GCE

Mathematics

Advanced GCE

Unit **4727:** Further Pure Mathematics 3

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Question			Answer	Marks	Guidance	
1	(i)		vectors in plane: two of $\begin{pmatrix}-4\\4\\1\end{pmatrix}, \begin{pmatrix}0\\6\\4\end{pmatrix}=2\begin{pmatrix}0\\3\\2\end{pmatrix}, \begin{pmatrix}4\\2\\3\end{pmatrix}$	M1	Differences between two pairs	Any multiple
			$\mathbf{r}=\begin{pmatrix}1\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\3\\2\end{pmatrix}+\mu\begin{pmatrix}4\\2\\3\end{pmatrix}$	A1	Aef of parametric equation	Must have “**r = ...**”
				[2]		
1	(ii)		$\begin{pmatrix}0\\3\\2\end{pmatrix}\times\begin{pmatrix}4\\2\\3\end{pmatrix}=\begin{pmatrix}5\\8\\-12\end{pmatrix}$	M1 A1	Calculate vector product or multiple	M1 can be awarded where vector product has method shown or only one term wrong
			$\left(\mathbf{r}-\begin{pmatrix}1\\6\\2\end{pmatrix}\right).\begin{pmatrix}5\\8\\-12\end{pmatrix}=0$	M1		Or Cartesian form = *d* with attempt to compute *d*
			$5x+8y-12z=29$	A1	Aef of cartesian equation, isw.	
				[4]		
			Alternate method			
					EITHER	
				M1	*x, y, z* in parametric form	
				A1	both parameters in terms of e.g. *x, y*	
				M1A1	substitute into parametric form of *z*	
					OR	
				M1	*x, y, z* in parametric form	
				A1	2 equations in *x, y, z* and one parameter	
				M1 A1	eliminate parameter	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>2</td><td>(i)</td><td></td><td>$$\begin{array}{c|cccc} & 1 & 3 & 5 & 7 \\ \hline 1 & 1 & 3 & 5 & 7 \\ 3 & 3 & 1 & 7 & 5 \\ 5 & 5 & 7 & 1 & 3 \\ 7 & 7 & 5 & 3 & 1 \end{array}$$</td><td>B2</td><td>–1 each error</td><td></td></tr>
<tr><td></td><td></td><td></td><td>From table clearly closed</td><td>B1</td><td></td><td>Must be clear they are referring to tabulated results</td></tr>
<tr><td></td><td></td><td></td><td>1 is identity</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>$3^{-1} \equiv 3,\ 5^{-1} \equiv 5,\ 7^{-1} \equiv 7 \pmod{8}$</td><td>B1</td><td></td><td>Or “1 appears in every row”</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[5]</td><td>Superfluous fact/s gets –1</td><td></td></tr>
<tr><td>2</td><td>(ii)</td><td></td><td>1 has order 1 and 3, 5, 7 all have order 2</td><td>B1
[1]</td><td></td><td></td></tr>
<tr><td>2</td><td>(iii)</td><td></td><td>{1, 3}, {1, 5}, {1, 7} (and {1})</td><td>B1
[1]</td><td>All correct, no extras</td><td>Allow {1} included or omitted</td></tr>
<tr><td>2</td><td>(iv)</td><td></td><td>in <i>H</i> $3^2 \equiv 9 \pmod{10}$ so 3 not order 2</td><td>M1</td><td>Shows and states that 3 or that 7 is not order 2 (or is order 4)</td><td></td></tr>
<tr><td></td><td></td><td></td><td>no element of order > 2 in <i>G</i> so not isomorphic</td><td>A1

[2]</td><td>Completely correct reasoning
Or, if zero, then SC1 for merely stating comparable orders and then saying that “orders don’t correspond, so not isomorphic"
Or
table for H with saying “not all elements self inverse, so not isomorphic”</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
3			$u = y^3 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1		Or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}u^{-\frac{2}{3}} \frac{\mathrm{d}u}{\mathrm{d}x}$
			in DE gives $x\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = \frac{\cos x}{x}$	A1		
			$\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{2}{x}u = \frac{\cos x}{x^2}$	B1	Divide	Both sides
			$I = \exp\left(\int \frac{2}{x}\mathrm{d}x\right) = \mathrm{e}^{2\ln x}$	M1	Correctly integrates	Must have form $\frac{\mathrm{d}u}{\mathrm{d}x} + \mathrm{f}(x)u = \mathrm{g}(x)$
			$= x^2$	A1		Can be implied by subsequent work
			$x^2 \frac{\mathrm{d}u}{\mathrm{d}x} + 2xu = \cos x$			
			$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 u\right) = \cos x$			
			$x^2 u = \sin x \quad (+A)$	M1	Integrate	
			$u = \frac{\sin x + A}{x^2}$	A1	Or gives GS in implicit form	Must include constant at this stage
			$y = \left(\frac{\sin x + A}{x^2}\right)^{\frac{1}{3}}$	A1		
				[8]		

Question			Answer	Marks	Guidance	
4	(i)		Sketch	B1		Must have axes, A shown 3 across and either scale (or co-ordinates) with B in rough position, or angle and distance on argand diagram. No inconsistencies
			$OA=\lvert 3\rvert=3,\ OB=\left\lvert 3\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\right\rvert=3$			
			and $\angle BOA=\frac{1}{3}\pi$	M1	Can be seen on diagram	Alt. Attempts AB or second angle
			hence ΔOAB equilateral	A1		
				[3]		
4	(ii)		$3\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$	M1A1	Or $3\mathrm{e}^{\frac{5}{3}\pi\mathrm{i}}$. Isw M1 for evidence they are considering BA, or for $\frac{3}{2}-\frac{3}{2}\sqrt{3}\,\mathrm{i}$	For full marks can use CiS form, or clear polar co-ordinates, in radians. Not C-iS
				[2]		
4	(iii)		$\left(3-3\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\right)^5=3^5\,\mathrm{e}^{-\frac{5}{3}\pi\mathrm{i}}$	M1	For mod^5 and arg × 5	“Hence” so must use ‘their $3\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$’
			$=243\left(\cos\frac{5}{3}\pi-\mathrm{i}\sin\frac{5}{3}\pi\right)$	A1ft	aef	
			$=\frac{243}{2}+\frac{243}{2}\sqrt{3}\,\mathrm{i}$	B1		Condone use of “121.5”.
				[3]		

Question			Answer	Marks	Guidance	
5			AE: $\lambda^2+2\lambda+5=0$	M1		
			$\lambda=-1\pm 2\mathrm{i}$	A1		
			CF: $\mathrm{e}^{-x}\left(A\cos 2x+B\sin 2x\right)$	A1ft		Or $A\mathrm{e}^{-x}\cos(2x+\alpha)$ Must be in real form
			PI: $y=a\mathrm{e}^{-x}$	B1		If PI $y=ax\mathrm{e}^{-x}$,then max of M1,A1,A1, B0,M1,A0,A0 (since cannot be consistent) M1, M1, A1.
			$a\mathrm{e}^{-x}-2a\mathrm{e}^{-x}+5a\mathrm{e}^{-x}=\mathrm{e}^{-x}$ $4a=1$	M1	Differentiate & substitute	Must have a constant in "their PI"
			$a=\frac{1}{4}$	A1		
			GS: $y=\mathrm{e}^{-x}\left(\frac{1}{4}+A\cos 2x+B\sin 2x\right)$	A1ft		Must have "y ="
			$\frac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^{-x}\left(\frac{1}{4}+A\cos 2x+B\sin 2x\right)$ $+\mathrm{e}^{-x}\left(-2A\sin 2x+2B\cos 2x\right)$	M1*	Differentiate their GS of form $y=\mathrm{e}^{-x}\left(P+A\cos nx+B\sin nx\right)$ where P is constant or linear term, n not 0 or 1	Allow one error
			$x=0,\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow-\left(\frac{1}{4}+A\right)+2B=0$ $x=0,y=0\Rightarrow\frac{1}{4}+A=0$	*M1	Use conditions	But M0 if leads to solution of y = 0
			$A=-\frac{1}{4},\ B=0$	A1ft	From their GS	
			$y=\frac{1}{4}\mathrm{e}^{-x}\left(1-\cos 2x\right)$	A1		Must have 'y =' and be in real form
				[11]		
6	**(i)**		$x=2t+1,\ y=5t-1,\ z=t+2$	B1	Parameterise	or B1 for y and z correctly in terms of x e.g. $2y = 5x – 7$, $2z = x + 3$ Then M1 for full simultaneous equations method.
			$\left(2t+1\right)+2\left(5t-1\right)-2\left(t+2\right)=5$		Substitute into plane	
			$\Rightarrow 10t=10\Rightarrow t=1$	M1	Solve	
			Intersect at (3, 4, 3)	A1	cao	Accept vector form
				[3]		

Question			Answer	Marks	Guidance	
6	**(ii)**		$\cos\left(\frac{1}{2}\pi-\theta\right)=\frac{\left\lvert\begin{pmatrix}2\\5\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-2\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}2\\5\\1\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}1\\2\\-2\end{pmatrix}\right\rvert}=\frac{10}{3\sqrt{30}}$	M1A1		Attempt to find angle or its complement
			$\theta=0.654$	A1 **[3]**	or 37.5°	
6	**(iii)**		If *P* is point of intersection and *Q* is required point, $\overrightarrow{PQ}=\lambda\begin{pmatrix}2\\5\\1\end{pmatrix}$ so $\sin\theta=\frac{2}{PQ}=\frac{2}{\lvert\lambda\rvert\sqrt{30}}$	M1*	or $\overrightarrow{PQ}.\hat{\mathbf{n}}=\pm2$ where $\mathbf{n}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Use $\overrightarrow{PQ}$ with right angled triangle or consider component of $\overrightarrow{PQ}$ in direction of normal vector.
			$\frac{10}{3\sqrt{30}}=\frac{2}{\lvert\lambda\rvert\sqrt{30}}$	M1		Valid method to set up equation in λ alone.
			$\lambda=\pm\frac{3}{5}$	A1		(May work from general point on original equation)
			points have position vectors $\begin{pmatrix}3\\4\\3\end{pmatrix}\pm\frac{3}{5}\begin{pmatrix}2\\5\\1\end{pmatrix}$	*M1	Dep on 1st M1	
			points at (1.8, 1, 2.4) and (4.2, 7, 3.6)	A1	cao	
			Alternative:			
			Distance $=\frac{\left\lvert 2t+1+2(5t-1)-2(t+2)-5\right\rvert}{\sqrt{1^2+2^2+2^2}}=2$	M1* A1		Zero if formula used without substitution in of parametric form.
				*M1	Solve	
			$\Rightarrow t=0.4$ or 1.6	A1	At least one value found	
			(1.8, 1, 2.4) and (4.2, 7, 3.6)	A1 **[5]**		

Question			Answer	Marks	Guidance	
7	**(i)**		$(ab)^6 = abab...ab = a^6b^6$ as commutative	M1	Must give reason	Some demonstration that they understand commutativity
			$= \left(a^2\right)^3 \left(b^3\right)^2 = e^3e^2 = e$ So *ab* has order 1, 2, 3, or 6	A1	Using orders of *a* and *b*	
			($b \neq a \Rightarrow ab \neq a^2 \Rightarrow ab \neq e$ so *ab* not order 1) $(ab)^2 = a^2b^2 = eb^2 = b^2$ and *b* not order 2, so *ab* not order 2 $(ab)^3 = a^3b^3 = aa^2e = aee = a \neq e$, so *ab* not order 3	M1	Consider other cases	Condone absence of this line Insufficient to merely have simplified **all** $(ab)^n$
			(So must be order 6)	A1 **[4]**	AG Complete argument	
7	**(ii)**		*ac* has order 18	B1		Or *abc* or generator
			18 is LCM of 2 and 9 , (so we can use a similar argument to part (i))	M1	or explicit consideration of other cases	
			So as *G* has an element of order 18 it must be cyclic.	A1 **[3]**	AG Complete argument	
8	**(i)**		$\cos 5\theta + \mathrm{i}\sin 5\theta = \left(\cos\theta + \mathrm{i}\sin\theta\right)^5$	B1	Or $\cos 5\theta = re\{\left(\cos\theta + \mathrm{i}\sin\theta\right)^5\}$	
			$= c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$	M1		No more than 1 error, can be unsimplified
			$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$	M1	Take real parts	
			$= c^5 - 10c^3\left(1-c^2\right) + 5c\left(1-c^2\right)^2$ $= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$	M1		
			$\cos 5\theta = 16c^5 - 20c^3 + 5c$	A1 **[5]**	AG	

Question			Answer	Marks	Guidance	
8	(ii)		Multiplying by x gives $16x^5 - 20x^3 + 5x = 0$ letting $x = \cos\alpha$ gives $\cos 5\alpha = 0$ hence $5\alpha = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi$ $\alpha = \frac{1}{10}\pi, \frac{3}{10}\pi, \frac{5}{10}\pi, \frac{7}{10}\pi, \frac{9}{10}\pi$ $\cos\frac{5}{10}\pi = 0$ which is not a root so roots $x = \cos\frac{1}{10}\pi, \cos\frac{3}{10}\pi, \cos\frac{7}{10}\pi, \cos\frac{9}{10}\pi$	M1 A1 A1 A1 **[4]**		Hence, so no marks for using quadratic at this stage.
8	(iii)		$16x^4 - 20x^2 + 5 = 0 \Leftrightarrow x^2 = \frac{20 \pm \sqrt{80}}{32}$ cos decreases between 0 and π so $\cos\frac{1}{10}\pi$ is greatest root so $\cos\frac{1}{10}\pi = \sqrt{\frac{20+\sqrt{80}}{32}} = \sqrt{\frac{5+\sqrt{5}}{8}}$	B1 M1 A1 **[3]**	Dep on full marks in (ii)	Can be gained if seen in (ii)

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