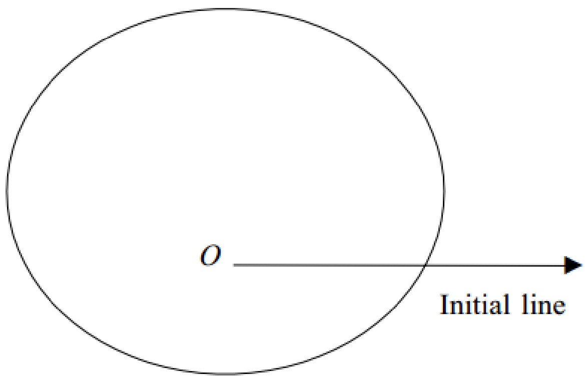
FP3 Polar Coordinates Questions

6 (a) A circle C_1 has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of C_1 is $r = 12 \sin\theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2\sin\theta + 5$, $0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.

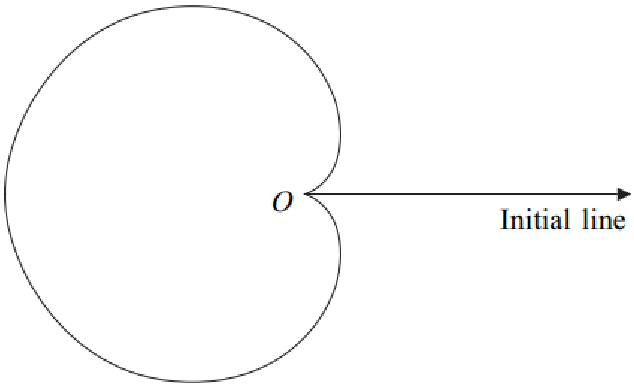


Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

4 The diagram shows the curve C with polar equation

$$r = 6(1 - \cos\theta), \qquad 0 \leqslant \theta < 2\pi$$



(a) Find the area of the region bounded by the curve C. *(6 marks)*

(b) The circle with cartesian equation $x^2 + y^2 = 9$ intersects the curve C at the points A and B.

(i) Find the polar coordinates of A and B. *(4 marks)*

(ii) Find, in surd form, the length of AB. *(2 marks)*

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2 A curve has polar equation $r(1 - \sin\theta) = 4$. Find its cartesian equation in the form $y = \mathrm{f}(x)$. *(6 marks)*

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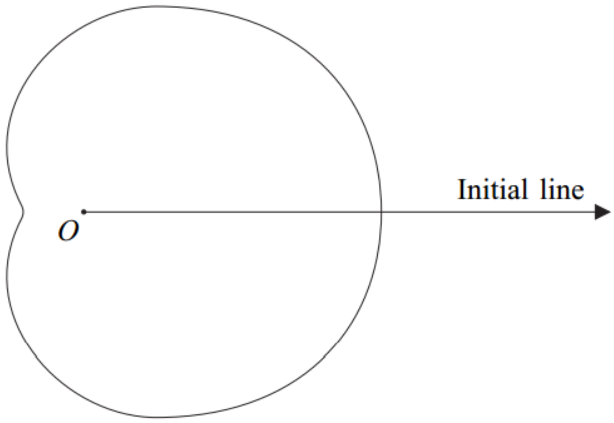
7 A curve C has polar equation

$$r = 6 + 4\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

The diagram shows a sketch of the curve C, the pole O and the initial line.



(a) Calculate the area of the region bounded by the curve C. *(6 marks)*

(b) The point P is the point on the curve C for which $\theta = \frac{2\pi}{3}$.

The point Q is the point on C for which $\theta = \pi$.

Show that QP is parallel to the line $\theta = \frac{\pi}{2}$. *(4 marks)*

(c) The line PQ intersects the curve C again at a point R.

The line RO intersects C again at a point S.

(i) Find, in surd form, the length of PS. *(4 marks)*

(ii) Show that the angle OPS is a right angle. *(1 mark)*

4 (a) Show that $(\cos\theta + \sin\theta)^2 = 1 + \sin 2\theta$. *(1 mark)*

(b) A curve has cartesian equation

$$(x^2 + y^2)^3 = (x + y)^4$$

Given that $r \geqslant 0$, show that the polar equation of the curve is

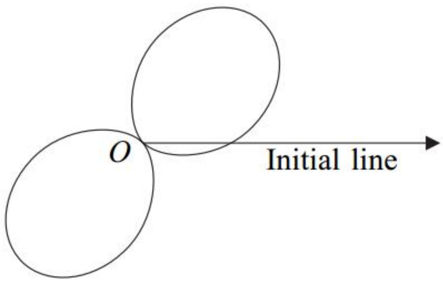
$$r = 1 + \sin 2\theta$$

(4 marks)

(c) The curve with polar equation

$$r = 1 + \sin 2\theta, \quad -\pi \leqslant \theta \leqslant \pi$$

is shown in the diagram.



(i) Find the two values of θ for which $r = 0$. *(3 marks)*

(ii) Find the area of one of the loops. *(6 marks)*

FP3 Polar Coordinates Answers

6(a)	$x^2+y^2-12y+36=36$	M1		Use of $y=r\sin\theta$ ($x=r\cos\theta$ PI)
		M1		Use of $x^2+y^2=r^2$
	$r^2-12r\sin\theta+36=36$	m1		
	$\Rightarrow r=12\sin\theta$	A1	4	**CSO AG**
(b)	Area = $\frac{1}{2}\int(2\sin\theta+5)^2\,d\theta$.	M1		Use of $\frac{1}{2}\int r^2\,d\theta$.
	.. = $\frac{1}{2}\int_0^{2\pi}(4\sin^2\theta+20\sin\theta+25)d\theta$	B1		Correct expn. of $(2\sin\theta+5)^2$
		B1		Correct limits
	$=\frac{1}{2}\int_0^{2\pi}(2(1-\cos 2\theta)+20\sin\theta+25)d\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$.
	$=\frac{1}{2}[27\theta-\sin 2\theta-20\cos\theta]_0^{2\pi}$	A1√		Correct integration ft wrong coeffs
	$=27\pi$.	A1	6	CSO
(c)	At intersection $12\sin\theta=2\sin\theta+5$	M1		OE eg $r=6(r-5)$
	$\Rightarrow \sin\theta=\frac{5}{10}$	A1		OE eg $r=6$
	Points $\left(6,\frac{\pi}{6}\right)$ and $\left(6,\frac{5\pi}{6}\right)$	A1		OE
	OPMQ is a rhombus of side 6			Or two equilateral triangles of side 6
	Area = $6\times6\times\sin\frac{2\pi}{3}$ oe	M1		Any valid complete method to find the area (or half area) of quadrilateral.
		A1		
	$=18\sqrt{3}$	A1	6	Accept unsimplified surd
	Total		**16**	

4(a)	Area $= \frac{1}{2}\int 36(1-\cos\theta)^2\,d\theta$	M1		use of $\frac{1}{2}\int r^2\,d\theta$
	$\ldots = \frac{1}{2}\int_0^{2\pi} 36(1-2\cos\theta+\cos^2\theta)\,d\theta$	B1 B1		for correct explanation of $[6(1-\cos\theta)]^2$ for correct limits
	$= 9\int_0^{2\pi} 2-4\cos\theta+(\cos 2\theta+1)\,d\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$.
	$= \left[27\theta - 36\sin\theta + \frac{9}{2}\sin 2\theta\right]_0^{2\pi}$	A1√		Correct integration; only ft if integrating $a + b\cos\theta + c\cos 2\theta$ with non-zero a, b, c.
	$= 54\pi$	A1	6	CSO
(b)(i)	$x^2+y^2=9 \Rightarrow r^2=9$	B1		PI
	A & B: $3=6-6\cos\theta \Rightarrow \cos\theta=\frac{1}{2}$	M1		
	Pts of intersection $\left(3,\frac{\pi}{3}\right)$; $\left(3,\frac{5\pi}{3}\right)$	A1 A1√	4	OE (accept 'different' values of θ not in the given interval)
(ii)	Length $AB = 2\times r\sin\theta$	M1		
	………… $= 2\times 3\times\frac{\sqrt{3}}{2} = 3\sqrt{3}$	A1	2	OE exact surd form
	Total		**12**	

2	$r - r\sin\theta = 4$	M1		
	$r - y = 4$	B1		$r\sin\theta = y$ stated or used
	$r = y+4$	A1		
	$x^2+y^2=(y+4)^2$	M1		$r^2=x^2+y^2$ used
	$x^2+y^2=y^2+8y+16$	A1F		ft one slip
	$y=\frac{x^2-16}{8}$	A1	6	
	Total		**6**	

7(a)	Area $= \frac{1}{2}\int (6+4\cos\theta)^2\,d\theta$	M1		use of $\frac{1}{2}\int r^2\,d\theta$
	$= \frac{1}{2}\left(\int_{-\pi}^{\pi} 36+48\cos\theta+16\cos^2\theta\right)d\theta$	B1 B1		for correct expansion of $[6+4\cos\theta)]^2$ for limits
	$= \left(\int_{-\pi}^{\pi} 18+24\cos\theta+4(\cos 2\theta+1)\right)d\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$= [22\theta+24\sin\theta+2\sin 2\theta]_{-\pi}^{\pi}$	A1F		correct integration ft wrong coefficients
	$= 44\pi$	A1	6	CSO
(b)	At P, $r=4$; At Q, $r=2$;	B1		PI
	P $\{x=\}$ $r\cos\theta = 4\cos\frac{2\pi}{3} = -2$	M1		Attempt to use $r\cos\theta$
	Q $\{x=\}$ $r\cos\theta = 2\cos\pi = -2$	A1		Both
	Since P and Q have same 'x', PQ is vertical so QP is parallel to the vertical line $\theta=\frac{\pi}{2}$	E1	4	

(c)(i)	$OP = 4$; $OS = 8$;	B1		
	Angle $POS = \frac{\pi}{3}$	B1		or $S\,(4, 4\sqrt{3})$ and $P\,(-2, 2\sqrt{3})$
	$PS^2 = 4^2 + 8^2 - 2\times 4\times 8\times \cos\frac{\pi}{3}$ oe	M1		Cosine rule used in triangle POS OE $PS^2 = (4+2)^2 + (4\sqrt{3} - 2\sqrt{3})^2$
	$PS = \sqrt{48}$ $\{= 4\sqrt{3}\}$	A1	4	
(ii)	Since $8^2 = 4^2 + \left(\sqrt{48}\right)^2$, $OS^2 = OP^2 + PS^2 \Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem)	E1	1	Accept valid equivalents e.g. $PR = 2PQ = 2(2\sqrt{3}) = PS$. $\angle SRP = \angle RSP = \angle RPO = \frac{\pi}{6}$ $\Rightarrow$ OPS is a right angle
	Total		**15**	

4(a)	$(\cos\theta + \sin\theta)^2 = \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta$ $= 1 + \sin 2\theta$	B1	1	AG (be convinced)
(b)	$(x^2 + y^2)^3 = (x + y)^4$			
	$(r^2)^3 = (r\cos\theta + r\sin\theta)^4$	M2,1,0		[M1 for one of $x^2 + y^2 = r^2$ OE, $x = r\cos\theta$, $y = r\sin\theta$ used]
	$r^6 = r^4\,(\cos\,\theta + \sin\,\theta)^4$			
	$r^6 = r^4\,(1 + \sin\,2\theta)^2$	M1		Uses (a) OE at any stage
	$r^2 = (1 + \sin\,2\theta)^2$			
	$\Rightarrow r = (1 + \sin\,2\theta)$ $\{r \geq 0\}$	A1	4	CSO; AG
(c)(i)	$r = 0 \Rightarrow \sin\,2\theta = -1$			
	$2\theta = \sin^{-1}(-1)$; $= -\frac{\pi}{2}, \frac{3\pi}{2}$	M1		

	$\theta = -\frac{\pi}{4};\ \frac{3\pi}{4}$	A1A1ft	3	A1 for either
(ii)	Area $= \frac{1}{2}\int(1+\sin 2\theta)^2 \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2 \mathrm{d}\theta$
	$= \frac{1}{2}\int(1+2\sin 2\theta+\sin^2 2\theta)\ \mathrm{d}\theta$	B1		Correct expansion of $(1+\sin 2\theta)^2$
	$= \frac{1}{2}\int\left(1+2\sin 2\theta+\frac{1}{2}(1-\cos 4\theta)\right)\mathrm{d}\theta$	M1		Attempt to write $\sin^2 2\theta$ in terms of $\cos 4\theta$
	$= \left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]$	A1ft		Correct integration ft wrong coefficients only
	$= \left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$			
	$= \left(\frac{9\pi}{16}\right)-\left(-\frac{3\pi}{16}\right)$	m1		Using c's values from (c)(i) as limits or the correct limits
	$= \frac{3\pi}{4}$	A1	6	CSO
	Total		**14**	