Edexcel Maths FP3

Topic Questions from Papers

Integration

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4. Given that $y = \text{arsinh}\,(\sqrt{x}),\quad x > 0,$

(a) find $\dfrac{dy}{dx}$, giving your answer as a simplified fraction. **(3)**

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,dx,$$

giving your answer in the form $\ln\left(\dfrac{a+b\sqrt{5}}{2}\right)$, where a and b are integers. **(6)**

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5.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25-x^2)}}\,\mathrm{d}x\,, \qquad n \geqslant 0$$

(a) Find an expression for $\int \frac{x}{\sqrt{(25-x^2)}}\mathrm{d}x$, $\quad 0 \leqslant x \leqslant 5$.

(2)

(b) Using your answer to part (a), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2} \qquad n \geqslant 2$$

(5)

(c) Find I_4 in the form $k\pi$, where k is a fraction.

(4)

Leave blank

Question 5 continued

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8. A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The curve is rotated through 2π radians about the x-axis.

(a) Show that the area of the surface generated is given by the integral

$$k\pi\int_0^{\alpha} \sqrt{(16c^2 + 9)}\,\mathrm{d}c, \quad \text{where } c = \cos\theta,$$

and where k and α are constants to be found.

(6)

(b) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of your working and giving the final answer to 3 significant figures.

(5)

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Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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2. Use calculus to find the exact value of $\int_{-2}^{1} \frac{1}{x^2+4x+13}\,dx$.

(5)

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4. $I_n = \int_0^a (a - x)^n \cos x \, dx, \quad a > 0, \quad n \geqslant 0$

(a) Show that, for $n \geqslant 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(b) Hence evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx$.

(3)

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Question 4 continued

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Turn over

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1. The curve C has equation $y = 2x^3$, $0 \leqslant x \leqslant 2$.

The curve C is rotated through 2π radians about the x-axis.

Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.

(5)

Leave blank

3. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2-10x+34}\,\mathrm{d}x = k\pi$, giving the value of the fraction k,

(5)

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2-10x+34)}}\,\mathrm{d}x = \ln(A+\sqrt{n})$, giving the values of the integers A and n.

(4)

Leave blank

4.

$$I_n = \int_1^e x^2 (\ln x)^n \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$$

(4)

(b) Find the exact value of I_3.

(4)

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Question 4 continued

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2.

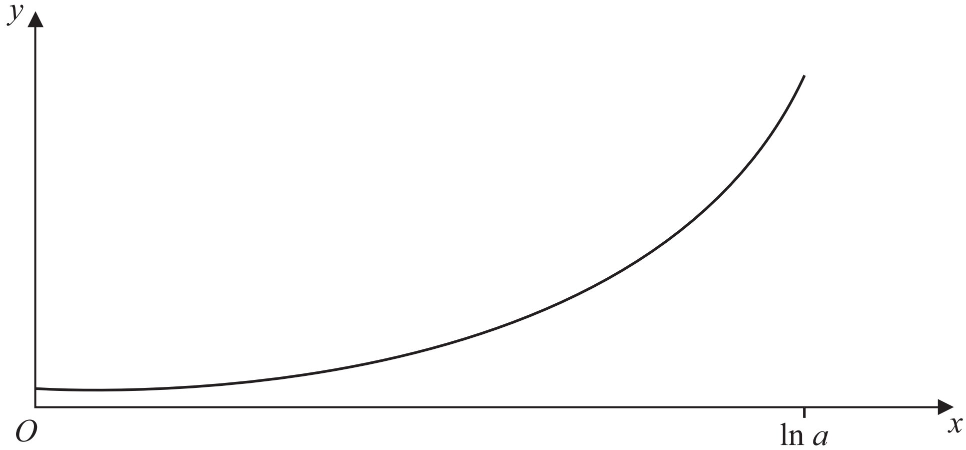

Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \leqslant x \leqslant \ln a$$

where a is a constant and $a > 1$

Using calculus, show that the length of curve C is

$$k\left(a^3 - \frac{1}{a^3}\right)$$

and state the value of the constant k.

(6)

Leave blank

4. $$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 2$,

$$I_n = \frac{1}{4}n\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)I_{n-2}$$

(5)

(b) Find the exact value of I_2

(4)

(c) Show that $I_4 = \frac{1}{64}(\pi^3 - 24\pi + 48)$

(2)

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Question 4 continued

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5. (a) Differentiate $x\,\text{arsinh}\,2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \text{arsinh}\,2x \; \mathrm{d}x$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

Leave blank

Question 5 continued

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7. $$f(x) = 5\cosh x - 4\sinh x, \qquad x \in \mathbb{R}$$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$ **(2)**

Hence

(b) solve $f(x) = 5$ **(4)**

(c) show that $\displaystyle\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,dx = \frac{\pi}{18}$ **(5)**

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Question 7 continued

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5.

$$I_n = \int_1^5 x^n (2x - 1)^{-\frac{1}{2}}\, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$(2n + 1)I_n = nI_{n-1} + 3 \times 5^n - 1$$

(5)

(b) Using the reduction formula given in part (a), find the exact value of I_2

(5)

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Question 5 continued

Turn over

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7.

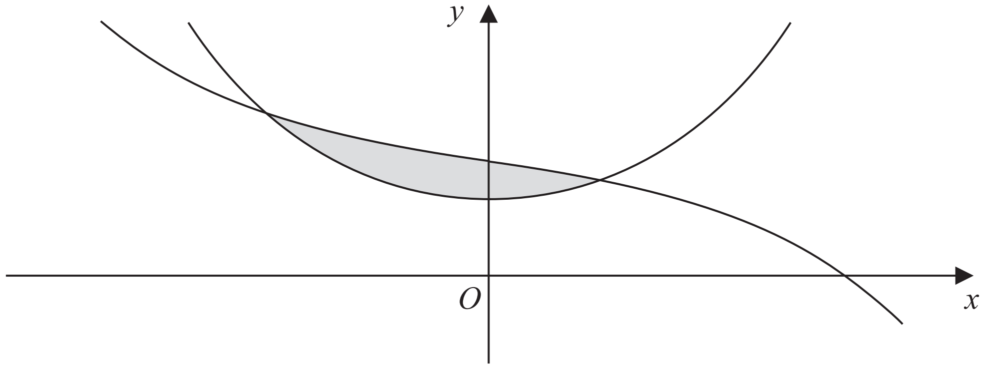

Figure 1

The curves shown in Figure 1 have equations

$$y = 6\cosh x \quad \text{and} \quad y = 9 - 2\sinh x$$

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)

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Question 7 continued

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8.

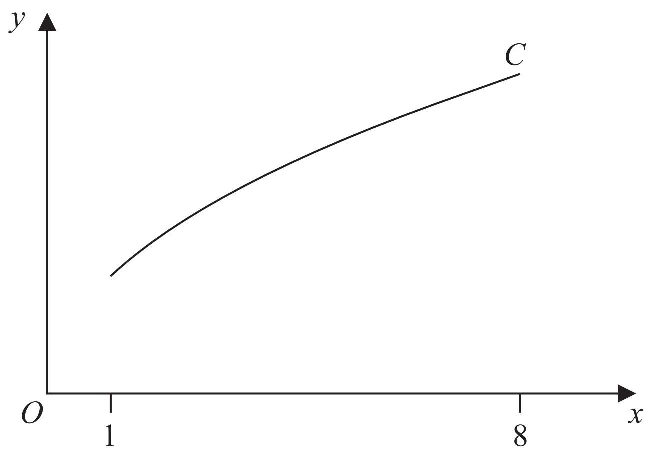

Figure 2

The curve C, shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \qquad 1 \leqslant x \leqslant 8$$

(a) Show that the length s of curve C is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)}\,\mathrm{d}x$$

(2)

(b) Using the substitution $x = \sinh^2 u$, or otherwise, find an exact value for s.

Give your answer in the form $a\sqrt{2} + \ln(b + c\sqrt{2})$ where a, b and c are integers.

(9)

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Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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2. (a) Find

$$\int \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

giving your answer in the form $k\ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

Leave blank

3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4 \sinh \theta, \quad 0 \leqslant \theta \leqslant 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

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6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,\mathrm{d}x, \quad n \geqslant 0,$$

(a) prove that, for $n \geqslant 2$,

$$(n + 2)I_n = 16(n - 1)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of I_5

(5)

Leave blank

Question 6 continued

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of **a** in the direction of **b** is $\frac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing *AB* in the ratio $\lambda:\mu$ is $\frac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{pmatrix}$

$$\mathbf{a.}(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\mathbf{b.}(\mathbf{c}\times\mathbf{a})=\mathbf{c.}(\mathbf{a}\times\mathbf{b})$$

If *A* is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\ (=\lambda)$$

The plane through *A* with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$$n_1x+n_2y+n_3z+d=0 \text{ where } d=-\mathbf{a.n}$$

The plane through non-collinear points *A*, *B* and *C* has vector equation

$$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$$

The perpendicular distance of (α,β,γ) from $n_1x+n_2y+n_3z+d=0$ is $\frac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$.

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\text{arcosh}\ x = \ln\{x + \sqrt{x^2 - 1}\} \qquad (x \geq 1)$

$\text{arsinh}\ x = \ln\{x + \sqrt{x^2 + 1}\}$

$\text{artanh}\ x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(*x*)	**f′(*x*)**
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\dfrac{1}{1-x^2}$

Integration **(*+ constant*; $a>0$ *where relevant*)**

f(*x*)	$\int$ **f(*x*) d*x***
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right),\quad \ln\left\{x+\sqrt{x^2-a^2}\right\} \quad (x>a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right),\quad \ln\left\{x+\sqrt{x^2+a^2}\right\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\operatorname{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \qquad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \;\mathrm{d}s \;=\; 2\pi \int y \sqrt{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\,\mathrm{d}x$$

$$= 2\pi \int y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4} n^2 (n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$