Edexcel Maths FP3

Topic Questions from Papers

Differentiation

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4. Given that $y = \text{arsinh}\,(\sqrt{x}), \quad x > 0,$

(a) find $\dfrac{dy}{dx}$, giving your answer as a simplified fraction.
(3)

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,dx,$$

giving your answer in the form $\ln\left(\dfrac{a+b\sqrt{5}}{2}\right)$, where a and b are integers.
(6)

Leave blank

5. Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(a) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y$,

(5)

(b) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$.

(4)

Leave blank

Question 5 continued

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2. (a) Given that $y = x \arcsin x$, $0 \leqslant x \leqslant 1$, find

(i) an expression for $\frac{dy}{dx}$,

(ii) the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$.

(3)

(b) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$

(5)

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5. (a) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, \mathrm{d}x$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

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Question 5 continued

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4.

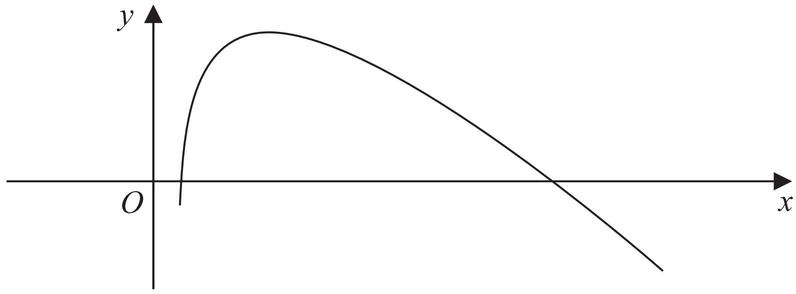

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \text{ arcosh } x - 9x, \qquad x \geqslant 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r\ln 3 + s\right)$, where p, q, r and s are integers.

(7)

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Question 4 continued

Q4

(Total 7 marks)

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7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b > 0$$

The line l is a normal to E at a point $P(a\cos\theta, b\sin\theta)$, $\quad 0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at A and the y-axis at B.

(b) Show that the area of the triangle OAB, where O is the origin, may be written as $k\sin 2\theta$, giving the value of the constant k in terms of a and b.

(4)

(c) Find, in terms of a and b, the exact coordinates of the point P, for which the area of the triangle OAB is a maximum.

(3)

Leave blank

Question 7 continued

P 4 3 1 4 3 A 0 2 1 2 8

Turn over

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing *AB* in the ratio $\lambda:\mu$ is $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{pmatrix}$

$$\mathbf{a}.(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\mathbf{b}.(\mathbf{c}\times\mathbf{a})=\mathbf{c}.(\mathbf{a}\times\mathbf{b})$$

If *A* is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\;(=\lambda)$$

The plane through *A* with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$$n_1x+n_2y+n_3z+d=0 \text{ where } d=-\mathbf{a.n}$$

The plane through non-collinear points *A*, *B* and *C* has vector equation

$$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$$

The perpendicular distance of (α,β,γ) from $n_1x+n_2y+n_3z+d=0$ is $\dfrac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$.

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\text{arcosh}\ x = \ln\{x + \sqrt{x^2 - 1}\} \qquad (x \geq 1)$

$\text{arsinh}\ x = \ln\{x + \sqrt{x^2 + 1}\}$

$\text{artanh}\ x = \tfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right) \qquad (|x| < 1)$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(*x*)	**f′(*x*)**
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\text{sech}^2\, x$
$\text{arsinh}\; x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\text{arcosh}\; x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\text{artanh}\, x$	$\dfrac{1}{1-x^2}$

***Integration* (+ *constant*; $a > 0$ *where relevant*)**

f(*x*)	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\text{arcosh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x+\sqrt{x^2-a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\text{arsinh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x+\sqrt{x^2+a^2}\right\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\text{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, \mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t \qquad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \ \ \mathrm{d}s = 2\pi \int y \sqrt{\left(1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\, \mathrm{d}x$$

$$= 2\pi \int y \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\, \mathrm{d}t$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B \equiv 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B \equiv 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B \equiv 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B \equiv -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\text{cosec}^2\, x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\dfrac{\text{f}(x)}{\text{g}(x)}$	$\dfrac{\text{f}'(x)\text{g}(x) - \text{f}(x)\text{g}'(x)}{(\text{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$