Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The plane p has equation $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 4$ and the line l_1 has equation $\mathbf{r} = 2\mathbf{j} - \mathbf{k} + t(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The line l_2 is parallel to p and perpendicular to l_1, and passes through the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Find the equation of l_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

2 **(i)** Solve the equation $z^4 = 2(1 + \mathrm{i}\sqrt{3})$, giving the roots exactly in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[5]**

(ii) Sketch an Argand diagram to show the lines from the origin to the point representing $2(1 + \mathrm{i}\sqrt{3})$ and from the origin to the points which represent the roots of the equation in part **(i)**. **[3]**

3 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\cot x = 2x$$

for which $y = 2$ when $x = \frac{1}{6}\pi$. Give your answer in the form $y = \mathrm{f}(x)$. **[9]**

4 The elements a, b, c, d are combined according to the operation table below, to form a group G of order 4.

	a	b	c	d
a	b	a	d	c
b	a	b	c	d
c	d	c	a	b
d	c	d	b	a

Group G is isomorphic **either** to the multiplicative group $H = \{e, r, r^2, r^3\}$ **or** to the multiplicative group $K = \{e, p, q, pq\}$. It is given that $r^4 = e$ in group H and that $p^2 = q^2 = e$ in group K, where e denotes the identity in each group.

(i) Write down the operation tables for H and K. **[4]**

(ii) State the identity element of G. **[1]**

(iii) Demonstrate the isomorphism between G and either H or K by listing how the elements of G correspond to the elements of the other group. If the correspondence can be shown in more than one way, list the alternative correspondence(s). **[4]**

5 **(i)** By expressing $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, prove that

$$\sin^3\theta\cos^2\theta \equiv -\tfrac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta).$$ **[6]**

(ii) Hence show that all the roots of the equation

$$\sin 5\theta = \sin 3\theta + 2\sin\theta$$

are of the form $\theta = \dfrac{n\pi}{k}$, where n is any integer and k is to be determined. **[3]**

6 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 12e^{2x}.$$

(i) Find the general solution of the differential equation. **[6]**

(ii) It is given that the curve which represents a particular solution of the differential equation has gradient 6 when $x = 0$, and approximates to $y = e^{2x}$ when x is large and positive. Find the equation of the curve. **[4]**

7 With respect to the origin O, the position vectors of the points U, V and W are $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively. The mid-points of the sides VW, WU and UV of the triangle UVW are M, N and P respectively.

(i) Show that $\overrightarrow{UM} = \frac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$. **[2]**

(ii) Verify that the point G with position vector $\frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$ lies on UM, and deduce that the lines UM, VN and WP intersect at G. **[5]**

(iii) Write down, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line through G which is perpendicular to the plane UVW. (It is not necessary to simplify the expression for $\mathbf{b}$.) **[2]**

(iv) It is now given that $\mathbf{u} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{v} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix}0\\0\\1\end{pmatrix}$. Find the perpendicular distance from O to the plane UVW. **[3]**

8 The set M of matrices $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$, where a, b, c and d are real and $ad - bc = 1$, forms a group $(M, \times)$ under matrix multiplication. R denotes the set of all matrices $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$.

(i) Prove that $(R, \times)$ is a subgroup of $(M, \times)$. **[6]**

(ii) By considering geometrical transformations in the x-y plane, find a subgroup of $(R, \times)$ of order 6. Give the elements of this subgroup in exact numerical form. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.