

# Mathematics

Advanced GCE

Unit **4727**: Further Pure Mathematics 3

## **Mark Scheme for January 2012**

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Annotations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions for GCE Mathematics Pure strand**

- a. Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

## g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$(y = xu \Rightarrow) \frac{dy}{dx} = x \frac{du}{dx} + u$ $x \frac{du}{dx} + u = \frac{2 + u^2}{u}$ $\Rightarrow x \frac{du}{dx} = \frac{2}{u}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For a correct statement</p> <p>For using the substitution to eliminate <math>y</math> (If B0, then <math>y</math> must be eliminated from LHS, but <math>\frac{d(uv)}{dx}</math> sufficient)</p> <p>For correct equation <b>AG</b></p>
1	(ii)	$\int u \, du = \int \frac{2}{x} \, dx$ $\Rightarrow \frac{1}{2}u^2 = 2\ln(kx) \text{ OR } \frac{1}{2}u^2 = 2\ln x + c$ $\Rightarrow \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln(kx) \text{ OR } \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln x + c$ $\Rightarrow y^2 = 4x^2 \ln(kx) \text{ OR } y^2 = 4x^2 \ln x + Cx^2$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For separating variables and writing/attempting integrals</p> <p>For correct integration both sides (<math>k</math> or <math>c</math> not required here)</p> <p>For substituting for <math>u</math> into integrated terms with constant (on either side)</p> <p>For correct solution <b>AEF</b> <math>y^2 = f(x)</math> Do not penalise “<math>c</math>” being used for different constants e.g. <math>2\ln x + c = 2\ln(cx)</math></p>
2	(i)	$(z^n - e^{i\theta})(z^n - e^{-i\theta}) \equiv z^{2n} - 2z^n \left( \frac{e^{i\theta} + e^{-i\theta}}{2} \right) + 1$ $\equiv z^{2n} - (2\cos\theta)z^n + 1$	<p>B1</p> <p>[1]</p>	<p>For multiplying out to <b>AG</b> with evidence of <math>\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})</math> (Can be implied by <math>2\cos\theta = (e^{i\theta} + e^{-i\theta})</math>)</p>

Question	Answer	Marks	Guidance
2 (ii)	<p>METHOD 1</p> $2 \cos \theta = 1 \Rightarrow \theta = \frac{1}{3} \pi$ $\Rightarrow z^4 - z^2 + 1 \equiv \left( z^2 - e^{\frac{1}{3}\pi i} \right) \left( z^2 - e^{-\frac{1}{3}\pi i} \right)$ $\equiv \left( z + e^{\frac{1}{6}\pi i} \right) \left( z - e^{\frac{1}{6}\pi i} \right) \left( z + e^{-\frac{1}{6}\pi i} \right) \left( z - e^{-\frac{1}{6}\pi i} \right)$ $\equiv \left( z - e^{\frac{1}{6}\pi i} \right) \left( z - e^{\frac{5}{6}\pi i} \right) \left( z - e^{\frac{7}{6}\pi i} \right) \left( z - e^{\frac{11}{6}\pi i} \right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>For using (i) to find <math>\theta</math></p> <p>For correct quadratic factors (Or <math>\frac{5\pi}{3}i</math> in place of <math>-\frac{\pi}{3}i</math>)</p> <p>For factorising <math>(z^2 - a^2)</math></p> <p>For correct linear factors</p> <p>For adjusting arguments (must attempt correct range <b>and</b> “(z – root)”)</p> <p>For correct factors <b>CAO</b></p> <p>Correct answer www gets 6</p>
	<p>METHOD 2</p> $z^4 - z^2 + 1 = 0 \Rightarrow z^2 = \frac{1}{2} \pm \frac{1}{2} \sqrt{3}i = e^{\frac{1}{3}\pi i}, e^{-\frac{1}{3}\pi i}$ $\Rightarrow z = \pm e^{\frac{1}{6}\pi i}, \pm e^{-\frac{1}{6}\pi i}$ $= e^{\frac{1}{6}\pi i}, e^{\frac{7}{6}\pi i}, e^{\frac{5}{6}\pi i}, e^{\frac{11}{6}\pi i}$ $\Rightarrow \left( z - e^{\frac{1}{6}\pi i} \right) \left( z - e^{\frac{5}{6}\pi i} \right) \left( z - e^{\frac{7}{6}\pi i} \right) \left( z - e^{\frac{11}{6}\pi i} \right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For solving quadratic</p> <p>For correct roots in exp form</p> <p>For attempt to find 4 roots</p> <p>For correct roots <math>\pm e^{i\alpha}</math></p> <p>For adjusting arguments</p> <p>For correct factors <b>CAO</b></p>
3 (i)	<p>METHOD 1</p> $(yx)(yx)^{-1} = e \Rightarrow x(yx)^{-1} = y^{-1}$ $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$ <p>METHOD 2</p> <p>Compare <math>(yx)(yx)^{-1} = e</math> with <math>yx x^{-1} y^{-1} = e</math></p> $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p>	<p>For starting point and appropriate multiplication</p> <p>For correct result <b>AG</b></p> <p>For appropriate comparison</p> <p>For correct result <b>AG</b></p> <p>For A1, proof cannot be written in the form ‘LHS = RHS <math>\rightarrow \dots \rightarrow e = e</math>’</p>



Question		Answer	Marks	Guidance
3	(ii)	$x^n y^n = (xy)^n = x(yx)^{n-1} y$ $\Rightarrow x^{-1} x^n y^n y^{-1} = x^{-1} x (yx)^{n-1} y y^{-1}$ $\Rightarrow x^{n-1} y^{n-1} = (yx)^{n-1}$	M1 M1 A1 [3]	For using associativity or an inverse with respect to LHS, RHS or initial equality <b>www beforehand</b> For using $(xy)^n = x(yx)^{n-1} y$ <b>oe</b> For correct result <b>AG</b> <b>SR</b> for numerical $n$ used, allow M1 M1 only
3	(iii)	METHOD 1 All steps in <b>(ii)</b> are reversible $\Rightarrow$ result follows  METHOD 2 Show working for <b>(ii)</b> in reverse $\Rightarrow$ result follows	B1*dep B1*dep [2]  B1* B1*dep	For correct reason. Dep on correct part(ii) For correct conclusion  For correct working For correct conclusion

Question	Answer	Marks	Guidance
4	<p>(i)</p> <p>METHOD 1 (<math>M</math>, then distance)  <math>M = (1 + 2t, 1 + 3t, -1 + 2t)</math>  <math>\mathbf{AM} = (\pm)[2t - 6, 3t - 2, 2t - 8]</math>  <math>\mathbf{AM}</math> perp. <math>l \Rightarrow 2(2t - 6) + 3(3t - 2) + 2(2t - 8) = 0</math>  <math>\Rightarrow t = 2, M = (5, 7, 3)</math>  <math>AM = \sqrt{2^2 + 4^2 + 4^2} = 6</math></p> <p>METHOD 2(a) (distance, then <math>M</math>)  <math>(C = (1, 1, -1)) \mathbf{AC} = \pm[6, 2, 8]</math>  <math>\mathbf{n} = \mathbf{AC} \times [2, 3, 2] = k[-20, 4, 14]</math>  <math>d = \frac{ \mathbf{n} }{ [2, 3, 2] } = \frac{\sqrt{612}}{\sqrt{17}} = 6</math>  <math>CM = \sqrt{(6^2 + 2^2 + 8^2)} - 6 = 2\sqrt{17}</math>  <math> [2, 3, 2]  = \sqrt{17} \Rightarrow t = 2, M = (5, 7, 3)</math></p> <p>METHOD 2(b)  <math>(C = (1, 1, -1)) \mathbf{AC} = \pm[6, 2, 8]</math>  <math>\cos \theta = \frac{\mathbf{AC} \cdot (2, 3, 2)}{ \mathbf{AC}   (2, 3, 2) }, \theta = 36.0(39..)</math> or <math>\sin \theta = \frac{153}{\sqrt{442}}</math>  <math> \mathbf{AM}  =  \mathbf{AC}  \sin \theta = 6</math>  <math>M = (5, 7, 3)</math></p>	<p>B1</p> <p>B1 FT</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[7]</p> <p>B1</p> <p>M1</p> <p>A1 FT</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>B1</p> <p>M1,A1</p> <p>M1,A1</p> <p>M1,A1</p>	<p>Coordinates or vectors allowed throughout</p> <p>For correct parametric form soi</p> <p>For correct vector. FT from <math>M</math></p> <p>For using perpendicular condition</p> <p>For correct equation</p> <p>For correct coordinates</p> <p>For using distance formula</p> <p>For correct distance</p> <p>For correct vector</p> <p>For finding <math>\mathbf{AC} \times</math> direction of <math>l</math></p> <p>For correct <math> \mathbf{n} </math>. FT from <math>\mathbf{n}</math></p> <p>For correct distance</p> <p>For a correct method for finding position of <math>M</math></p> <p>For <math> [2, 3, 2]  = \sqrt{17}</math> soi</p> <p>For correct vector</p> <p>As above</p>

Question		Answer	Marks	Guidance
4	(ii)	$\mathbf{AM} = [-2, 4, -4]$ or $\mathbf{MA} = [2, -4, 4]$ $\Rightarrow B = (7, 3, 7) + \frac{3}{4}(-2, 4, -4) = \left(7 - \frac{3}{2}, 3 + 3, 7 - 3\right)$ OR $B = (5, 7, 3) + \frac{1}{4}(2, -4, 4) = \left(5 + \frac{1}{2}, 7 - 1, 3 + 1\right)$ OR $B = \frac{3}{4}(5, 7, 3) + \frac{1}{4}(7, 3, 7) = \left(\frac{15}{4} + \frac{7}{4}, \frac{21}{4} + \frac{3}{4}, \frac{9}{4} + \frac{7}{4}\right)$ $B = \left(\frac{11}{2}, 6, 4\right)$	M1  M1  A1  [3]	For using $A + k_1 \vec{AM}$ or $M + k_2 \vec{MA}$ or ratio theorem or equivalent  For $B = (7, 3, 7) + \frac{3}{4}x$ their $(-2, 4, -4)$ oe (or M1 for quadratic in parameter for line AM, followed by M1 for attempt to use <b>correct</b> value of parameter to find B)  For correct coordinates
5	(i)	$(2m^2 + 3m - 2 = 0) \Rightarrow m = \frac{1}{2}, -2$ $CF = Ae^{\frac{1}{2}x} + Be^{-2x}$	M1  A1  [2]	For attempt to solve correct auxiliary equation  For correct CF
5	(ii)	$\frac{dy}{dx} = pe^{-2x} - 2pxe^{-2x}$ $\frac{d^2y}{dx^2} = -4pe^{-2x} + 4pxe^{-2x}$ $\Rightarrow (-8p + 3p + 8px - 6px - 2px)e^{-2x} = 5e^{-2x}$ $\Rightarrow p = -1$	M1  A1  M1  A1  [4]	For differentiating PI twice, using product rule  $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ For correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ For substituting into DE  For correct p

Question		Answer	Marks	Guidance
5	(iii)	$GS (y =) Ae^{\frac{1}{2}x} + Be^{-2x} - xe^{-2x}$ $(0, 0) \Rightarrow A + B = 0$ $\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} - 2Be^{-2x} - e^{-2x} + 2xe^{-2x}$ $\left(0, \frac{dy}{dx} = 4\right) \Rightarrow \frac{1}{2}A - 2B = 5$ $\Rightarrow A = 2, B = -2$ $\Rightarrow y = 2e^{\frac{1}{2}x} - 2e^{-2x} - xe^{-2x}$	B1 FT B1 FT  M1 M1 A1 [5]	For GS soi. FT from CF (2 constants) and $p$ For correct equation. FT from GS of form $Ae^{\alpha x} + Be^{\beta x} - Cxe^{-2x}$ For differentiating GS and substituting values, using GS of form $Ae^{\alpha x} + Be^{\beta x} - Cxe^{-2x}$ For solving for $A$ and $B$ (can be gained from incorrect GS) For correct solution, including $y =$
6	(i)	METHOD 1 $\mathbf{n} = [2, -1, -1] \times [2, -3, -5] = [2, 8, -4]$ $\mathbf{n} = k[1, 4, -2]$ $\Pi \text{ is } \mathbf{r} \cdot \mathbf{n} = [1, 6, 7] \cdot \mathbf{n}$ $\Rightarrow \mathbf{r} \cdot [1, 4, -2] = 11$ METHOD 2 $y - z = -1 + 2\mu$ $\mu = \frac{y - z + 1}{2}$ $\lambda = 7 - z - 5 \frac{y - z + 1}{2}$ $x = 11 + 2z - 4y$ $r \cdot (1, 4, -2) = 11$	M1 A1 M1 A1  M1  M1 A1 A1 [4]	For finding vector product of 2 vectors in $\Pi$ (or 2 scalar products = 0, with attempt to solve) For correct $\mathbf{n}$ For attempt to find equation of $\Pi$ , including cartesian equation For correct equation (allow multiples)  for finding $\lambda$ or $\mu$ in terms of two from $x, y, z$ .  For both $\lambda$ & $\mu$ AEF

Question		Answer	Marks	Guidance
6	(ii)	$[7 + 3t, 4, 1 - t] \cdot \mathbf{n} = 11 \Rightarrow t = -2$ $\Rightarrow [1, 4, 3]$	M1 A1 [2]	For attempt to find $t$ , (or to find $\lambda$ and $\mu$ by equating original equations) For correct position vector <i>OR</i> point
6	(iii)	METHOD 1 $\mathbf{c} = [1, 4, -2] \times [2, -1, -1]$  $\mathbf{c} = k[2, 1, 3]$  METHOD 2 $\mathbf{c} = [2, -3, -5] + s[2, -1, -1]$ $\mathbf{c} \cdot [2, -1, -1] = 0 \Rightarrow$ $2(2 + 2s) - 1(-3 - s) - 1(-5 - s) = 0$ $\Rightarrow s = -2 \Rightarrow \mathbf{c} = k[2, 1, 3]$	M1 M1  A1 [3]  M1 M1  A1	For using given vector product (or 2 correct 'scalar products = 0')  For calculating given vector product (or 2 correct scalar products = 0, with attempt to solve) (or M1 for using vector product of $\mathbf{c}$ with $\mathbf{n}$ or $(2, -1, -1)$ in an equation, followed by M1 for calculating vector product and attempting to solve)  For correct $\mathbf{c}$  For $\mathbf{c} =$ linear combination of $[2, -3, -5]$ and $[2, -1, -1]$ For an equation in $s$ from $\mathbf{c} \cdot [2, -1, -1] = 0$  For correct $\mathbf{c}$

Question		Answer	Marks	Guidance
7	(i)	$\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ n+m & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ m+n & 1 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \Rightarrow \text{commutative}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>For multiplying 2 distinct matrices of the correct form both ways, or generalised form at least one way,</p> <p>For stating or implying that addition is commutative <b>and</b> correct conclusion</p> <p><b>SR</b> Use of numerical matrices must be generalised for any credit</p>
7	(ii)	$(I \Rightarrow) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>EITHER</p> $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$ <p>OR</p> $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 2+n=0 \Rightarrow \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For correct identity</p> <p>For using inverse property</p> <p>For correct inverse</p>
7	(iii)	$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ has order 2 4 is not a factor of 6	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>For correct order</p> <p>For correct reason (Award B0 for “Lagrange” only). Must be explicit about the ‘6’</p>
7	(iv)	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ OR $\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ has order 6, (or > 3) OR $(M, \times)$ is cyclic, $G$ is non-cyclic (having no element of order 6) OR $(M, \times)$ is commutative $G$ is not commutative (being the non-cyclic group) $\Rightarrow$ groups are not isomorphic	<p>B1*</p> <p>B1*dep</p> <p>[2]</p>	<p>For stating (that there is) an element of <math>M</math> with order 6</p> <p>Award B1* for a relevant statement about <math>M</math> and <math>G</math></p> <p>For correct conclusion and no false statements attached to conclusion</p>

Question		Answer	Marks	Guidance
8	(i)	$\cos 5\theta + i \sin 5\theta =$ $c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$ $\Rightarrow \tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$  Division of numerator & denominator by $c^5$ . $\Rightarrow \tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$	B1 M1  M1 A1 [4]	For explicit use of de Moivre with $n = 5$ For correct expressions for $\sin 5\theta$ and $\cos 5\theta$  For $\frac{\sin 5\theta}{\cos 5\theta}$ in terms of $c$ and $s$ For simplifying to <b>AG, www</b> with explicit mention of division by $c^5$
8	(ii)	$5\theta = \{1, 5, 9, 13, 17\} \frac{1}{4} \pi$ $\theta = \{1, 5, 9, 13, 17\} \frac{1}{20} \pi$	M1 A1 A1 [3]	For at least 2 of given values and no extras. For at least 3 values of $\theta$ and no extras in range For all 5 values and no extras outside range
8	(iii)	$\tan 5\theta = 1 \Rightarrow t^5 - 5t^4 - 10t^3 + 10t^2 + 5t - 1 = 0$ $\Rightarrow (t-1)(t^4 - 4t^3 - 14t^2 - 4t + 1) = 0$ $\tan \alpha = 1$ OR $\alpha = \frac{1}{4}\pi$ is not included in roots of the quartic $\Rightarrow t = \tan \alpha$ for $\alpha = \{1, 9, 13, 17\} \frac{1}{20} \pi$	M1* A1 B1 M1*dep A1 [5]	For $\tan 5\theta = 1$ and equation in $t$ For correct factors For solution rejected (may be implied by $\frac{5}{20}\pi$ not appearing in set of solutions) For 2 correct values of $t$ For all 4 values and no more in range

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