RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4727

Further Pure Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 29 January 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \qquad \text{and} \qquad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$

intersect or are skew. **[5]**

2 H denotes the set of numbers of the form $a + b\sqrt{5}$, where a and b are rational. The numbers are combined under multiplication.

(i) Show that the product of any two members of H is a member of H. **[2]**

It is now given that, for a and b not both zero, H forms a group under multiplication.

(ii) State the identity element of the group. **[1]**

(iii) Find the inverse of $a + b\sqrt{5}$. **[2]**

(iv) With reference to your answer to part **(iii)**, state a property of the number 5 which ensures that every number in the group has an inverse. **[1]**

3 Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{-3x}$$

for which $y = 1$ when $x = 0$. Express your answer in the form $y = f(x)$. **[6]**

4 **(i)** Write down, in cartesian form, the roots of the equation $z^4 = 16$. **[2]**

(ii) Hence solve the equation $w^4 = 16(1 - w)^4$, giving your answers in cartesian form. **[5]**

5 A regular tetrahedron has vertices at the points

$$A\left(0, 0, \tfrac{2}{3}\sqrt{6}\right), \quad B\left(\tfrac{2}{3}\sqrt{3}, 0, 0\right), \quad C\left(-\tfrac{1}{3}\sqrt{3}, 1, 0\right), \quad D\left(-\tfrac{1}{3}\sqrt{3}, -1, 0\right).$$

(i) Obtain the equation of the face ABC in the form

$$x + \sqrt{3}y + \left(\tfrac{1}{2}\sqrt{2}\right)z = \tfrac{2}{3}\sqrt{3}.$$ **[5]**

(Answers which only verify the given equation will not receive full credit.)

(ii) Give a geometrical reason why the equation of the face ABD can be expressed as

$$x - \sqrt{3}y + \left(\tfrac{1}{2}\sqrt{2}\right)z = \tfrac{2}{3}\sqrt{3}.$$ **[2]**

(iii) Hence find the cosine of the angle between two faces of the tetrahedron. **[4]**

6 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 16y = 8\cos 4x.$$

(i) Find the complementary function of the differential equation. **[2]**

(ii) Given that there is a particular integral of the form $y = px\sin 4x$, where p is a constant, find the general solution of the equation. **[6]**

(iii) Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[4]**

7 **(i)** Solve the equation $\cos 6\theta = 0$, for $0 < \theta < \pi$. **[3]**

(ii) By using de Moivre's theorem, show that

$$\cos 6\theta \equiv (2\cos^2\theta - 1)(16\cos^4\theta - 16\cos^2\theta + 1).$$ **[5]**

(iii) Hence find the exact value of

$$\cos\left(\tfrac{1}{12}\pi\right)\cos\left(\tfrac{5}{12}\pi\right)\cos\left(\tfrac{7}{12}\pi\right)\cos\left(\tfrac{11}{12}\pi\right),$$

justifying your answer. **[5]**

8 The function f is defined by $\mathrm{f} : x \mapsto \frac{1}{2-2x}$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$. The function g is defined by $\mathrm{g}(x) = \mathrm{ff}(x)$.

(i) Show that $\mathrm{g}(x) = \frac{1-x}{1-2x}$ and that $\mathrm{gg}(x) = x$. **[4]**

It is given that f and g are elements of a group K under the operation of composition of functions. The element e is the identity, where $\mathrm{e} : x \mapsto x$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$.

(ii) State the orders of the elements f and g. **[2]**

(iii) The inverse of the element f is denoted by h. Find $\mathrm{h}(x)$. **[2]**

(iv) Construct the operation table for the elements e, f, g, h of the group K. **[4]**