

# OCR

Oxford Cambridge and RSA

## Tuesday 9 June 2015 – Morning

### A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x. \quad [8]$$

- 2 The elements of a group  $G$  are polynomials of the form  $a + bx + cx^2$ , where  $a, b, c \in \{0, 1, 2, 3, 4\}$ . The group operation is addition, where the coefficients are added modulo 5.

(i) State the identity element. [1]

(ii) State the inverse of  $3 + 2x + x^2$ . [2]

(iii) State the order of  $G$ . [1]

The proper subgroup  $H$  contains  $2 + x$  and  $1 + x$ .

(iv) Find the order of  $H$ , justifying your answer. [4]

- 3 The plane  $\Pi$  passes through the points  $(1, 2, 1)$ ,  $(2, 3, 6)$  and  $(4, -1, 2)$ .

(i) Find a cartesian equation of the plane  $\Pi$ . [5]

The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$ .

(ii) Find the coordinates of the point of intersection of  $\Pi$  and  $l$ . [3]

(iii) Find the acute angle between  $\Pi$  and  $l$ . [3]

- 4 In an Argand diagram, the complex numbers  $0$ ,  $z$  and  $ze^{\frac{1}{6}i\pi}$  are represented by the points  $O$ ,  $A$  and  $B$  respectively.

(i) Sketch a possible Argand diagram showing the triangle  $OAB$ . Show that the triangle is isosceles and state the size of angle  $AOB$ . [4]

The complex numbers  $1 + i$  and  $5 + 2i$  are represented by the points  $C$  and  $D$  respectively. The complex number  $w$  is represented by the point  $E$ , such that  $CD = CE$  and angle  $DCE = \frac{1}{6}\pi$ .

(ii) Calculate the possible values of  $w$ , giving your answers exactly in the form  $a + bi$ . [5]

- 5 Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 3y = x^2 + x$$

for which  $y = 1$  when  $x = 1$ , giving  $y$  in terms of  $x$ . [8]

- 6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4} = \frac{y-1}{-2} = \frac{z+1}{3}. \quad [7]$$

- 7 (i) Use de Moivre's theorem to show that  $\tan 4\theta \equiv \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$ . [4]

(ii) Hence find the exact roots of  $t^4 + 4\sqrt{3}t^3 - 6t^2 - 4\sqrt{3}t + 1 = 0$ . [5]

- 8 Let  $G$  be any multiplicative group.  $H$  is a subset of  $G$ .  $H$  consists of all elements  $h$  such that  $hg = gh$  for every element  $g$  in  $G$ .

- (i) Prove that  $H$  is a subgroup of  $G$ . [8]

Now consider the case where  $G$  is given by the following table:

	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>e</i>	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>p</i>	<i>p</i>	<i>q</i>	<i>e</i>	<i>s</i>	<i>t</i>	<i>r</i>
<i>q</i>	<i>q</i>	<i>e</i>	<i>p</i>	<i>t</i>	<i>r</i>	<i>s</i>
<i>r</i>	<i>r</i>	<i>t</i>	<i>s</i>	<i>e</i>	<i>q</i>	<i>p</i>
<i>s</i>	<i>s</i>	<i>r</i>	<i>t</i>	<i>p</i>	<i>e</i>	<i>q</i>
<i>t</i>	<i>t</i>	<i>s</i>	<i>r</i>	<i>q</i>	<i>p</i>	<i>e</i>

- (ii) Show that  $H$  consists of just the identity element. [4]

**END OF QUESTION PAPER**

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