GCE

Mathematics

Unit **4727:** Further Pure Mathematics 3

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

1 **Subject-specific Marking Instructions for GCE Mathematics Pure strand**

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

PMT

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$z = 1, e^{2\pi i/5}, e^{4\pi i/5}, e^{6\pi i/5}, e^{8\pi i/5}$	M1 A1 [2]	$e^{2\pi i/5}$ soi	
	(ii)		$z^5 = -32$ has a root -2, so roots are $-2, -2e^{2\pi i/5}, -2e^{4\pi i/5}, -2e^{6\pi i/5}, -2e^{8\pi i/5}$ Roots $-2, 2e^{7\pi i/5}, 2e^{9\pi i/5}, 2e^{\pi i/5}, 2e^{3\pi i/5}$ Argand diagram	M1 A1 M1 A1 [4]	Use part **(i)** or from scratch cao with $r > 0, 0 < \theta < 2\pi$ (allow $2e^{\pi i}$ for -2) one root in each quadrant plus one on real axis axes and roots labelled. Roots equal moduli and equiangular spacing	
2			$\begin{pmatrix}1\\2\\-1\end{pmatrix} \times \begin{pmatrix}3\\0\\1\end{pmatrix} = \begin{pmatrix}2\\-4\\-6\end{pmatrix} = -2\begin{pmatrix}-1\\2\\3\end{pmatrix}$ $\begin{pmatrix}2\\1\\0\end{pmatrix} - \begin{pmatrix}-1\\1\\2\end{pmatrix} = \begin{pmatrix}3\\0\\-2\end{pmatrix}$ Shortest distance $= \dfrac{\left\|\begin{pmatrix}3\\0\\-2\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\3\end{pmatrix}\right\|}{\sqrt{1^2+2^2+3^2}}$ $= \frac{9}{\sqrt{14}}$ or 2.41	M1 A1 M1 A1 [4]	at least 2 correct values for the cross product or method shown Any multiple	
3	(i)		$\frac{dy}{dx} = -u^{-2}\frac{du}{dx}$ $2u - xu^2\left(-u^{-2}\frac{du}{dx}\right) = \frac{1}{x^2}$ $x\frac{du}{dx} + 2u = \frac{1}{x^2}$	M1 M1 A1 [3]	Differentiate Substitute **ag** Convincingly shown	

PMT

Question			Answer	Marks	Guidance	
	(ii)		$\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2u}{x}=\frac{1}{x^3}$			
			$I=\exp\left(\int\frac{2}{x}\mathrm{d}x\right)=\mathrm{e}^{2\ln x}$	M1	$\mathrm{e}^{k\ln x}$	
			$=x^2$	A1		incorrect IF means no further marks can be gained
			$\frac{\mathrm{d}}{\mathrm{d}x}(x^2u)=x^{-1}$	M1	for LHS, multiply and recognise derivative	if RHS is not multiplied by IF then no further marks can be gained
			$x^2u=\ln x+A$	A1		or $\ldots=lnkx$
			$u=(\ln x+A)/x^2$			
			$y=x^2/(\ln x+A)$	M1	for y = reciprocal of 'their u'	
			$x=1,y=1\Rightarrow 1=\frac{1}{0+A}\Rightarrow A=1$	M1		or $k=e$
			$y=\frac{x^2}{\ln x+1}$	A1 [7]	oe without fractions within fractions	or $y=\frac{x^2}{\ln ex}$
4	(i)		$\forall n, 1n=n1=n$ so 1 is identity	M1	Identify identity	can be implicit for M1
			But not all integers have an inverse, e.g. $2n\neq 1$ for any n	A1 [2]	Complete argument (example or general)	
	(ii)		$\{-1,1\}$	B1*		
			Demonstrates closure,			
			references associativity		...without contradiction	
			references identity			
			$(-1)^{-1}=-1$ (and $1^{-1}=1$) so inverses	*B2 [3]	B1 for any two of these Dep on 1st B1	

Question			Answer	Marks	Guidance	
5			AE: $\lambda^2 + 2\lambda + 10 = 0$ $\lambda = -1 \pm 3i$ CF: $e^{-x}(A\cos 3x + B\sin 3x)$ PI: $y = a\cos x + b\sin x$ $y' = -a\sin x + b\cos x$ $y'' = -a\cos x - b\sin x$ In DE: $-a\cos x - b\sin x + 2(-a\sin x + b\cos x) + 10(a\cos x + b\sin x) = 85\cos x$ $-a + 2b + 10a = 85$ $-b - 2a + 10b = 0$ $a = 9, b = 2$ GS: $y = 9\cos x + 2\sin x + e^{-x}(A\cos 3x + B\sin 3x)$	B1 B1 B1ft B1 M1* M1* A1 *A1ft [8]	condone $Ae^{(-1+3i)x} + Be^{(-1-3i)x}$ Differentiate twice and substitute Compare coefficients PI correct Their CF (of standard form) + their PI	ft on complex λ only trial function $y = a\cos x$ scores max of B0 M1 M0 at this stage dep on gaining both M1 marks
6	(i)		$\begin{pmatrix}1\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}7\\-2\\-3\end{pmatrix}$ finds point on both planes $\frac{x}{-7} = \frac{y-1}{2} = \frac{z-1}{3}$	M1A1 B1 A1 [4]	e.g. (0,1,1) oe	or $\left(\frac{7}{3}, \frac{1}{3}, 0\right)$ or $\left(\frac{7}{2}, 0, -\frac{1}{2}\right)$
	ALT		$x + 2y + z = 3$ $2x + y + 4z = 5$ $3x + 7z = 7$ $2x + 7y = 7$ $\frac{x}{-7} = \frac{y-1}{2} = \frac{z-1}{3}$	M1 A1 M1A1 [4]	Attempts to find at least 1 equation 2 correct equations oe of the form f(x) = g(y)= h(z)	or $3y - 2z = 1$

Question			Answer	Marks	Guidance	
	(ii)		$\begin{pmatrix}-7\\2\\3\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=-7+10-3=0$ $\Rightarrow l \parallel \Pi_3$ $(0,1,1)$ is on line, but $\begin{pmatrix}0\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=4\neq 1$ so not on plane	M1 A1 B1 [3]	For scalar product, either shows method or gives answer of zero for A1 must have working out line for scalar product	
	ALT		$x+5y-z=1$ $7\lambda+5(1-2\lambda)-(1-3\lambda)=1$ $\Rightarrow 4=1$ inconsistent, so l is parallel and not on plane	 M1A1 A1 [3]		
	(iii)		$2+2\times 0+1=3$ (so on Π_1) $\begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=1$ (so on Π_3) Line has equation $\mathbf{r}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}-7\\2\\3\end{pmatrix}$	 B1 M1 A1 [3]	 Verify both oe vector form in cartesian form M1 only	must show working for at least one plane if cross product calculated incorrectly then M0A0
7	(i)		$\cos 6\theta + i\sin 6\theta = (\cos\theta + i\sin\theta)^6$ $= \cos^6\theta + 6i\sin\theta\cos^5\theta - 15\sin^2\theta\cos^4\theta - 20i\sin^3\theta\cos^3\theta + 15\sin^4\theta\cos^2\theta + 6i\sin^5\theta\cos\theta - \sin^6\theta$ $\sin 6\theta = 6\sin\theta\cos^5\theta - 20\sin^3\theta\cos^3\theta + 6\sin^5\theta\cos\theta$ $=\cos\theta\,(6\sin\theta\,(1-\sin^2\theta)^2 - 20\sin^3\theta\,(1-\sin^2\theta) + 6\sin^5\theta)$ $=\cos\theta\,(32\sin^5\theta - 32\sin^3\theta + 6\sin\theta)$	B1 B1 M1 M1 A1 [5]	Use de Moivre All terms correct Compare imaginary parts Take out factor of $\cos\theta$ and give other factor in terms of $\sin\theta$ only **ag** Convincingly shown, having been explicit about taking imaginary parts	or $\sin 6\theta = Im(\cos\theta + i\sin\theta)^6$ must have $\sin 6\theta = \cdots$'final line'

Question			Answer	Marks	Guidance	
	(ii)		$\frac{\sin 6\theta}{\sin 2\theta} = \frac{\cos\theta\,(32\sin^5\theta - 32\sin^3\theta + 6\sin\theta)}{2sin\theta cos\theta}$ $= 16\sin^4\theta - 16\sin^2\theta + 3$	M1		
			$= 4(2\sin^2\theta - 1)^2 - 1$	M1 A1	Complete the square	**M1A1** for showing stationary points occur when $sin^2\theta = 0,1\ or\ \frac{1}{2}$
			$\therefore \frac{\sin 6\theta}{\sin 2\theta} \geq -1$	M1	deduces lower bound	if using calculus, must convince for nature of stationary points for each M1 here
			$0 \leq 2\sin^2\theta \leq 2$ $\therefore (2\sin^2\theta - 1)^2 \leq 1$	M1	deduces upper bound	can omit line 1or 2 from the workings here, but not for final A mark
			$\therefore 4(2\sin^2\theta - 1)^2 - 1 \leq 3$ $\Rightarrow -1 \leq \frac{\sin 6\theta}{\sin 2\theta} \leq 3$		**SC** if none of marks 2 to 5 (M1A1M1M1) gained then **SC M1A1** for any valid method of deducing upper bound, and similarly for lower bound	
			But upper bound attained $\Rightarrow \sin^2\theta = 0$ or 1 $\Rightarrow \sin 2\theta = 0$	M1	Dep on showing valid method for UB<=3	Or independent proof that not equal to 3
			So $\sin 2\theta \neq 0 \Rightarrow -1 \leq \frac{\sin 6\theta}{\sin 2\theta} < 3$	A1 [7]	full convincing overall argument	

Question			Answer	Marks	Guidance	
8	(i)		$ba = a^n \Rightarrow b = a^{n-1}$ But these are distinct elements so $ba \neq a^n$	M1 A1 [2]		
	(ii)		$ba = a^2b$ $\Rightarrow a^2ba = a^4b$ $\Rightarrow a^2ba = b$	M1	or $b = a^2ba^3$, $a = ba^2b$, $a^2 = bab$ or $b = ba^2$	
			$\Rightarrow a^2ba^4 = ba^3$ $\Rightarrow a^2b = ba^3$ $\Rightarrow ba = ba^3$ $\Rightarrow e = a^2$	M1	validly reach any equality which gives 2 distinct elements of the group as equal	
			Which is false, hence $ba \neq a^2b$	A1	Complete argument	
			If $ba = ab$ then (all element pairs would have to be commutative and so) G would be abelian.	M1	Do not award for G non-abelian $\Rightarrow ba \neq ab$	
			If $ba = b$ then $a = e$ so $ba \neq b$.	M1		
			So, by elimination of other possibilities, $ba = a^3b$	A1 [6]	Dependent on all previous marks	
	(iii)		$ba^2 = baa = a^3ba$ $= a^3a^3b = a^2b$	M1 A1 [2]	Use previous result Complete argument	

Question		Answer	Marks	Guidance	
	(iv)	<table><tr><td></td><td>e</td><td>a</td><td>a^2</td><td>a^3</td></tr><tr><td>e</td><td>e</td><td>a</td><td>a^2</td><td>a^3</td></tr><tr><td>a</td><td>a</td><td>a^2</td><td>a^3</td><td>e</td></tr><tr><td>a^2</td><td>a^2</td><td>a^3</td><td>e</td><td>a</td></tr><tr><td>a^3</td><td>a^3</td><td>e</td><td>a</td><td>a^2</td></tr></table>	B1	All correct	
		<table><tr><td></td><td>e</td><td>a^2</td><td>b</td><td>a^2b</td></tr><tr><td>e</td><td>e</td><td>a^2</td><td>b</td><td>a^2b</td></tr><tr><td>a^2</td><td>a^2</td><td>e</td><td>a^2b</td><td>b</td></tr><tr><td>b</td><td>b</td><td>a^2b</td><td>e</td><td>a^2</td></tr><tr><td>a^2b</td><td>a^2b</td><td>b</td><td>a^2</td><td>e</td></tr></table>	B1 M1 A1	Correct elements At least 12 out of 16 entries correct All correct	
		<table><tr><td></td><td>e</td><td>a^2</td><td>ab</td><td>a^3b</td></tr><tr><td>e</td><td>e</td><td>a^2</td><td>ab</td><td>a^3b</td></tr><tr><td>a^2</td><td>a^2</td><td>e</td><td>a^3b</td><td>ab</td></tr><tr><td>ab</td><td>ab</td><td>a^3b</td><td>e</td><td>a^2</td></tr><tr><td>a^3b</td><td>a^3b</td><td>ab</td><td>a^2</td><td>e</td></tr></table>	B1 M1 A1 [7]	Correct elements At least 12 out of 16 entries correct All correct	
		Total	**72**		

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