(\dfrac{x}{a}\right)$, $\ln\left\{x+\sqrt{x^2-a^2}\right\}$ $(x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\text{arsinh}\left(\dfrac{x}{a}\right)$, $\ln\left\{x+\sqrt{x^2+a^2}\right\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\text{artanh}\left(\dfrac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \quad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \; ds = 2\pi \int y \sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)}\, dx$$

$$= 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4} n^2 (n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \ \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of $45°$.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ *constant*)

$f(x)$	$\int f(x)\ dx$
$\sec^2 kx$	$\dfrac{1}{k}\tan kx$
$\tan x$	$\ln\left|\sec x\right|$
$\cot x$	$\ln\left|\sin x\right|$
$\operatorname{cosec} x$	$-\ln\left|\operatorname{cosec} x + \cot x\right|, \quad \ln\left|\tan(\tfrac{1}{2}x)\right|$
$\sec x$	$\ln\left|\sec x + \tan x\right|, \quad \ln\left|\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\right|$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\displaystyle\int_a^b y \ dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Arithmetic series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$