

GCE

Mathematics

Unit **4727**: Further Pure Mathematics 3

Advanced GCE

Mark Scheme for June 2014

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Question	Answer	Marks	Guidance
1 (i)	$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \\ 7 \end{pmatrix} = 7 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ <p>(eg) $z = 0 \Rightarrow 2x + y = 4, 3x + 5y = 13 \Rightarrow x = 1, y = 2$</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	<p>M1 A1</p> <p>M1</p> <p>A1</p>	<p>M1 requires evidence of method for cross product or at least 2 correct values calculated</p> <p>or any valid point e.g.(0, 3, -1), (3, 0, 2)</p> <p>Must have full equation including 'r ='</p> <p>oe vector form</p>
	<p>Alternative: Find one point Find a second point and vector between points</p> <p>multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ <p>Alternative: Solve simultaneously</p> <p>Point and direction found</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>to at least expressions for x,y,z parametrically, or two relationship between 2 variables.</p>

Question	Answer	Marks	Guidance
1 (ii)	$\frac{ 2 \times 2 + 5 - -2 - 4 }{\sqrt{2^2 + 1^2 + 1^2}} = \frac{7}{\sqrt{6}}$	M1 A1	Condone lack of absolute signs for M1 2.86 with no workings scores M1 oe surd form. isw
	Alternative: find parameter for perpendicular meets plane and use to find distance	M1 [2]	For complete method with calculation errors look for $\lambda = -7/6$
2	$u = y^2 \Rightarrow \frac{du}{dx} = 2y \frac{dy}{dx}$ <p>so DE $\Rightarrow 2y \frac{dy}{dx} - 4y^2 = 2e^x$</p> $\Rightarrow \frac{du}{dx} - 4u = 2e^x$ $I = \exp \int -4 dx = e^{-4x}$ $e^{-4x} \frac{du}{dx} - 4e^{-4x} u = 2e^{-3x}$ $u e^{-4x} = -\frac{2}{3} e^{-3x} (+A)$ $u = -\frac{2}{3} e^x + A e^{4x}$ $y = \sqrt{-\frac{2}{3} e^x + A e^{4x}}$ <p>Alternative from 4th mark to 6th mark CF: $(u = \dots) A e^{4x}$</p> <p>PI: $u = k e^x, \frac{du}{dx} = k e^x$</p> $k e^x - 4k e^x = 2e^x, \quad k = -\frac{2}{3}$	M1 M1 A1 A1ft M1* *M1dep* M1dep* A1 A1 M1* M1 dep* [8]	Correctly finds Or $\frac{dy}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \frac{du}{dx}$ or for complete unsimplified substitution Can be implied by next A1 Must have form $\frac{du}{dx} + f(x)u = g(x)$ for this mark and any further marks Can be implied by subsequent work No more than 1 numerical error at this step ignore use of '±' Cao PI chosen & differentiated correctly Substitutes and solves

Question	Answer	Marks	Guidance
3 (i)	$z^6 = 1 \Rightarrow z = e^{2k\pi i/6}$ $k = 0, 1, 2, 3, 4, 5$ Diagram	M1 A1 B1 B1 [4]	Oe exactly 6 roots 6 roots in right quadrant, correct angles and moduli accept roots 1, -1 given as integers. as evidenced by labels, circles, or accurate diagram, or by co-ordinates
3 (ii)	$(1+i)^6 = \left(\sqrt{2} e^{\frac{1}{4}\pi i}\right)^6$ $8e^{\frac{6}{4}\pi i}$ $= -8i$ <p>Alternative:</p> $(1+i)^6 = 1 + 6i + 15i^2 + 20i^3 + 15i^4 + 6i^5 + i^6$ $= 1 + 6i - 15 - 20i + 15 + 6i - 1$ $= -8i$ <p>Alternative: $(1+i)^2 = 2i$</p> $(1+i)^6 = (2i)^3$ $= -8i$	M1 M1 A1 M1 M1 A1 M1 M1 A1 [3]	Attempts modulus-argument form, getting at least 1 correct for $(\text{mod})^6$ and $\arg x$ 6 ag complete argument including start line no more than 1 term wrong ag ag Sc 2 for only lines 2 & 3 correct

Question	Answer	Marks	Guidance
3 (iii)	$z^6 = -8i \Rightarrow z = (1+i)e^{2k\pi i/6}$ $= \sqrt{2}e^{i\pi/4} e^{2k\pi i/6}$ $\sqrt{2}e^{i\pi(1/4+k/3)}, k = 0,1,2,3,4,5$ <p>Alternative: $z^6 = 8e^{i\pi(\frac{3}{2}+2k)}$</p> $\sqrt{2}e^{i\pi(1/4+k/3)}, k = 0,1,2,3,4,5$	M1 M1 A1 M1 M1 A1 [3]	 or equivalent k or equivalent: e.g. $\sqrt{2}e^{i\pi(-1/12+k/3)}$ accept unsimplified modulus

Question		Answer									Marks	Guidance	
4	(i)										B1	2 or more	Ignore 1
		element	(1)	3	7	9	11	13	17	19	B1	4 or more	
		inverse	(1)	7	3	9	11	17	13	19	B1	all 7 correct	
											[3]		
4	(ii)	<p>(1 has order 1) 9,11,19 have order 2</p> <p>$3^2 = 9 \Rightarrow 3^4 = 1$ so order 4 similarly 7,13,17 order 4</p> <p>no element of order 8 so not cyclic</p>									M1	Correctly identifies order of all elements	Allow one error must show workings towards a^4 for demonstration that these elements are order 4` condone “no generator” in place of “no element of order 8”
											B1	justifies order for at least 1 element of order 4	
											A1	www	
											[3]		
4	(iii)										M1	For two sets which both contain “1” and all (4) elements’ inverses	
		{1,13, 9, 17} and {1, 3, 9, 7}									B1	One subgroup of order 4	
		1 ↔ 1, 9 ↔ 9, 3 ↔ 13, 7 ↔ 17									A1		
											M1	for correspondence of “their” elements of same order	
											A1	or 3 ↔ 17, 7 ↔ 13	
											[5]		

Question	Answer	Marks	Guidance
5	AE: $\lambda^2 + 5\lambda + 6 = 0$ $\lambda = -2, -3$ CF: $Ae^{-2x} + Be^{-3x}$ PI: $y = ae^{-x}$ $ae^{-x} - 5ae^{-x} + 6ae^{-x} = e^{-x}$ $2a = 1$ $a = \frac{1}{2}$ GS: $(y =) \frac{1}{2}e^{-x} + Ae^{-2x} + Be^{-3x}$ $x = 0, y = 0 \Rightarrow \frac{1}{2} + A + B = 0$ $y' = -\frac{1}{2}e^{-x} - 2Ae^{-2x} - 3Be^{-3x}$ $x = 0, y' = 0 \Rightarrow -\frac{1}{2} - 2A - 3B = 0$ $A = -1, B = \frac{1}{2}$ $y = \frac{1}{2}e^{-x} - e^{-2x} + \frac{1}{2}e^{-3x}$	B1 B1ft B1ft M1 A1 A1ft M1 M1* M1dep* A1 [10]	Differentiate and substitute Differentiate their GS of form $y = ke^{-x} + Ae^{mx} + Be^{nx}$ where k, m, n are non-zero constants and m, n not 1 Use condition and attempt to find A, B www fit must be of form “ ke^{-x} plus a standard CF form” with 2 arbitrary constants Must have 2 arbitrary constants Must have ‘y =’

Question	Answer	Marks	Guidance
6 (i)	$l \parallel \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \quad \Pi \perp \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \quad \text{so} \quad \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = 0 \Rightarrow l \parallel \Pi$ <p>$(1, -2, 7)$ on l but $4 \times 1 - 2 - 7 = -1 \neq 8$ so not in Π</p> <p>hence l not in Π</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>dot product of correct vectors = 0</p> <p>substitute point on line into Π and calculate d</p> <p>Full argument includes key components</p> <p>Argument can be about a general point on line</p>
6 (ii)	$(\mathbf{r} =) \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$ <p>closest point where meets Π</p> $4(1 + 4\lambda) - (-2 - \lambda) - (7 - \lambda) = 8$ $\Rightarrow \lambda = \frac{1}{2}$ $\Rightarrow \mathbf{r} = \begin{pmatrix} 3 \\ -\frac{5}{2} \\ \frac{13}{2} \end{pmatrix}$	<p>B1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[4]</p>	<p>parametric form of (x, y, z) substituted into plane</p>
6 (iii)	$\mathbf{r} = \begin{pmatrix} 3 \\ -\frac{5}{2} \\ \frac{13}{2} \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$	<p>B1ft</p> <p>[1]</p>	<p>oe</p> <p>must have “$\mathbf{r} =$”</p>

Question	Answer	Marks	Guidance
7 (i)	$2i \sin \theta = e^{i\theta} - e^{-i\theta}$ $2i \sin n\theta = e^{in\theta} - e^{-in\theta}$ $(2i \sin \theta)^5 = (e^{i\theta} - e^{-i\theta})^5$ $= e^{i5\theta} - 5e^{i3\theta} + 10e^{i\theta} - 10e^{-i\theta} + 5e^{-i3\theta} - e^{-i5\theta}$ $32i \sin^5 \theta = (e^{5i\theta} - e^{-5i\theta}) - 5(e^{3i\theta} - e^{-3i\theta}) + 10(e^{i\theta} - e^{-i\theta})$ $= 2i \sin 5\theta - 5(2i \sin 3\theta) + 10(2i \sin \theta)$ $\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$	B1 M1* M1dep* A1 [4]	any equivalent form binomial expansion grouping terms AG If use z , must define it can be unsimplified Award B1 then sc M1A1 for candidates who omit this stage from otherwise complete argument must convince on the $\frac{1}{16}$ and on the elimination of i
7 (ii)	$16 \sin^5 \theta - 10 \sin \theta = \sin 5\theta - 5 \sin 3\theta$ $16 \sin^5 \theta - 6 \sin \theta = 0$ $\sin \theta = 0, \pm \sqrt[4]{\frac{3}{8}}$ $\theta = 0, \pm 0.899$	M1* A1 M1dep* A1 [4]	Attempts to eliminate $\sin 5\theta$ and $\sin 3\theta$ must have 3 values for $\sin \theta$ Or $16 \sin^5 \theta = 6 \sin \theta$

Question	Answer	Marks	Guidance
8 (i)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is identity}$ $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}^{-1} = \frac{1}{a^2 + b^2} \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in G$ $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} ac - bd & -bc - ad \\ bc + ad & ac - bd \end{pmatrix}$ <p>and</p> $(ac - bd)^2 + (bc + ad)^2 = a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2$ $= (a^2 + b^2)(c^2 + d^2) \neq 0$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>for M1, must at least get matrix in form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, or state existence of inverse due to non-singularity</p> <p>Must not attempt to prove commutativity in (i)</p>
8 (ii)	$\begin{pmatrix} c & -d \\ d & c \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} ac - bd & -bc - ad \\ bc + ad & ac - bd \end{pmatrix}$ $= \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} \text{ so commutative}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>must also consider matrices reversed, but may be seen in (i)</p>
8 (iii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>order 4</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>g^2 must be correct</p> <p>allow 1 error in getting g^4</p>

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