Exercise 1A

1 a $\sinh 4 = 27.29 \quad (2 \text{ d.p.})$

$\left(\dfrac{e^4 - e^{-4}}{2} = 27.29\right)$

Direct from calculator.

b $\cosh\left(\dfrac{1}{2}\right) = 1.13 \ (2 \text{ d.p.})$

$\left(\dfrac{e^{0.5} + e^{-0.5}}{2} = 1.13\right)$

Direct from calculator.

c $\tanh(-2) = -0.96 \ (2 \text{ d.p.})$

$\left(\dfrac{e^{-4} - 1}{e^{-4} + 1} = -0.96\right)$

Direct from calculator.

d $\operatorname{sech} 5 = 0.01347\ldots$

$= 0.01 \ (2 \text{ d.p.})$

2 a $\sinh 1 = \dfrac{e^1 - e^{-1}}{2} = \dfrac{e - e^{-1}}{2}$

b $\cosh 4 = \dfrac{e^4 + e^{-4}}{2}$

c $\tanh 0.5 = \dfrac{e^1 - 1}{e^1 + 1}$

$= \dfrac{e - 1}{e + 1}$

Use $\tanh x = \dfrac{e^{2x} - 1}{e^{2x} + 1}$.

d $\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$

$\operatorname{sech}(-1) = \dfrac{2}{e^{-1} + e^1}$

$= \dfrac{2}{\frac{1}{e} + e}$

$= \dfrac{2e}{1 + e^2}$

3 a $\sinh(\ln 2) = \dfrac{e^{\ln 2} - e^{-\ln 2}}{2}$

$= \dfrac{2 - \frac{1}{2}}{2} = \dfrac{3}{4}$

$e^{h2} = 2$, and $e^{-\ln 2} = e^{\ln 2^{-1}} = \frac{1}{2}$

3 b $\cosh(\ln 3) = \dfrac{e^{\ln 3} + e^{-\ln 3}}{2}$

$= \dfrac{3 + \frac{1}{3}}{2} = \dfrac{5}{3}$

$e^{\ln 3} = 3$, and $e^{-\ln 3} = e^{\ln 3^{-1}} = \frac{1}{3}$

c $\tanh(\ln 2) = \dfrac{e^{2\ln 2} - 1}{e^{2\ln 2} + 1}$

$= \dfrac{4-1}{4+1} = \dfrac{3}{5}$

$e^{2\ln 2} = e^{\ln 2^2} = 4$

d $\operatorname{cosech} x = \dfrac{2}{e^x - e^{-x}}$

$\operatorname{cosech} x = \dfrac{2}{e^x - \dfrac{1}{e^x}}$

$= \dfrac{2e^x}{e^{2x} - 1}$

$\operatorname{cosech}(\ln \pi) = \dfrac{2e^{\ln \pi}}{e^{2\ln \pi} - 1}$

$= \dfrac{2\pi}{\pi^2 - 1}$

4

$\dfrac{e^x + e^{-x}}{2} = 2$

$e^x + e^{-x} = 4$

$e^{2x} + 1 = 4e^x$

$e^{2x} - 4e^x + 1 = 0$

$e^x = \dfrac{4 \pm \sqrt{16 - 4}}{2}$

$e^x = 3.732$ or $e^x = 0.268$

$x = \ln 3.732 = 1.32$ (2 d.p.)

$x = \ln 0.268 = -1.32$ (2 d.p.)

Multiply throughout by e^x.

Solve as a quadratic in e^x.

5

$$\frac{e^x - e^{-x}}{2} = 1$$

$$e^x - e^{-x} = 2$$

$$e^{2x} - 1 = 2e^x$$

Multiply throughout by e^x.

$$e^{2x} - 2e^x - 1 = 0$$

$$e^x = \frac{2 \pm \sqrt{4+4}}{2}$$

Solve as a quadratic in e^x.

$$e^x = 2.414 \text{ or } e^x = -0.414$$

$$e^x = 2.414$$

e^x cannot be negative.

$$x = \ln\ 2.414 = 0.88 \text{ (2 d.p.)}$$

6

$$\frac{e^{2x} - 1}{e^{2x} + 1} = -\frac{1}{2}$$

$$2\left(e^{2x} - 1\right) = -\left(e^{2x} + 1\right)$$

$$2e^{2x} - 2 = -e^{2x} - 1$$

$$3e^{2x} = 1$$

$$e^{2x} = \tfrac{1}{3}$$

$$2x = \ln\left(\tfrac{1}{3}\right)$$

$$x = \tfrac{1}{2}\ln\left(\tfrac{1}{3}\right) = -0.55 \text{ (2 d.p.)}$$

7 $\coth x = \dfrac{e^{2x} + 1}{e^{2x} - 1}$

If $\coth x = 10$, then:

$$\frac{e^{2x} + 1}{e^{2x} - 1} = 10$$

$$e^{2x} + 1 = 10e^{2x} - 10$$

$$9e^{2x} = 11$$

$$e^{2x} = \frac{11}{9}$$

$$2x = \ln\left(\frac{11}{9}\right)$$

$$x = \frac{1}{2}\ln\left(\frac{11}{9}\right)$$

$$= 0.10033...$$

$$= 0.10 \text{ (2 d.p.)}$$

8 $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

If $\operatorname{sech} x = \frac{1}{8}$, then:

$$\frac{2}{e^x + e^{-x}} = \frac{1}{8}$$
$$e^x + e^{-x} = 16$$
$$e^x + \frac{1}{e^x} = 16$$
$$\frac{e^{2x} + 1}{e^x} = 16$$
$$e^{2x} + 1 = 16e^x$$
$$e^{2x} - 16e^x + 1 = 0$$

Let $y = e^x$

$$y^2 - 16y + 1 = 0$$
$$y = \frac{16 \pm \sqrt{16^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{16 \pm 6\sqrt{7}}{2}$$
$$= 8 \pm 3\sqrt{7}$$

Since $y = e^x$

$e^x = 8 + 3\sqrt{7}$ or $e^x = 8 - 3\sqrt{7}$

When $e^x = 8 + 3\sqrt{7}$:

$$x = \ln\left(8 + 3\sqrt{7}\right)$$
$$= 2.76865...$$
$$= 2.77 \text{ (2 d.p.)}$$

When $e^x = 8 - 3\sqrt{7}$:

$$x = \ln\left(8 - 3\sqrt{7}\right)$$
$$= -2.76865...$$
$$= -2.77 \text{ (2 d.p.)}$$