OCR
RECOGNISING ACHIEVEMENT

Friday 27 January 2012 – Morning

A2 GCE MATHEMATICS

4727 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x^2 + y^2}{xy}. \qquad \text{(A)}$$

(i) Use the substitution $y = ux$, where u is a function of x, to obtain the differential equation

$$x\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{u}. \qquad \textbf{[3]}$$

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y^2 = \mathrm{f}(x)$. **[4]**

2 **(i)** Show that $\left(z^n - \mathrm{e}^{\mathrm{i}\theta}\right)\left(z^n - \mathrm{e}^{-\mathrm{i}\theta}\right) \equiv z^{2n} - (2\cos\theta)\, z^n + 1$. **[1]**

(ii) Express $z^4 - z^2 + 1$ as the product of four factors of the form $\left(z - \mathrm{e}^{\mathrm{i}\alpha}\right)$, where $0 \leqslant \alpha < 2\pi$. **[6]**

3 A multiplicative group contains the distinct elements e, x and y, where e is the identity.

(i) Prove that $x^{-1}y^{-1} = (yx)^{-1}$. **[2]**

(ii) Given that $x^n y^n = (xy)^n$ for some integer $n \geqslant 2$, prove that $x^{n-1}y^{n-1} = (yx)^{n-1}$. **[3]**

(iii) If $x^{n-1}y^{n-1} = (yx)^{n-1}$, does it follow that $x^n y^n = (xy)^n$? Give a reason for your answer. **[2]**

4 The line l has equations $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{2}$ and the point A is (7, 3, 7). M is the point where the perpendicular from A meets l.

(i) Find, in either order, the coordinates of M and the perpendicular distance from A to l. **[7]**

(ii) Find the coordinates of the point B on AM such that $\overrightarrow{AB} = 3\overrightarrow{BM}$. **[3]**

5 The variables x and y satisfy the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 5\mathrm{e}^{-2x}.$$

(i) Find the complementary function of the differential equation. **[2]**

(ii) Given that there is a particular integral of the form $y = px\mathrm{e}^{-2x}$, find the constant p. **[4]**

(iii) Find the solution of the equation for which $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ when $x = 0$. **[5]**

6 The plane Π has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -5 \end{pmatrix}$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$.

(i) Express the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. **[4]**

(ii) Find the point of intersection of l and Π. **[2]**

(iii) The equation of Π may be expressed in the form $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \mu\mathbf{c}$, where $\mathbf{c}$ is perpendicular to $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$. Find $\mathbf{c}$. **[3]**

7 The set M consists of the six matrices $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$, where $n \in \{0, 1, 2, 3, 4, 5\}$. It is given that M forms a group $(M, \times)$ under matrix multiplication, with numerical addition and multiplication both being carried out modulo 6.

(i) Determine whether $(M, \times)$ is a commutative group, justifying your answer. **[2]**

(ii) Write down the identity element of the group and find the inverse of $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. **[3]**

(iii) State the order of $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ and give a reason why $(M, \times)$ has no subgroup of order 4. **[2]**

(iv) The multiplicative group G has order 6. All the elements of G, apart from the identity, have order 2 or 3. Determine whether G is isomorphic to $(M, \times)$, justifying your answer. **[2]**

8 **(i)** Use de Moivre's theorem to prove that

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$ **[4]**

(ii) Solve the equation $\tan 5\theta = 1$, for $0 \leqslant \theta < \pi$. **[3]**

(iii) Show that the roots of the equation

$$t^4 - 4t^3 - 14t^2 - 4t + 1 = 0$$

may be expressed in the form $\tan\alpha$, stating the exact values of α, where $0 \leqslant \alpha < \pi$. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE