Mark Scheme 4727
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1 (a) Identity = $1+0\mathrm{i}$	B1	For correct identity. Allow 1
Inverse = $\frac{1}{1+2\mathrm{i}}$	B1	For $\frac{1}{1+2\mathrm{i}}$ seen or implied
$= \frac{1}{1+2\mathrm{i}} \times \frac{1-2\mathrm{i}}{1-2\mathrm{i}} = \frac{1}{5}-\frac{2}{5}\mathrm{i}$	B1 **3**	For correct inverse **AEFcartesian**
(b) Identity = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	B1	For correct identity
Inverse = $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$	B1 **2**	For correct inverse
	$\boxed{5}$	
2 (a) $(z_1 z_2 =)\ 6\mathrm{e}^{\frac{5}{12}\pi\mathrm{i}}$	B1 B1	For modulus = 6 For argument = $\frac{5}{12}\pi$
$\left(\frac{z_1}{z_2} = \frac{2}{3}\mathrm{e}^{-\frac{1}{12}\pi\mathrm{i}} =\right) \frac{2}{3}\mathrm{e}^{\frac{23}{12}\pi\mathrm{i}}$	M1 A1 **4**	For subtracting arguments For correct answer
(b) $\left(w^{-5} =\right) 2^{-5}\operatorname{cis}\left(-\frac{5}{8}\pi\right)$	M1 A1	For use of de Moivre For $-\frac{5}{8}\pi$ seen or implied
$= \frac{1}{32}\left(\cos\frac{11}{8}\pi + \mathrm{i}\sin\frac{11}{8}\pi\right)$	A1 **3**	For correct answer (allow 2^{-5} and $\operatorname{cis}\frac{11}{8}\pi$)
	$\boxed{7}$	

3 *EITHER* $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a})\times[8, 3, -6]$	M1*	For attempt at vector product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\mathbf{n} = \pm[-12, 50, 9]$	A1 √	For obtaining **n**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \dfrac{\lvert\mathbf{n}\rvert}{\lvert[8, 3, -6]\rvert}$	M1 (dep*)	For dividing $\lvert\mathbf{n}\rvert$ by magnitude of $[8, 3, -6]$
$= \dfrac{\sqrt{2725}}{\sqrt{109}}$	A1	For either magnitude correct
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a}).[8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\cos\theta = \pm\dfrac{109}{\sqrt{134}\sqrt{109}} = \pm\sqrt{\dfrac{109}{134}}$	A1 √	For correct $\cos\theta$ **AEF**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134}\sin\theta$	M1 (dep*) A1	For using trigonometry for perpendicular distance For correct expression for d in terms of θ
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a}).[8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$x = \dfrac{109}{\sqrt{109}} = \sqrt{109}$	A1 √	For finding projection of $\mathbf{c}-\mathbf{a}$ onto $[8, 3, -6]$ f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134-109}$	M1 (dep*) A1	For using Pythagoras for perpendicular distance For correct expression for d
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{CP} = \pm[-11+8t, -3+3t, 2-6t]$	B1	For finding a vector from $C(12, 5, 3)$ to a point on the line
$\mathbf{CP}.[8, 3, -6] = 0$	M1*	For using scalar product for perpendicularity
$t = \pm 1$ *OR* $P = (9, 5, -1)$	A1 √	For correct point. f.t. from incorrect **CP**
$d = \sqrt{3^2+0^2+4^2}$	M1 (dep*) A1	For finding magnitude of **CP** For correct expression for d
(d =) 5	A1 **6**	For correct distance **CAO**
		SR Obtain $\mathbf{CP} = [11, 3, -2]-[8, 3, -6] = \pm[3, 0, 4]$ B1 Verify $[3, 0, 4].[8, 3, -6] = 0$ M1* $d = \sqrt{3^2+0^2+4^2}$ = 5 M1(dep*) A1 A1 (maximum 5 / 6)
	$\boxed{6}$	

4 Integrating factor $e^{\int -\frac{x^2}{1+x^3}dx}$	M1	For correct process for finding integrating factor
$= e^{-\frac{1}{3}\ln(1+x^3)} = \left(1+x^3\right)^{-\frac{1}{3}}$	A1	For correct IF, simplified (here or later)
$\Rightarrow \frac{d}{dx}\left(y\left(1+x^3\right)^{-\frac{1}{3}}\right) = \frac{x^2}{\left(1+x^3\right)^{\frac{1}{3}}}$	M1	For multiplying through by their IF
$\Rightarrow y\left(1+x^3\right)^{-\frac{1}{3}} = \frac{1}{2}\left(1+x^3\right)^{\frac{2}{3}}\ (+c)$	M1	For integrating RHS to obtain $A\left(1+x^3\right)^k$ *OR* $\ln A\left(1+x^3\right)^k$
	A1	For correct integration (+*c* not required here)
$\Rightarrow 1 = \frac{1}{2}+c \quad \Rightarrow \quad c = \frac{1}{2}$	M1 A1 √	For substituting (0, 1) into GS (including + *c*) For correct *c*. f.t. from their GS
$\Rightarrow y = \frac{1}{2}\left(1+x^3\right)+\frac{1}{2}\left(1+x^3\right)^{\frac{1}{3}}$	A1	For correct solution. **AEF** in form $y = f(x)$
	$\boxed{8}$	
5 **(i)** *EITHER* $\mathbf{a} = [2, 3, 5],\ \mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \pm k[-10, 10, -2]$	M1 A1 √	For finding perpendicular to plane For correct **n**. f.t. from incorrect **b**
Use (2, 1, 5) *OR* (0, –1, 5)	M1	For substituting a point into equation $ax+by+cz=d$ where $[a, b, c]$ = their **n**
$\Rightarrow 5x-5y+z=10$	A1	For correct cartesian equation **AEF**
OR $\mathbf{a} = [2, 3, 5],\ \mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
e.g. $\mathbf{r} = [2, 1, 5]+\lambda[2, 2, 0]+\mu[2, 3, 5]$	M1	For stating parametric equation of plane
$[x, y, z] = [2+2\lambda+2\mu, 1+2\lambda+3\mu, 5+5\mu]$	A1 √	For writing 3 equations in *x*, *y*, *z* f.t. from incorrect **b**
	M1	For eliminating λ and μ
$\Rightarrow 5x-5y+z=10$	A1 **5**	For correct cartesian equation **AEF**
(ii) $[2t, 3t-4, 5t-9]$	B1 **1**	For stating a point *A* on l_1 with parameter *t* **AEF**
(iii) $\pm[2t+5, 3t-7, 5t-13]$	M1	For finding direction of l_2 from *A* and (–5,3, 4)
$\pm[2t+5, 3t-7, 5t-13].[2, 3, 5]=0$	M1	For using scalar product for perpendicularity with any vector involving *t*
$\Rightarrow t=2$	A1	For correct value of *t*
$\frac{x+5}{9}=\frac{y-3}{-1}=\frac{z-4}{-3}$ *OR* $\frac{x-4}{9}=\frac{y-2}{-1}=\frac{z-1}{-3}$	A1 **4**	For a correct equation **AEFcartesian**
		SR For $2p+3q+5r=0$ and no further progress award B1
	$\boxed{10}$	

6 (i) $\left(m^2+4=0\Rightarrow\right) m=\pm 2\mathrm{i}$	B1	For correct solutions of auxiliary equation (may be implied by correct CF)
CF = $A\cos 2x+B\sin 2x$	B1	For correct CF (**AEtrig** but not $A\mathrm{e}^{2\mathrm{i}x}+B\mathrm{e}^{-2\mathrm{i}x}$ only)
PI = $p\sin x\ (+\ q\cos x)$	B1	State a trial PI with at least $p\sin x$
$-p\sin x\ (-q\cos x)+4p\sin x\ (+4q\cos x)=\sin x$	M1	For substituting PI into DE
$\Rightarrow p=\frac{1}{3},\quad q=0$	A1	For correct *p* and *q* (which may be implied)
$\Rightarrow y=A\cos 2x+B\sin 2x+\frac{1}{3}\sin x$	B1 √ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
(ii) $(0,0)\Rightarrow A=0$	B1 √	For correct equation in *A* and/or *B* f.t. from their GS
$\frac{\mathrm{d}y}{\mathrm{d}x}=2B\cos 2x+\frac{1}{3}\cos x\Rightarrow\frac{4}{3}=2B+\frac{1}{3}$	M1	For differentiating their GS and substituting values for *x* and $\frac{\mathrm{d}y}{\mathrm{d}x}$
$A=0,\ B=\frac{1}{2}$	A1	For correct *A* and *B* Allow $A=-\frac{1}{4}\mathrm{i},\ B=\frac{1}{4}\mathrm{i}$ from CF $A\mathrm{e}^{2\mathrm{i}x}+B\mathrm{e}^{-2\mathrm{i}x}$
$\Rightarrow y=\frac{1}{2}\sin 2x+\frac{1}{3}\sin x$	A1 **4** $\boxed{10}$	For stating correct solution **CAO**
7 (i) $C+\mathrm{i}S=1+\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{2\mathrm{i}\theta}+\mathrm{e}^{3\mathrm{i}\theta}+\mathrm{e}^{4\mathrm{i}\theta}+\mathrm{e}^{5\mathrm{i}\theta}$	M1	For using de Moivre, showing at least 3 terms
$=\frac{\mathrm{e}^{6\mathrm{i}\theta}-1}{\mathrm{e}^{\mathrm{i}\theta}-1}$	M1 A1	For recognising GP For correct GP sum
$=\frac{\mathrm{e}^{3\mathrm{i}\theta}-\mathrm{e}^{-3\mathrm{i}\theta}}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}-\mathrm{e}^{-\frac{1}{2}\mathrm{i}\theta}}\cdot\frac{\mathrm{e}^{3\mathrm{i}\theta}}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}}=\frac{\mathrm{e}^{3\mathrm{i}\theta}-\mathrm{e}^{-3\mathrm{i}\theta}}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}-\mathrm{e}^{-\frac{1}{2}\mathrm{i}\theta}}\mathrm{e}^{\frac{5}{2}\mathrm{i}\theta}$	A1 **4**	For obtaining correct expression **AG**
(ii) $C+\mathrm{i}S=\frac{2\mathrm{i}\sin 3\theta}{2\mathrm{i}\sin\frac{1}{2}\theta}\cdot\mathrm{e}^{\frac{5}{2}\mathrm{i}\theta}$	M1 A1	For expressing numerator and denominator in terms of sines For $k\sin 3\theta$ and $k\sin\frac{1}{2}\theta$
$\mathrm{Re}\Rightarrow C=\sin 3\theta\cos\frac{5}{2}\theta\,\mathrm{cosec}\frac{1}{2}\theta$	A1	For correct expression **AG**
$\mathrm{Im}\Rightarrow S=\sin 3\theta\sin\frac{5}{2}\theta\,\mathrm{cosec}\frac{1}{2}\theta$	B1 **4**	For correct expression
(iii) $C=S\ \Rightarrow\ \sin 3\theta=0,\ \tan\frac{5}{2}\theta=1$	M1	For either equation deduced **AEF** Ignore values outside $0<\theta<\pi$
$\theta=\frac{1}{3}\pi,\frac{2}{3}\pi$	A1	For both values correct and no extras
$\theta=\frac{1}{10}\pi,\frac{1}{2}\pi,\frac{9}{10}\pi$	A2 **4** $\boxed{12}$	For all values correct and no extras. Allow A1 for any 1 value *OR* all correct with extras

8 **(i)** $r^4.a \neq a.r^4$	B1 **1**	For stating the non-commutative product in the given table, or justifying another correct one
(ii) Possible subgroups order 2, 5	B1 B1 **2**	For either order stated For both orders stated, and no more (Ignore 1)
(iii) **(a)** $\{e, a\}$ **(b)** $\{e, r, r^2, r^3, r^4\}$	B1 B1 **2**	For correct subgroup For correct subgroup
(iv) order of $r^3 = 5$ $(ar)^2 = ar.ar = r^4a.ar = e$ ⇒ order of $ar = 2$ $(ar^2)^2 = ar^2ar.r = ar^2r^4a.r = ara.r = e$ ⇒ order of $ar^2 = 2$	B1 M1 A1 A1 **4**	For correct order For attempt to find $(ar)^m = e$ *OR* $(ar^2)^m = e$ For correct order For correct order
(v) $\begin{array}{c\|cccc} & ar & ar^2 & ar^3 & ar^4 \\ \hline ar & e & r & r^2 & r^3 \\ ar^2 & r^4 & e & r & r^2 \\ ar^3 & r^3 & r^4 & e & r \\ ar^4 & r^2 & r^3 & r^4 & e \end{array}$	 B1 B1 B1 B1 B1 **5** $\boxed{14}$	If the border elements ar ar^2 ar^3 ar^4 are not written, it will be assumed that the products arise from that order For all 16 elements of the form *e* or r^m For all 4 elements in leading diagonal = *e* For no repeated elements in any completed row or column For any two rows or columns correct For all elements correct