OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Further Pure Mathematics 3

4727

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 29 January 2009
Morning

Duration: 1 hour 30 minutes

4727

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 In this question G is a group of order n, where $3 \leqslant n < 8$.

(i) In each case, write down the smallest possible value of n:

(a) if G is cyclic, **[1]**

(b) if G has a proper subgroup of order 3, **[1]**

(c) if G has at least two elements of order 2. **[1]**

(ii) Another group has the same order as G, but is not isomorphic to G. Write down the possible value(s) of n. **[2]**

2 **(i)** Express $\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[3]**

(ii) Hence find the smallest positive value of n for which $\left(\dfrac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}}\right)^n$ is real and positive. **[2]**

3 Two skew lines have equations

$$\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3} \qquad \text{and} \qquad \frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}.$$

(i) Find the direction of the common perpendicular to the lines. **[2]**

(ii) Find the shortest distance between the lines. **[4]**

4 Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 5y = 65\sin 2x. \quad \textbf{[9]}$$

5 The variables x and y are related by the differential equation

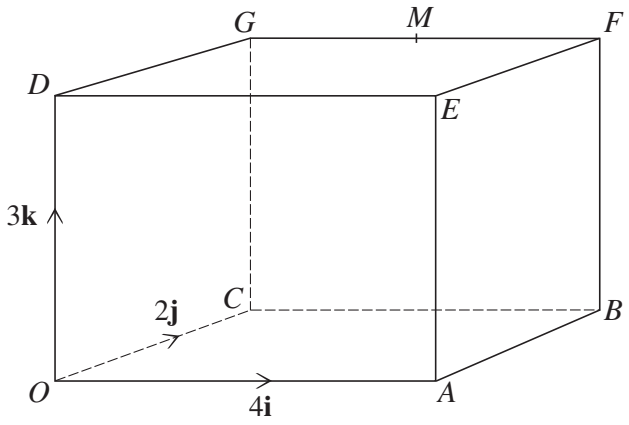
$$x^3\frac{\mathrm{d}y}{\mathrm{d}x} = xy + x + 1. \qquad \text{(A)}$$

(i) Use the substitution $y = u - \dfrac{1}{x}$, where u is a function of x, to show that the differential equation may be written as

$$x^2\frac{\mathrm{d}u}{\mathrm{d}x} = u. \quad \textbf{[4]}$$

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form $y = \mathrm{f}(x)$. **[5]**

6



The cuboid $OABCDEFG$ shown in the diagram has $\overrightarrow{OA} = 4\mathbf{i}$, $\overrightarrow{OC} = 2\mathbf{j}$, $\overrightarrow{OD} = 3\mathbf{k}$, and M is the mid-point of GF.

(i) Find the equation of the plane $ACGE$, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. **[4]**

(ii) The plane $OEFC$ has equation $\mathbf{r}.(3\mathbf{i} - 4\mathbf{k}) = 0$. Find the acute angle between the planes $OEFC$ and $ACGE$. **[4]**

(iii) The line AM meets the plane $OEFC$ at the point W. Find the ratio $AW : WM$. **[5]**

7 **(i)** The operation $*$ is defined by $x * y = x + y - a$, where x and y are real numbers and a is a real constant.

(a) Prove that the set of real numbers, together with the operation $*$, forms a group. **[6]**

(b) State, with a reason, whether the group is commutative. **[1]**

(c) Prove that there are no elements of order 2. **[2]**

(ii) The operation $\circ$ is defined by $x \circ y = x + y - 5$, where x and y are **positive** real numbers. By giving a numerical example in each case, show that two of the basic group properties are not necessarily satisfied. **[4]**

8 **(i)** By expressing $\sin\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^6\theta \equiv -\tfrac{1}{32}(\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10). \quad \textbf{[5]}$$

(ii) Replace θ by $\left(\frac{1}{2}\pi - \theta\right)$ in the identity in part **(i)** to obtain a similar identity for $\cos^6\theta$. **[3]**

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \left(\sin^6\theta - \cos^6\theta\right) \mathrm{d}\theta$. **[4]**

There are no questions printed on this page.