OCR Maths FP3

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1 Directions $[1, 1, -1]$ and $[2, -3, 1]$	B1	For identifying both directions (may be implied by working)
$\theta = \cos^{-1} \frac{\lvert[1, 1, -1] . [2, -3, 1]\rvert}{\sqrt{3} \quad \sqrt{14}}$	M1	For using scalar product of their direction vectors
$= \cos^{-1} \frac{\lvert -2 \rvert}{\sqrt{42}}$	M1	For completely correct process for their angle
= 72.0°, 72° or 1.26 rad	A1 **4**	For correct answer
	$\boxed{4}$	
2 **(i)** Identities b, 6	B1 B1	For correct identities
Subgroups $\{b, d\}$, $\{6, 4\}$	B1 B1 **4**	For correct subgroups
(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$	B1 B1	For $b \leftrightarrow 6$, $d \leftrightarrow 4$
	B1 **3**	For $a, c \leftrightarrow 2, 8$ in either order
		SR If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H
	$\boxed{7}$	
3 **(i)** $3y^2 \frac{dy}{dx} = \frac{dz}{dx}$	M1	For differentiating substitution
$\Rightarrow \frac{dz}{dx} + 2xz = e^{-x^2}$	A1	For resulting equation in z and x
Integrating factor $\left(e^{\int 2x\,dx} = \right) e^{x^2}$	B1 √	For correct IF f.t. for an equation in suitable form
$\Rightarrow \frac{d}{dx}\left(ze^{x^2}\right)$ *OR* $\frac{d}{dx}\left(y^3 e^{x^2}\right) = 1$	M1	For using IF correctly
$\Rightarrow z\,e^{x^2}$ *OR* $y^3 e^{x^2} = x\ (+c)$	A1	For correct integration (+c not required here)
$\Rightarrow y = (x+c)^{\frac{1}{3}}\ e^{-\frac{1}{3}x^2}$	A1 **6**	For correct answer **AEF**
(ii) As $x \to \infty$, $y \to 0$	B1 **1**	For correct statement
	$\boxed{7}$	
4 **(i)** $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$, $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$	B1	For either expression, seen or implied z may be used for $e^{i\theta}$ throughout
$\Rightarrow \cos^2\theta \sin^4\theta = \frac{1}{4}\left(e^{i\theta} + e^{-i\theta}\right)^2 \frac{1}{16}\left(e^{i\theta} - e^{-i\theta}\right)^4$		
$= \frac{1}{4}\left(e^{2i\theta} + 2 + e^{-2i\theta}\right).\frac{1}{16}\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$	M1 A1 A1	For expanding terms For the 2 correct expansions **SR** Allow A1 A0 for $k\left(e^{2i\theta} + 2 + e^{-2i\theta}\right)\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$, $k \ne \frac{1}{64}$
$= \frac{1}{64}\left(\left(e^{6i\theta} + e^{-6i\theta}\right) - 2\left(e^{4i\theta} + e^{-4i\theta}\right) - \left(e^{2i\theta} + e^{-2i\theta}\right) + 4\right)$	M1	For grouping terms and using multiple angles
$= \frac{1}{32}\left(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2\right)$ **AG**	A1 **6**	For answer obtained correctly

(ii) $\int_0^{\frac{1}{3}\pi}\cos^2\theta\sin^4\theta\,d\theta =$		
$= \frac{1}{32}\left[\frac{1}{6}\sin 6\theta - \frac{1}{2}\sin 4\theta - \frac{1}{2}\sin 2\theta + 2\theta\right]_0^{\frac{1}{3}\pi}$	M1 A1	For integrating answer to **(i)** For all terms correct
$= \frac{1}{32}\left[0 + \frac{1}{4}\sqrt{3} - \frac{1}{4}\sqrt{3} + \frac{2}{3}\pi - 0\right] = \frac{1}{48}\pi$	A1 **3**	For correct answer
	$\boxed{9}$	
5 (i) *EITHER* $z = \sqrt{8}\,\mathrm{cis}(2k+1)\frac{\pi}{4},\ k = 0, 1, 2, 3$	B1	For correct modulus **AEF**
OR $z = \sqrt{8}\,e^{(2k+1)\frac{\pi}{4}i},\ k = 0, 1, 2, 3$	B1 **2**	For correct arguments **AEF**
(ii)		
$z = 2\sqrt{2}\left\{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right\}$	B1	For any of $\pm\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$
$z = 2 + 2i,\ -2 + 2i,\ -2 - 2i,\ 2 - 2i$	B1	For any one value of z correct
	B1	For all values of z correct **AEFcartesian** (may be implied from symmetry or factors)
$(z - \alpha), (z - \beta), (z - \gamma), (z - \delta)$	B1 √ **4**	f.t., where $\alpha, \beta, \gamma, \delta$ are answers above
(iii) *EITHER* $(z - (2 + 2i))(z - (2 - 2i))$	M1	For combining factors from **(ii)** in pairs
$\times (z - (-2 + 2i))(z - (-2 - 2i))$	M1	Use of complex conjugate pairs
$= (z^2 + 4z + 8)(z^2 - 4z + 8)$	A1	For correct answer
OR $z^4 + 64 = (z^2 + az + b)(z^2 + cz + d)$	M1	For equating coefficients
$\Rightarrow a + c = 0,\ b + ac + d = 0,\ ad + bc = 0,\ bd = 64$	M1	For solving equations
Obtain $(z^2 + 4z + 8)(z^2 - 4z + 8)$	A1 **3**	For correct answer
	$\boxed{9}$	
6 (i) **MB** $= [2, 1, -2]$, **OF** $= [4, 1, 2]$	B1	For either vector correct (allow multiples)
MB × **OF**	M1	For finding vector product of their **MB** and **OF**
$= [4, -12, -2]$ *OR* $k[2, -6, -1]$	A1 **3**	For correct vector
(ii) *EITHER* Find vector product of any two of $\pm[2, -1, 2]$, $\pm[0, 0, 2]$, $\pm[2, -1, 0]$	M1	For finding two relevant vector products
and any two of $\pm[4, 0, 2]$, $\pm[4, -1, 0]$, $\pm[0, 1, 2]$		
Obtain $k[1, 2, 0]$	A1	For correct LHS of plane *CMG*
Obtain $k[1, 4, -2]$	A1	For correct LHS of plane *OEG*
	M1	For substituting a point into each equation
$x + 2y = 2$ and $x + 4y - 2z = 0$	A1	For both equations correct **AEF**
OR Use $ax + by + cz = d$ with coordinates of *C*, *M*, *G* *OR* *O*, *E*, *G* substituted	M1	For use of cartesian equation of plane
Obtain $a : b : c = 1 : 2 : 0$ for *CMG*	A1	For correct ratio
Obtain $a : b : c = 1 : 4 : -2$ for *OEG*	A1	For correct ratio
	M1	For substituting a point into each equation
$x + 2y = 2$ and $x + 4y - 2z = 0$	A1 **5**	For both equations correct **AEF**

(iii) *EITHER* Put x, y *OR* $z = t$ in planes *OR* evaluate $k[1, 2, 0] \times k[1, 4, -2]$	M1	For solving plane equations in terms of a parameter *OR* for finding vector product of normals to planes from **(ii)**
Obtain $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = [0, 1, 2], [2, 0, 1]$ *OR* $[4, -1, 0]$	A1	Obtain a correct point **AEF**
$\mathbf{b} = k[-2, 1, 1]$	A1 **3** $\boxed{11}$	Obtain correct direction **AEF**
7 **(i)** $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\ldots(x^{-1}ax)$	M1	For considering powers of $x^{-1}ax$
$= x^{-1}aa\ldots ax$, associativity, $xx^{-1} = e$	A1 A1	For using associativity and inverse properties
$= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$	B1	For using order of *a* correctly
$= x^{-1}x$	A1	For using property of identity
$= e \Rightarrow$ order n	A1 **6**	For correct conclusion
(ii) ***EITHER*** $(x^{-1}ax)z = e$	M1	For attempt to solve for *z* **AEF**
$\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$	A1	For using pre- or post multiplication
$\Rightarrow z = x^{-1}a^{-1}x$	A1	For correct answer
OR Use $(pq)^{-1} = q^{-1}p^{-1}$ *OR* $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$	M1	For applying inverse of a product of elements
State $(x^{-1})^{-1} = x$	A1	For stating this property
Obtain $x^{-1}a^{-1}x$	A1 **3**	For correct answer with no incorrect working **SR** correct answer with no working scores B1 only
(iii) $ax = xa \Rightarrow x = a^{-1}xa$	M1	Start from commutative property for *ax*
$\Rightarrow xa^{-1} = a^{-1}x$	A1 **2** $\boxed{11}$	Obtain commutative property for $a^{-1}x$
8 **(i)** $m^2 + 2km + 4 = 0$	M1	For stating and attempting to solve auxiliary eqn
$\Rightarrow m = -k \pm \sqrt{k^2 - 4}$	A1 **2**	For correct solutions, at any stage **AEF**
(a) $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\sqrt{k^2-4}\,t} + B\mathrm{e}^{-\sqrt{k^2-4}\,t}\right)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with distinct real roots of aux eqn For correct answer **AEF**
(b) $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\mathrm{i}\sqrt{4-k^2}\,t} + B\mathrm{e}^{-\mathrm{i}\sqrt{4-k^2}\,t}\right)$	M1	For using $\mathrm{e}^{\mathrm{f}(t)}$ with complex roots of aux eqn This form may not be seen explicitly but if stated as final answer earns M1 A0
$x = \mathrm{e}^{-kt}\left(A'\cos\sqrt{4-k^2}\,t + B'\sin\sqrt{4-k^2}\,t\right)$ *OR* $x = \mathrm{e}^{-kt}\left(C'\begin{matrix}\cos\\ \sin\end{matrix}\left(\sqrt{4-k^2}\,t + \alpha\right)\right)$	A1 **2**	For correct answer
(c) $x = \mathrm{e}^{-2t}\left(A'' + B''t\right)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with equal roots of aux eqn For correct answer. Allow *k* for 2

(ii)(a) $x = B'\mathrm{e}^{-t}\sin\sqrt{3}\,t$	B1 √	For using $t = 0, x = 0$ correctly. f.t. from **(b)**
$\dot{x} = B'\mathrm{e}^{-t}\left(\sqrt{3}\cos\sqrt{3}\,t - \sin\sqrt{3}\,t\right)$	M1 A1 √	For differentiating *x* For correct expression. f.t. from their *x*
$t = 0, \dot{x} = 6 \Rightarrow B' = 2\sqrt{3}$, $x = 2\sqrt{3}\mathrm{e}^{-t}\sin\sqrt{3}\,t$	A1 **4**	For correct solution **AEF** **SR** √ and **AEF** OK for $x = C'\mathrm{e}^{-t}\cos\left(\sqrt{3}\,t + \frac{1}{2}\pi\right)$
(b) $x \to 0$	B1	For correct statement
$\mathrm{e}^{-t} \to 0$ and sin( ) is bounded	B1 **2** $\boxed{14}$	For both statements

1 **(a)** Identity = $1+0\text{i}$	B1	For correct identity. Allow 1
Inverse = $\dfrac{1}{1+2\text{i}}$	B1	For $\dfrac{1}{1+2\text{i}}$ seen or implied
$= \dfrac{1}{1+2\text{i}}\times\dfrac{1-2\text{i}}{1-2\text{i}} = \frac{1}{5}-\frac{2}{5}\text{i}$	B1 **3**	For correct inverse **AEFcartesian**
(b) Identity = $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$	B1	For correct identity
Inverse = $\begin{pmatrix}-3 & 0\\ 0 & 0\end{pmatrix}$	B1 **2**	For correct inverse
	$\boxed{5}$	
2 **(a)** $(z_1 z_2 =)\ 6\,\text{e}^{\frac{5}{12}\pi\text{i}}$	B1 B1	For modulus = 6 For argument = $\frac{5}{12}\pi$
$\left(\dfrac{z_1}{z_2} = \frac{2}{3}\text{e}^{-\frac{1}{12}\pi\text{i}} =\right)\frac{2}{3}\text{e}^{\frac{23}{12}\pi\text{i}}$	M1 A1 **4**	For subtracting arguments For correct answer
(b) $\left(w^{-5} =\right) 2^{-5}\,\text{cis}\left(-\frac{5}{8}\pi\right)$	M1 A1	For use of de Moivre For $-\frac{5}{8}\pi$ seen or implied
$= \frac{1}{32}\left(\cos\frac{11}{8}\pi + \text{i}\sin\frac{11}{8}\pi\right)$	A1 **3**	For correct answer (allow 2^{-5} and $\text{cis}\,\frac{11}{8}\pi$)
	$\boxed{7}$	

3 *EITHER* $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a}) \times [8, 3, -6]$	M1*	For attempt at vector product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\mathbf{n} = \pm[-12, 50, 9]$	A1 √	For obtaining **n**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \dfrac{\lvert\mathbf{n}\rvert}{\lvert[8, 3, -6]\rvert}$	M1 (dep*)	For dividing $\lvert\mathbf{n}\rvert$ by magnitude of $[8, 3, -6]$
$= \dfrac{\sqrt{2725}}{\sqrt{109}}$	A1	For either magnitude correct
(*d* =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a}) \,.\, [8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\cos\theta = \pm\dfrac{109}{\sqrt{134}\sqrt{109}} = \pm\sqrt{\dfrac{109}{134}}$	A1 √	For correct $\cos\theta$ **AEF**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134}\sin\theta$	M1 (dep*)	For using trigonometry for perpendicular distance
	A1	For correct expression for *d* in terms of θ
(*d* =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a}) \,.\, [8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$x = \dfrac{109}{\sqrt{109}} = \sqrt{109}$	A1 √	For finding projection of $\mathbf{c}-\mathbf{a}$ onto $[8, 3, -6]$ f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134-109}$	M1 (dep*)	For using Pythagoras for perpendicular distance
	A1	For correct expression for *d*
(*d* =) 5	A1	For correct distance **CAO**
OR $\mathbf{CP} = \pm[-11+8t, -3+3t, 2-6t]$	B1	For finding a vector from *C*(12, 5, 3) to a point on the line
$\mathbf{CP} \,.\, [8, 3, -6] = 0$	M1*	For using scalar product for perpendicularity
$t = \pm1$ *OR* $P = (9, 5, -1)$	A1 √	For correct point. f.t. from incorrect **CP**
$d = \sqrt{3^2 + 0^2 + 4^2}$	M1 (dep*)	For finding magnitude of **CP**
	A1	For correct expression for *d*
(*d* =) 5	A1 **6**	For correct distance **CAO**
		SR Obtain $\mathbf{CP} = [11, 3, -2] - [8, 3, -6] = \pm[3, 0, 4]$ B1 Verify $[3, 0, 4] \,.\, [8, 3, -6] = 0$ M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5$ M1(dep*) A1 A1 (maximum 5 / 6)
	$\boxed{6}$	

4 Integrating factor $e^{\int -\frac{x^2}{1+x^3}dx}$	M1	For correct process for finding integrating factor
$= e^{-\frac{1}{3}\ln(1+x^3)} = \left(1+x^3\right)^{-\frac{1}{3}}$	A1	For correct IF, simplified (here or later)
$\Rightarrow \frac{d}{dx}\left(y\left(1+x^3\right)^{-\frac{1}{3}}\right) = \frac{x^2}{\left(1+x^3\right)^{\frac{1}{3}}}$	M1	For multiplying through by their IF
$\Rightarrow y\left(1+x^3\right)^{-\frac{1}{3}} = \frac{1}{2}\left(1+x^3\right)^{\frac{2}{3}}\ (+c)$	M1 A1	For integrating RHS to obtain $A\left(1+x^3\right)^k$ *OR* $\ln A\left(1+x^3\right)^k$ For correct integration (+*c* not required here)
$\Rightarrow 1 = \frac{1}{2} + c \quad \Rightarrow \quad c = \frac{1}{2}$	M1 A1 √	For substituting (0, 1) into GS (including + *c*) For correct *c*. f.t. from their GS
$\Rightarrow y = \frac{1}{2}\left(1+x^3\right) + \frac{1}{2}\left(1+x^3\right)^{\frac{1}{3}}$	A1	For correct solution. **AEF** in form $y = f(x)$
	$\boxed{8}$	
5 **(i)** *EITHER* $\mathbf{a} = [2, 3, 5],\ \mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \pm k[-10, 10, -2]$	M1 A1 √	For finding perpendicular to plane For correct **n**. f.t. from incorrect **b**
Use (2, 1, 5) *OR* (0, –1, 5)	M1	For substituting a point into equation $ax+by+cz=d$ where $[a, b, c]$ = their **n**
$\Rightarrow 5x - 5y + z = 10$	A1	For correct cartesian equation **AEF**
OR $\mathbf{a} = [2, 3, 5],\ \mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
e.g. $\mathbf{r} = [2, 1, 5] + \lambda[2, 2, 0] + \mu[2, 3, 5]$	M1	For stating parametric equation of plane
$[x, y, z] = [2+2\lambda+2\mu, 1+2\lambda+3\mu, 5+5\mu]$	A1 √	For writing 3 equations in *x*, *y*, *z* f.t. from incorrect **b**
	M1	For eliminating λ and μ
$\Rightarrow 5x - 5y + z = 10$	A1 **5**	For correct cartesian equation **AEF**
(ii) $[2t, 3t-4, 5t-9]$	B1 **1**	For stating a point *A* on l_1 with parameter *t* **AEF**
(iii) $\pm[2t+5, 3t-7, 5t-13]$	M1	For finding direction of l_2 from *A* and (–5,3, 4)
$\pm[2t+5, 3t-7, 5t-13] \mathbf{.} [2, 3, 5] = 0$	M1	For using scalar product for perpendicularity with any vector involving *t*
$\Rightarrow t = 2$	A1	For correct value of *t*
$\frac{x+5}{9} = \frac{y-3}{-1} = \frac{z-4}{-3}$ *OR* $\frac{x-4}{9} = \frac{y-2}{-1} = \frac{z-1}{-3}$	A1 **4**	For a correct equation **AEFcartesian**
		SR For $2p + 3q + 5r = 0$ and no further progress award B1
	$\boxed{10}$	

6 **(i)** $\left(m^2+4=0\Rightarrow\right)m=\pm 2\text{i}$	B1	For correct solutions of auxiliary equation (may be implied by correct CF)
CF = $A\cos 2x+B\sin 2x$	B1	For correct CF (**AEtrig** but not $A\text{e}^{2\text{i}x}+B\text{e}^{-2\text{i}x}$ only)
PI = $p\sin x\ (+\ q\cos x)$	B1	State a trial PI with at least $p\sin x$
$-p\sin x\ (-q\cos x)+4p\sin x\ (+4q\cos x)\ =\sin x$	M1	For substituting PI into DE
$\Rightarrow p=\frac{1}{3},\quad q=0$	A1	For correct *p* and *q* (which may be implied)
$\Rightarrow y=A\cos 2x+B\sin 2x+\frac{1}{3}\sin x$	B1 √ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
(ii) $(0,0)\Rightarrow A=0$	B1 √	For correct equation in *A* and/or *B* f.t. from their GS
$\frac{\text{d}y}{\text{d}x}=2B\cos 2x+\frac{1}{3}\cos x\Rightarrow\frac{4}{3}=2B+\frac{1}{3}$	M1	For differentiating their GS and substituting values for *x* and $\frac{\text{d}y}{\text{d}x}$
$A=0,\ B=\frac{1}{2}$	A1	For correct *A* and *B* Allow $A=-\frac{1}{4}\text{i},\ B=\frac{1}{4}\text{i}$ from CF $A\text{e}^{2\text{i}x}+B\text{e}^{-2\text{i}x}$
$\Rightarrow y=\frac{1}{2}\sin 2x+\frac{1}{3}\sin x$	A1 **4** $\boxed{10}$	For stating correct solution **CAO**
7 **(i)** $C+\text{i}S=1+\text{e}^{\text{i}\theta}+\text{e}^{2\text{i}\theta}+\text{e}^{3\text{i}\theta}+\text{e}^{4\text{i}\theta}+\text{e}^{5\text{i}\theta}$	M1	For using de Moivre, showing at least 3 terms
$=\frac{\text{e}^{6\text{i}\theta}-1}{\text{e}^{\text{i}\theta}-1}$	M1 A1	For recognising GP For correct GP sum
$=\frac{\text{e}^{3\text{i}\theta}-\text{e}^{-3\text{i}\theta}}{\text{e}^{\frac{1}{2}\text{i}\theta}-\text{e}^{-\frac{1}{2}\text{i}\theta}}\cdot\frac{\text{e}^{3\text{i}\theta}}{\text{e}^{\frac{1}{2}\text{i}\theta}}=\frac{\text{e}^{3\text{i}\theta}-\text{e}^{-3\text{i}\theta}}{\text{e}^{\frac{1}{2}\text{i}\theta}-\text{e}^{-\frac{1}{2}\text{i}\theta}}\text{e}^{\frac{5}{2}\text{i}\theta}$	A1 **4**	For obtaining correct expression **AG**
(ii) $C+\text{i}S=\frac{2\text{i}\sin 3\theta}{2\text{i}\sin\frac{1}{2}\theta}\cdot\text{e}^{\frac{5}{2}\text{i}\theta}$	M1 A1	For expressing numerator and denominator in terms of sines For $k\sin 3\theta$ and $k\sin\frac{1}{2}\theta$
$\text{Re}\Rightarrow C=\sin 3\theta\cos\frac{5}{2}\theta\ \text{cosec}\frac{1}{2}\theta$	A1	For correct expression **AG**
$\text{Im}\Rightarrow S=\sin 3\theta\sin\frac{5}{2}\theta\ \text{cosec}\frac{1}{2}\theta$	B1 **4**	For correct expression
(iii) $C=S\ \Rightarrow\ \sin 3\theta=0,\ \tan\frac{5}{2}\theta=1$	M1	For either equation deduced **AEF** Ignore values outside $0<\theta<\pi$
$\theta=\frac{1}{3}\pi,\frac{2}{3}\pi$	A1	For both values correct and no extras
$\theta=\frac{1}{10}\pi,\frac{1}{2}\pi,\frac{9}{10}\pi$	A2 **4** $\boxed{12}$	For all values correct and no extras. Allow A1 for any 1 value *OR* all correct with extras

8 (i) $r^4 . a \neq a . r^4$	B1 **1**	For stating the non-commutative product in the given table, or justifying another correct one
(ii) Possible subgroups order 2, 5	B1 B1 **2**	For either order stated For both orders stated, and no more (Ignore 1)
(iii) (a) $\{e, a\}$ **(b)** $\{e, r, r^2, r^3, r^4\}$	B1 B1 **2**	For correct subgroup For correct subgroup
(iv) order of $r^3 = 5$ $(ar)^2 = ar.ar = r^4a.ar = e$ $\Rightarrow$ order of $ar = 2$ $(ar^2)^2 = ar^2ar.r = ar^2r^4a.r = ara.r = e$ $\Rightarrow$ order of $ar^2 = 2$	B1 M1 A1 A1 **4**	For correct order For attempt to find $(ar)^m = e$ *OR* $(ar^2)^m = e$ For correct order For correct order
(v) $\begin{array}{c\|cccc} & ar & ar^2 & ar^3 & ar^4 \\ \hline ar & e & r & r^2 & r^3 \\ ar^2 & r^4 & e & r & r^2 \\ ar^3 & r^3 & r^4 & e & r \\ ar^4 & r^2 & r^3 & r^4 & e \end{array}$	B1 B1 B1 B1 B1 **5** $\boxed{14}$	If the border elements $ar\ ar^2\ ar^3\ ar^4$ are not written, it will be assumed that the products arise from that order For all 16 elements of the form *e* or r^m For all 4 elements in leading diagonal = *e* For no repeated elements in any completed row or column For any two rows or columns correct For all elements correct

1 **(i)** Attempt to show no closure	M1	For showing operation table or otherwise
$3\times 3=1,\ 5\times 5=1$ *OR* $7\times 7=1$	A1	For a convincing reason
OR Attempt to show no identity	M1	For attempt to find identity *OR* for showing operation table
Show $a\times e=a$ has no solution	A1 **2**	For showing identity is not 3, not 5, and not 7 by reference to operation table or otherwise
(ii) (a =) 1	B1 **1**	For value of a stated
(iii) *EITHER*: $\{e, r, r^2, r^3\}$ is cyclic, **(ii)** group is not cyclic	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 2 self-inverse elements, **(ii)** group has 4 self-inverse elements	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 1 element of order 2 **(ii)** group has 3 elements of order 2	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has element(s) of order 4 **(ii)** group has no element of order 4	B1*	For a pair of correct statements
Not isomorphic	B1 (dep*) **2** $\boxed{5}$	For correct conclusion
2 *EITHER*: $[3, 1, -2]\times[1, 5, 4]$	M1	For attempt to find vector product of both normals
$\Rightarrow \mathbf{b}=k[1, -1, 1]$	A1	For correct vector identified with **b**
e.g. put x *OR* y *OR* z = 0	M1	For giving a value to one variable
and solve 2 equations in 2 unknowns	M1	For solving the equations in the other variables
Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$	A1	For a correct vector identified with **a**
OR: Solve $3x+y-2z=4,\ x+5y+4z=6$		
e.g. $y+z=1$ *OR* $x-z=1$ *OR* $x+y=2$	M1	For eliminating one variable between 2 equations
Put x *OR* y *OR* z = t	M1	For solving in terms of a parameter
$[x, y, z]=[t, 2-t, -1+t]$ *OR* $[2-t, t, 1-t]$ *OR* $[1+t, 1-t, t]$	M1	For obtaining a parametric solution for x, y, z
Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$	A1	For a correct vector identified with **a**
Obtain $k[1, -1, 1]$	A1 **5** $\boxed{5}$	For correct vector identified with **b**
3 **(i)** $z=\dfrac{6\pm\sqrt{36-144}}{2}$	M1	For using quadratic equation formula or completing the square
$z=3\pm 3\sqrt{3}\,\mathrm{i}$	A1	For obtaining cartesian values **AEF**
Obtain $(r=)\ 6$	A1	For correct modulus
Obtain $(\theta=)\ \frac{1}{3}\pi$	A1 **4**	For correct argument
(ii) *EITHER*: 6^{-3} *OR* $\frac{1}{216}$ seen	B1√	f.t. from their r^{-3}
$Z^{-3}=6^{-3}(\cos(-\pi)\pm \mathrm{i}\sin(-\pi))$	M1	For using de Moivre with $n=\pm 3$
Obtain $-\frac{1}{216}$	A1	For correct value
OR: $z^3=6z^2-36z=6(6z-36)-36z$	M1	For using equation to find z^3
216 seen	B1	Ignore any remaining z terms
Obtain $-\frac{1}{216}$	A1 **3** $\boxed{7}$	For correct value

4 **(i)** $(y = xz \Rightarrow)\ \frac{\mathrm{d}y}{\mathrm{d}x} = x\frac{\mathrm{d}z}{\mathrm{d}x} + z$	B1	For a correct statement
$x\frac{\mathrm{d}z}{\mathrm{d}x} + z = \frac{x^2(1-z^2)}{x^2 z} = \frac{1}{z} - z$	M1	For substituting into differential equation and attempting to simplify to a variables separable form
$x\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{1}{z} - 2z = \frac{1-2z^2}{z}$	A1 **3**	For correct equation **AG**
(ii) $\int \frac{z}{1-2z^2}\,\mathrm{d}z = \int \frac{1}{x}\,\mathrm{d}x \Rightarrow -\frac{1}{4}\ln(1-2z^2) = \ln cx$	M1 M1* A1	For separating variables and writing integrals For integrating both sides to ln forms For correct result (*c* not required here)
$1-2z^2 = (cx)^{-4}$	A1√	For exponentiating their ln equation including a constant (this may follow the next M1)
$\frac{x^2-2y^2}{x^2} = \frac{c^{-4}}{x^4}$	M1 (dep*)	For substituting $z = \frac{y}{x}$
$x^2(x^2-2y^2) = k$	A1 **6**	For correct solution properly obtained, including dealing with any necessary change of constant to *k* as given **AG**
	$\boxed{9}$	
5 **(i)** **(a)** $e,\ p,\ p^2$	B1	For correct elements
(b) $e,\ q,\ q^2$	B1 **2**	For correct elements **SR** If the answers to parts **(i)** and **(iv)** are reversed, full credit may be earned for both parts
(ii) $p^3 = q^3 = e \Rightarrow (pq)^3 = p^3q^3 = e$	M1	For finding $(pq)^3$ or $(pq^2)^3$
$\Rightarrow$ order 3	A1	For correct order
$(pq^2)^3 = p^3q^6 = p^3(q^3)^2 = e \Rightarrow$ order 3	A1 **3**	For correct order **SR** For answer(s) only allow B1 for either or both
(iii) 3	B1 **1**	For correct order and no others
(iv)	B1	For stating *e* and either pq or p^2q^2
$e,\ pq,\ p^2q^2$ *OR* $e,\ pq,\ (pq)^2$	B1	For all 3 elements and no more
	B1	For stating *e* and either pq^2 or p^2q
$e,\ pq^2,\ p^2q$ *OR* $e,\ pq^2,\ (pq^2)^2$ *OR* $e,\ p^2q,\ (p^2q)^2$	B1 **4**	For all 3 elements and no more
	$\boxed{10}$	

6 **(i)** (CF $m=-3\Rightarrow$) Ae^{-3x}	B1 **1**	For correct CF
(ii) $(y=)\ px+q$	B1	For stating linear form for PI (may be implied)
$\Rightarrow p+3(px+q)=2x+1$	M1	For substituting PI into DE (needs y and $\frac{\mathrm{d}y}{\mathrm{d}x}$)
$\Rightarrow p=\frac{2}{3},\quad q=\frac{1}{9}$	A1 A1	For correct values
$\Rightarrow$ GS $\ y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√	For correct GS. f.t. from their CF + PI
		SR Integrating factor method may be used, but CF must be stated somewhere to earn the mark in **(i)**
I.F. $e^{3x}\quad\Rightarrow\frac{\mathrm{d}}{\mathrm{d}x}\left(y\,e^{3x}\right)=(2x+1)e^{3x}$	B1	For stating integrating factor
$\Rightarrow y\,e^{3x}=\frac{1}{3}e^{3x}(2x+1)-\int\frac{2}{3}e^{3x}\mathrm{d}x$	M1	For attempt at integrating by parts the right way round
$\Rightarrow y\,e^{3x}=\frac{2}{3}x\,e^{3x}+\frac{1}{3}e^{3x}-\frac{2}{9}e^{3x}+A$	A2 *	For correct integration, including constant Award A1 for any 2 algebraic terms correct
$\Rightarrow$ GS $\ y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√ **5**	For correct GS. f.t. from their * with constant
(iii) *EITHER* $\frac{\mathrm{d}y}{\mathrm{d}x}=-3Ae^{-3x}+\frac{2}{3}$	M1	For differentiating their GS
$\Rightarrow -3A+\frac{2}{3}=0$	M1	For putting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=0$
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1	For correct solution
OR $\frac{\mathrm{d}y}{\mathrm{d}x}=0,\ x=0\ \Rightarrow 3y=1$	M1	For using original DE with $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and $x=0$ to find y
$\Rightarrow\frac{1}{3}=A+\frac{1}{9}$	M1	For using their GS with y and $x=0$ to find A
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1 **3**	For correct solution
(iv) $y=\frac{2}{3}x+\frac{1}{9}$	B1√ **1** $\boxed{\mathbf{10}}$	For correct function. f.t. from linear part of **(iii)**

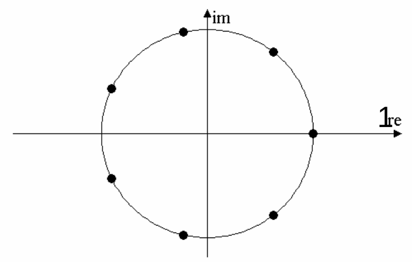
7 (i) *EITHER*: (**AG** is $\mathbf{r}$ =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
Normal to *BCD* is	M1	For finding vector product of any two of $\pm[1, -4, -1], \pm[2, 1, 1], \pm[1, 5, 2]$
$\mathbf{n} = k[1, 1, -3]$	A1	For correct **n**
Equation of *BCD* is $\mathbf{r}.[1, 1, -3] = -6$	A1	For correct equation (or in cartesian form)
Intersect at $(6+t)+4+(-3)(8+t) = -6$	M1	For substituting point on *AG* into plane
$t = -4$ ($t = -1$ using $[3, 4, 5]$) $\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1	For correct position vector of *M* **AG**
OR: (**AG** is $\mathbf{r}$ =) $[6, 4, 8] + tk[1, 0, 1]$ *or* $[3, 4, 5] + tk[1, 0, 1]$	B1	For a correct equation
$\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$, where $\mathbf{u} = [2, 1, 3]$ *or* $[1, 5, 4]$ *or* $[3, 6, 5]$ $\mathbf{v}, \mathbf{w} =$ two of $[1, -4, -1], [1, 5, 2], [2, 1, 1]$	M1 A1	For a correct parametric equation of *BCD*
e.g. $\begin{aligned} (x =)\ \ 6+t &= 2+\lambda+\mu \\ (y =)\ \ 4 &= 1-4\lambda+5\mu \\ (z =)\ \ 8+t &= 3-\lambda+2\mu \end{aligned}$	M1	For forming 3 equations in *t*, λ, μ from line and plane, and attempting to solve them
$t = -4$ *or* $\lambda = -\frac{1}{3}, \mu = \frac{1}{3}$	A1	For correct value of *t* *or* λ, μ
$\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1 **6**	For correct position vector of *M* **AG**
(ii) $\left.\begin{array}{l} A, G, M \text{ have } t = 0, -3, -4 \quad OR \\ AG = 3\sqrt{2},\ AM = 4\sqrt{2} \quad OR \\ \mathbf{AG} = [-3, 0, -3],\ \mathbf{AM} = [-4, 0, -4] \end{array}\right\} \Rightarrow AG : AM = 3 : 4$	B1 **1**	For correct ratio **AEF**
(iii) $\mathbf{OP} = \mathbf{OC} + \frac{4}{3}\mathbf{CG}$	M1	For using given ratio to find position vector of *P*
$= \left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right]$	A1 **2**	For correct vector
(iv) *EITHER*: Normal to *ABD* is	M1	For finding vector product of any two of $\pm[4, 3, 5], \pm[1, 5, 2], \pm[3, -2, 3]$
$\mathbf{n} = k[19, 3, -17]$	A1	For correct **n**
Equation of *ABD* is $\mathbf{r}.[19, 3, -17] = -10$	M1	For finding equation (or in cartesian form)
$19.\frac{11}{3} + 3.\frac{11}{3} - 17.\frac{16}{3} = -10$	A1	For verifying that *P* satisfies equation
OR: Equation of *ABD* is $\mathbf{r} = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$ (etc.)	M1	For finding equation in parametric form
$\left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right] = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$	M1	For substituting *P* and solving 2 equations for λ, μ
$\lambda = -\frac{2}{3}, \quad \mu = \frac{1}{3}$	A1	For correct λ, μ
	A1	For verifying 3rd equation is satisfied
OR: $\mathbf{AP} = \left[-\frac{7}{3}, -\frac{1}{3}, -\frac{8}{3}\right]$	M1 A1	For finding 3 relevant vectors in plane *ABDP* For correct **AP** *or* **BP** *or* **DP**
$\mathbf{AB} = [-4, -3, -5],\ \mathbf{AD} = [-3, 2, -3]$ $\Rightarrow \mathbf{AB} + \mathbf{AD} = [-7, -1, -8]$	M1	For finding **AB, AD** *or* **BA, BD** *or* **DB, DA**
$\Rightarrow \mathbf{AP} = \frac{1}{3}(\mathbf{AB} + \mathbf{AD})$	A1 **4**	For verifying linear relationship
	$\boxed{13}$	

8 **(i)** $\cos 4\theta + \mathrm{i}\sin 4\theta =$ $c^4 + 4\mathrm{i}\,c^3 s - 6c^2 s^2 - 4\mathrm{i}\,cs^3 + s^4$	M1	For using de Moivre with $n = 4$
$\Rightarrow \sin 4\theta = 4c^3 s - 4cs^3$ and $\cos 4\theta = c^4 - 6c^2 s^2 + s^4$	A1	For both expressions
$\Rightarrow \tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	M1	For expressing $\dfrac{\sin 4\theta}{\cos 4\theta}$ in terms of c and s
	A1 **4**	For simplifying to correct expression
(ii) $\cot 4\theta = \dfrac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$	B1 **1**	For inverting **(i)** and using $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$. **AG**
(iii) $\cot 4\theta = 0$	B1	For putting $\cot 4\theta = 0$ (can be awarded in **(iv)** if not earned here)
Put $x = \cot^2\theta$	B1	For putting $x = \cot^2\theta$ in the numerator of **(ii)**
$\theta = \frac{1}{8}\pi \Rightarrow x^2 - 6x + 1 = 0$ *OR* $x^2 - 6x + 1 = 0 \Rightarrow \theta = \frac{1}{8}\pi$	B1 **3**	For deducing quadratic from **(ii)** and $\theta = \frac{1}{8}\pi$ *OR* For deducing $\theta = \frac{1}{8}\pi$ from **(ii)** and quadratic
(iv) $4\theta = \frac{3}{2}\pi$ *OR* $\frac{1}{2}(2n+1)\pi$	M1	For attempting to find another value of θ
2nd root is $x = \cot^2\left(\frac{3}{8}\pi\right)$	A1	For the other root of the quadratic
$\Rightarrow \cot^2\left(\frac{1}{8}\pi\right) + \cot^2\left(\frac{3}{8}\pi\right) = 6$	M1	For using sum of roots of quadratic
$\Rightarrow \operatorname{cosec}^2\left(\frac{1}{8}\pi\right) + \operatorname{cosec}^2\left(\frac{3}{8}\pi\right) = 8$	M1 A1 **5** $\boxed{13}$	For using $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$ For correct value

1 **(i)** $zz^* = r\,\mathrm{e}^{\mathrm{i}\theta}.r\,\mathrm{e}^{-\mathrm{i}\theta} = r^2 = \|z\|^2$	B1 **1**	For verifying result **AG**
(ii) Circle	B1	For stating circle
Centre $0\,(+0\mathrm{i})$ *OR* (0, 0) *OR* *O*, radius 3	B1 **2** $\boxed{3}$	For stating correct centre and radius
2 *EITHER*: $(\mathbf{r} =)\,[3+t, 1+4t, -2+2t]$	M1	For parametric form of *l* seen or implied
$8(3+t)-7(1+4t)+10(-2+2t)=7$	M1 A1	For substituting into plane equation
$\Rightarrow (0t)+(-3)=7 \Rightarrow$ contradiction	A1	For obtaining a contradiction
l is parallel to Π, no intersection	B1 **5**	For conclusion from correct working
OR: $[1, 4, 2]\,.\,[8, -7, 10]=0$	M1	For finding scalar product of direction vectors
$\Rightarrow$ *l* is parallel to Π	A1	For correct conclusion
$(3, 1, -2)$ into Π	M1	For substituting point into plane equation
$\Rightarrow\ 24-7-20 \neq 7$	A1	For obtaining a contradiction
l is parallel to Π, no intersection	B1	For conclusion from correct working
OR:Solve $\dfrac{x-3}{1}=\dfrac{y-1}{4}=\dfrac{z+2}{2}$ and $8x-7y+10z=7$		
eg $y-2z=3,\ 2y-2=4z+8$	M1 A1	For eliminating one variable
	M1	For eliminating another variable
eg $4z+4=4z+8$	A1	For obtaining a contradiction
l is parallel to Π, no intersection	B1 $\boxed{5}$	For conclusion from correct working
3 Aux. equation $m^2-6m+8\,(=0)$	M1	For auxiliary equation seen
$m=2, 4$	A1	For correct roots
CF $(y=)\ A\mathrm{e}^{2x}+B\mathrm{e}^{4x}$	A1√	For correct CF. f.t. from their *m*
PI $(y=)\ C\mathrm{e}^{3x}$	M1	For stating and substituting PI of correct form
$9C-18C+8C=1 \Rightarrow C=-1$	A1	For correct value of *C*
GS $y=A\mathrm{e}^{2x}+B\mathrm{e}^{4x}-\mathrm{e}^{3x}$	B1√ **6** $\boxed{6}$	For GS. f.t. from their CF + PI with 2 arbitrary constants in CF and none in PI

4 **(i)** $q(st) = qp = s$	B1	For obtaining s
$(qs)t = tt = s$	B1 **2**	For obtaining s
(ii) METHOD 1		
Closed: see table	B1	For stating closure with reason
Identity = r	B1	For stating identity r
Inverses: $p^{-1} = s,\ q^{-1} = t,\ (r^{-1} = r),$	M1	For checking for inverses
$s^{-1} = p,\ t^{-1} = q$	A1 **4**	For stating inverses *OR* For giving sufficient explanation to justify each element has an inverse eg r occurs once in each row and/or column
METHOD 2		
Identity = r	B1	For stating identity r
	M1	For attempting to establish a generator $\neq r$
eg $p^2 = t$, $p^3 = q$, $p^4 = s$	A1	For showing powers of p (*OR* q, s or t) are different elements of the set
$\Rightarrow\ p^5 = r$, so p is a generator	A1	For concluding p^5 (*OR* q^5 , s^5 or t^5) = r
(iii) e, d, d^2, d^3, d^4	B2 **2**	For stating all elements **AEF** eg d^{-1}, d^{-2} , dd
	$\boxed{8}$	

5 **(i)** $(\cos 6\theta =)\ \text{Re}\left(c + \text{i}\,s\right)^6$	M1	For expanding (real part of) $\left(c + \text{i}\,s\right)^6$ at least 4 terms and 1 evaluated binomial coefficient needed
$(\cos 6\theta =)\ c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1	For correct expansion
$(\cos 6\theta =)$ $c^6 - 15c^4\left(1-c^2\right) + 15c^2\left(1-c^2\right)^2 - \left(1-c^2\right)^3$	M1	For using $s^2 = 1 - c^2$
$(\cos 6\theta =)\ 32c^6 - 48c^4 + 18c^2 - 1$	A1 **4**	For correct result **AG**
(ii) $64x^6 - 96x^4 + 36x^2 - 3 = 0 \Rightarrow \cos 6\theta = \frac{1}{2}$	M1	For obtaining a numerical value of $\cos 6\theta$
$\Rightarrow (\theta =)\ \frac{1}{18}\pi, \frac{5}{18}\pi, \frac{7}{18}\pi$ etc.	A1	For any correct solution of $\cos 6\theta = \frac{1}{2}$
$\cos 6\theta = \frac{1}{2}$ has multiple roots	M1	For stating or implying at least 2 values of θ
largest x requires smallest θ $\Rightarrow$ largest positive root is $\cos\frac{1}{18}\pi$	A1 **4**	For identifying $\cos\frac{1}{18}\pi$ **AEF** as the largest positive root from a list of 3 positive roots *OR* from general solution *OR* from consideration of the cosine function
	$\boxed{8}$	

6 (i) $\mathbf{n}=l_1\times l_2$	B1	For stating or implying in **(i)** or **(ii)** that **n** is perpendicular to l_1 and l_2
$\mathbf{n}=[2,\,-1,\,1]\times[4,\,3,\,2]$	M1*	For finding vector product of direction vectors
$\mathbf{n}=k[-1,\,0,\,2]$	A1	For correct vector (any k)
$[3,\,4,\,-1]\,.\,k[-1,\,0,\,2]=-5k$	M1 (*dep)	For substituting a point of l_1 into $\mathbf{r}\,.\,\mathbf{n}$
$\mathbf{r}\,.[-1,\,0,\,2]=-5$	A1 5	For obtaining correct p. **AEF** in this form
(ii) $[5,1,1]\,.\,k[-1,\,0,\,2]=-3k$	M1	For using same **n** and substituting a point of l_2
$\mathbf{r}\,.[-1,\,0,\,2]=-3$	A1√ 2	For obtaining correct p. **AEF** in this form f.t. on incorrect **n**
(iii) $d=\dfrac{\lvert -5+3\rvert}{\sqrt{5}}$ *OR* $d=\dfrac{\lvert[2,\,-3,\,2]\,.\,[-1,\,0,\,2]\rvert}{\sqrt{5}}$ *OR* d from $(5,1,1)$ to $\Pi_1=\dfrac{\lvert 5(-1)+1(0)+1(2)+5\rvert}{\sqrt{5}}$ *OR* d from $(3,\,4,\,-1)$ to $\Pi_2=\dfrac{\lvert 3(-1)+4(0)-1(2)+3\rvert}{\sqrt{5}}$	M1	For using a distance formula from their equations Allow omission of $\lvert\ \rvert$
OR $[3-t,\,4,\,-1+2t]\,.[-1,\,0,\,2]=-3\ \Rightarrow t=\frac{2}{5}$ *OR* $[5-t,1,1+2t]\,.[-1,\,0,\,2]=-5\ \Rightarrow t=-\frac{2}{5}$		*OR* For finding intersection of $\mathbf{n}_1$ and Π_2 or $\mathbf{n}_2$ and Π_1
$d=\dfrac{2}{\sqrt{5}}=\dfrac{2\sqrt{5}}{5}=0.894427...$	A1√ 2	For correct distance **AEF** f.t. on incorrect **n**
(iv) d is the shortest *OR* perpendicular distance between l_1 and l_2	B1 1 $\boxed{10}$	For correct statement
7 (i) $(z-e^{i\phi})(z-e^{-i\phi})\equiv z^2-(2)z\dfrac{(e^{i\phi}+e^{-i\phi})}{(2)}+1$ $\equiv z^2-(2\cos\phi)z+1$	B1 1	For correct justification **AG**
(ii) $z=e^{\frac{2}{7}k\pi i}$	B1	For general form *OR* any one non-real root
for $k=0,1,2,3,4,5,6$ *OR* $0,\pm1,\pm2,\pm3$	B1	For other roots specified (k=0 may be seen in any form, eg 1, e^0, $e^{2\pi i}$) For answers in form $\cos\theta+i\sin\theta$ allow maximum B1 B0
	B1	For any 7 points equally spaced round unit circle (circumference need not be shown)
	B1 4	For 1 point on $+^{ve}$ real axis, and other points in correct quadrants
(iii) $\left(z^7-1=\right)(z-1)(z-e^{\frac{2}{7}\pi i})(z-e^{\frac{4}{7}\pi i})$ $(z-e^{\frac{6}{7}\pi i})(z-e^{\frac{-2}{7}\pi i})(z-e^{\frac{-4}{7}\pi i})(z-e^{\frac{-6}{7}\pi i})$	M1	For using linear factors from **(ii),** seen or implied
$=(z-e^{\frac{2}{7}\pi i})(z-e^{\frac{-2}{7}\pi i})\times(z-e^{\frac{4}{7}\pi i})(z-e^{\frac{-4}{7}\pi i})$ $(z-e^{\frac{6}{7}\pi i})(z-e^{\frac{-6}{7}\pi i})\times$	M1	For identifying at least one pair of complex conjugate factors
$\times(z-1)$	B1	For linear factor seen
$=(z^2-(2\cos\frac{2}{7}\pi)z+1)\times$	A1	For any one quadratic factor seen
$(z^2-(2\cos\frac{4}{7}\pi)z+1)\times(z^2-(2\cos\frac{6}{7}\pi)z+1)\times$ $\times(z-1)$	A1 5 $\boxed{10}$	For the other 2 quadratic factors and expression written as product of 4 factors

8 **(i)** Integrating factor $e^{\int \tan x\,(\mathrm{d}x)}$	B1	For correct IF
$= e^{-\ln\cos x}$	M1	For integrating to ln form
$= (\cos x)^{-1}$ *OR* $\sec x$	A1	For correct simplified IF **AEF**
$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left(y(\cos x)^{-1}\right) = \cos^2 x$	B1√	For $\frac{\mathrm{d}}{\mathrm{d}x}\left(y.\text{their IF}\right) = \cos^3 x$. their IF
$y(\cos x)^{-1} = \int \frac{1}{2}(1+\cos 2x)\,(\mathrm{d}x)$	M1 M1	For integrating LHS For attempting to use $\cos 2x$ formula *OR* parts for $\int \cos^2 x\,\mathrm{d}x$
$y(\cos x)^{-1} = \frac{1}{2}x + \frac{1}{4}\sin 2x\ (+c)$	A1	For correct integration both sides **AEF**
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c\right)\cos x$	A1 **8**	For correct general solution **AEF**
(ii) $2 = \left(\frac{1}{2}\pi + c\right).-1 \Rightarrow c = -2 - \frac{1}{2}\pi$	M1	For substituting $(\pi, 2)$ into their GS and solve for c
$y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x - 2 - \frac{1}{2}\pi\right)\cos x$	A1 **2**	For correct solution **AEF**
	$\boxed{\mathbf{10}}$	
9 **(i)** $3^n \times 3^m = 3^{n+m},\ n+m \in \mathbb{Z}$	B1	For showing closure
$\left(3^p \times 3^q\right) \times 3^r = \left(3^{p+q}\right) \times 3^r = 3^{p+q+r}$	M1	For considering 3 distinct elements, seen bracketed 2+1 or 1+2
$= 3^p \times \left(3^{q+r}\right) = 3^p \times \left(3^q \times 3^r\right) \Rightarrow$ associativity	A1	For correct justification of associativity
Identity is 3^0	B1	For stating identity. Allow 1
Inverse is 3^{-n}	B1	For stating inverse
$3^n \times 3^m = 3^{n+m} = 3^{m+n} = 3^m \times 3^n \Rightarrow$ commutativity	B1 **6**	For showing commutativity
(ii) (a) $3^{2n} \times 3^{2m} = 3^{2n+2m}\ \left(= 3^{2(n+m)}\right)$	B1*	For showing closure
Identity, inverse OK	B1 (*dep) **2**	For stating other two properties satisfied and hence a subgroup
(b) For 3^{-n},	M1	For considering inverse
$-n \notin$ subset	A1 **2**	For justification of not being a subgroup 3^{-n} must be seen here or in **(i)**
(c) *EITHER*: eg $3^{1^2} \times 3^{2^2} = 3^5$	M1	For attempting to find a specific counter-example of closure
$\neq 3^{r^2} \Rightarrow$ not a subgroup	A1 **2**	For a correct counter-example and statement that it is not a subgroup
OR: $3^{n^2} \times 3^{m^2} = 3^{n^2+m^2}$	M1	For considering closure in general
$\neq 3^{r^2}$ eg $1^2 + 2^2 = 5 \Rightarrow$ not a subgroup	A1	For explaining why $n^2 + m^2 \neq r^2$ in general and statement that it is not a subgroup
	$\boxed{\mathbf{12}}$	

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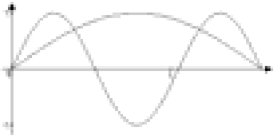
1 **(a) (i)** e.g. $ap \neq pa \Rightarrow$ not commutative	B1 **1**	For correct reason and conclusion
(ii) 3	B1 **1**	For correct number
(iii) e, a, b	B1 **1**	For correct elements
(b) c^3 has order 2	B1	For correct order
c^4 has order 3	B1	For correct order
c^5 has order 6	B1 **3**	For correct order
	$\boxed{6}$	
2 $m^2-8m+16=0$	M1	For stating and attempting to solve auxiliary eqn
$\Rightarrow m=4$	A1	For correct solution
$\Rightarrow$ CF $(y=)\ (A+Bx)e^{4x}$	A1√	For CF of correct form. f.t. from m
For PI try $y=px+q$	M1	For using linear expression for PI
$\Rightarrow -8p+16(px+q)=4x$		
$\Rightarrow p=\frac{1}{4}\quad q=\frac{1}{8}$	A1 A1	For correct coefficients
$\Rightarrow$ GS $y=(A+Bx)e^{4x}+\frac{1}{4}x+\frac{1}{8}$	B1√ **7**	For GS = CF + PI. Requires $\boxed{y=}$. f.t. from CF and PI with 2 arbitrary constants in CF and none in PI
	$\boxed{7}$	
3 **(i)** line segment OA	B1 B1 **2**	For stating line through O *OR* A For correct description **AEF**
(ii) $(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\overrightarrow{AP}\times\overrightarrow{BP}$	B1	For identifying $\mathbf{r}-\mathbf{a}$ with $\overrightarrow{AP}$ and $\mathbf{r}-\mathbf{b}$ with $\overrightarrow{BP}$ Allow direction errors
$=\lvert AP\rvert\lvert BP\rvert\sin\pi\,.\,\hat{\mathbf{n}}=\mathbf{0}$	B1 **2**	For using × of 2 parallel vectors = **0** *OR* sin π = 0 *or* sin 0 = 0 in an appropriate vector expression
(iii) line through O parallel to AB	B1 B1 B1 **3**	For stating line For stating through O For stating correct direction
		SR For $\overrightarrow{AB}$ or $\overrightarrow{BA}$ allow B1 B0 B1
	$\boxed{7}$	
4 $(C+\mathrm{i}S=)\ \int_0^{\frac{1}{2}\pi}e^{2x}(\cos 3x+\mathrm{i}\sin 3x)(\mathrm{d}x)$		
$\cos 3x+\mathrm{i}\sin 3x=e^{3\mathrm{i}x}$	B1	For using de Moivre, seen or implied
$\int_0^{\frac{1}{2}\pi}e^{(2+3\mathrm{i})x}(\mathrm{d}x)=\frac{1}{2+3\mathrm{i}}\left[e^{(2+3\mathrm{i})x}\right]_0^{\frac{1}{2}\pi}$	M1* A1	For writing as a single integral in exp form For correct integration (ignore limits)
$=\frac{2-3\mathrm{i}}{4+9}\left(e^{(2+3\mathrm{i})\frac{1}{2}\pi}-e^0\right)=\frac{2-3\mathrm{i}}{13}\left(-\mathrm{i}\,e^{\pi}-1\right)$	A1	For substituting limits correctly (unsimplified) (may be earned at any stage)
	M1 (dep*)	For multiplying by complex conjugate of 2+3i
$=\left\{\frac{1}{13}\left(-2-3e^{\pi}+\mathrm{i}(3-2e^{\pi})\right)\right\}$	M1 (dep*)	For equating real and/or imaginary parts
$C=-\frac{1}{13}\left(2+3e^{\pi}\right)$	A1	For correct expression **AG**
$S=\frac{1}{13}\left(3-2e^{\pi}\right)$	A1	For correct expression
	$\boxed{8}$	

5 **(i)** IF $e^{\int\frac{1}{x}\mathrm{d}x}=e^{\ln x}=x$ *OR* $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=x\sin 2x$	M1	For correct process for finding integrating factor *OR* for multiplying equation through by x
$\Rightarrow\ \frac{\mathrm{d}}{\mathrm{d}x}(xy)=x\sin 2x$	A1	For writing DE in this form (may be implied)
$\Rightarrow xy=\int x\sin 2x(\mathrm{d}x)$	M1	For integration by parts the correct way round
$xy=-\frac{1}{2}x\cos 2x+\frac{1}{2}\int\cos 2x(\mathrm{d}x)$	A1	For 1st term correct
$xy=-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x\ (+c)$	M1	For their 1st term and attempt at integration of $\genfrac{}{}{0pt}{}{\cos}{\sin}kx$
$\Rightarrow y=-\frac{1}{2}\cos 2x+\frac{1}{4x}\sin 2x+\frac{c}{x}$	A1 **6**	For correct expression for y
(ii) $\left(\frac{1}{4}\pi,\frac{2}{\pi}\right)\Rightarrow\frac{2}{\pi}=\frac{1}{\pi}+\frac{4c}{\pi}\Rightarrow c=\frac{1}{4}$	M1	For substituting $\left(\frac{1}{4}\pi,\frac{2}{\pi}\right)$ in solution
$\Rightarrow y=-\frac{1}{2}\cos 2x+\frac{1}{4x}\sin 2x+\frac{1}{4x}$	A1 **2**	For correct solution. Requires $\boxed{y=}$.
(iii) $(y\approx)-\frac{1}{2}\cos 2x$	B1√ **1** $\boxed{9}$	For correct function **AEF** f.t. from **(ii)**
6 **(i)**		***Either coordinates or vectors may be used*** Methods 1 and 2 may be combined, for a maximum of 5 marks
METHOD 1		
State $B=(-1,-7,2)+t(1,2,-2)$	M1	For using vector normal to plane
On plane $\Rightarrow(-1+t)+2(-7+2t)-2(2-2t)=-1$	M1 M1	For substituting parametric form into plane For solving a linear equation in t
$\Rightarrow t=2\Rightarrow B=(1,-3,-2)$	A1	For correct coordinates
$AB=\sqrt{2^2+4^2+4^2}$ *OR* $2\sqrt{1^2+2^2+2^2}=6$	A1 **5**	For correct length of AB
METHOD 2		
$AB=\left\vert\frac{-1-14-4+1}{\sqrt{1^2+2^2+2^2}}\right\vert=6$	M1	For using a correct distance formula
OR $AB=\mathbf{AC}\,.\,\mathbf{AB}=\frac{[6,7,1]\,.\,[1,2,-2]}{\sqrt{1^2+2^2+2^2}}=6$	A1	For correct length of AB
$B=(-1,-7,2)\pm 6\frac{(1,2,-2)}{\sqrt{1^2+2^2+2^2}}$	M1	For using $B=A$ + length of AB × unit normal
$B=(-1,-7,2)\pm(2,4,-4)$	B1	For checking whether + or – is needed (substitute into plane equation)
$B=(1,-3,-2)$	A1	For correct coordinates (allow even if B0)
(ii) Find vector product of any two of $\pm[6,7,1],\quad\pm[6,-3,0],\quad\pm(0,10,1)$	M1	For finding vector product of two relevant vectors
Obtain $k[1,2,-20]$	A1	For correct vector **n**
$\theta=\cos^{-1}\frac{\vert[1,2,-2]\,.\,[1,2,-20]\vert}{\sqrt{1^2+2^2+2^2}\sqrt{1^2+2^2+20^2}}$	M1* M1 (dep*)	For using scalar product of two normal vectors For stating both moduli in denominator
$\theta=\cos^{-1}\frac{45}{\sqrt{9}\sqrt{405}}=41.8°\ (41.810...°,\ 0.72972...)$	A1√ A1 **6** $\boxed{11}$	For correct scalar product. f.t. from **n** For correct angle

7 (i) (a) $\sin\frac{6}{8}\pi=\frac{1}{\sqrt{2}}$, $\sin\frac{2}{8}\pi=\frac{1}{\sqrt{2}}$	B1	**1**	For verifying $\theta=\frac{1}{8}\pi$
(b) 	M1		For sketching $y=\sin 6\theta$ and $y=\sin 2\theta$ for $0 „ \theta „ \frac{1}{2}\pi$ *OR* any other correct method for solving $\sin 6\theta=\sin 2\theta$ for $\theta \neq k\frac{\pi}{2}$ *OR* appropriate use of symmetry *OR* attempt to verify a reasonable guess for θ
$\theta=\frac{3}{8}\pi$	A1	**2**	For correct θ
(ii) $\text{Im}\,(c+\text{i}\,s)^6=6c^5s-20c^3s^3+6cs^5$	M1 A1		For expanding $(c+\text{i}\,s)^6$; at least 3 terms and 3 binomial coefficients needed For 3 correct terms
$\sin 6\theta=\sin\theta\left(6c^5-20c^3(1-c^2)+6c(1-c^2)^2\right)$	M1		For using $s^2=1-c^2$
$\sin 6\theta=\sin\theta\left(32c^5-32c^3+6c\right)$	A1		For any correct intermediate stage
$\sin 6\theta=2\sin\theta\cos\theta\left(16c^4-16c^2+3\right)$	A1		For obtaining this expression correctly
$\sin 6\theta=\sin 2\theta\left(16\cos^4\theta-16\cos^2\theta+3\right)$		**5**	**AG**
(iii) $16c^4-16c^2+3=1$	M1		For stating this equation **AEF**
$\Rightarrow c^2=\frac{2\pm\sqrt{2}}{4}$	A1		For obtaining both values of c^2
– sign requires larger $\theta=\frac{3}{8}\pi$	A1	**3**	For stating and justifying $\theta=\frac{3}{8}\pi$ Calculator OK if figures seen
		$\boxed{11}$	

8 **(i)** Group A: $e = 6$ Group B: $e = 1$ Group C: $e = 2^0$ *OR* 1 Group D: $e = 1$	B1 B1 **2**	For any two correct identities For two other correct identities **AEF** for D, but not " $m = n$ "
(ii) *EITHER* *OR* $\begin{array}{c\|cccc} A & 2 & 4 & 6 & 8 \\ \hline 2 & 4 & 8 & 2 & 6 \\ 4 & 8 & 6 & 4 & 2 \\ 6 & 2 & 4 & 6 & 8 \\ 8 & 6 & 2 & 8 & 4 \end{array}$ orders of elements 1, 2, 4, 4 *OR* cyclic group $\begin{array}{c\|cccc} B & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$ orders of elements 1, 2, 2, 2 *OR* non-cyclic group *OR* Klein group $\begin{array}{c\|cccc} C & 2^0 & 2^1 & 2^2 & 2^3 \\ \hline 2^0 & 2^0 & 2^1 & 2^2 & 2^3 \\ 2^1 & 2^1 & 2^2 & 2^3 & 2^0 \\ 2^2 & 2^2 & 2^3 & 2^0 & 2^1 \\ 2^3 & 2^3 & 2^0 & 2^1 & 2^2 \end{array}$ orders of elements 1, 2, 4, 4 *OR* cyclic group	B1* B1*	For showing group table *OR* sufficient details of orders of elements *OR* stating cyclic / non-cyclic / Klein group (as appropriate) for one of groups A, B, C for another of groups A, B, C
$A \ncong B$	B1 (dep*)	For stating non-isomorphic
$B \ncong C$	B1 (dep*)	For stating non-isomorphic with sufficient detail relating to the first 2 marks
$A \cong C$	B1 (dep*) **5**	For stating isomorphic
(iii) $\frac{1+2m}{1+2n} \times \frac{1+2p}{1+2q} = \frac{1+2m+2p+4mp}{1+2n+2q+4nq}$ $= \frac{1+2(m+p+2mp)}{1+2(n+q+2nq)} \equiv \frac{1+2r}{1+2s}$	M1* M1 (dep*) A1 A1 **4**	For considering product of 2 distinct elements of this form For multiplying out For simplifying to form shown For identifying as correct form, so closed **SR** $\frac{\text{odd}}{\text{odd}} \times \frac{\text{odd}}{\text{odd}} = \frac{\text{odd}}{\text{odd}}$ earns full credit **SR** If clearly attempting to prove commutativity, allow at most M1
(iv) Closure not satisfied Identity and inverse not satisfied	B1 B1 **2** $\boxed{13}$	For stating closure For stating identity and inverse **SR** If associativity is stated as not satisfied, then award at most B1 B0 *OR* B0 B1

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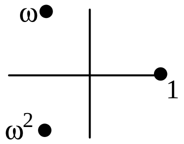
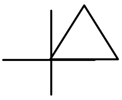
1 (a)(i)	e, r^3, r^6, r^9	M1		For stating e, r^m (any $m \,..\, \mathbf{2}$), and 2 other different elements in terms of e and r
		A1	2	For all elements correct
(ii)	r generates G	B1	1	For this or any statement equivalent to: all elements of G are included in a group with e and r *OR* order of r > order of all possible proper subgroups
(b)	m, n, p, mn, np, pm	B1		For any 3 orders correct
		B1	2	For all 6 correct and no extras (Ignore 1 and *mnp*)
		$\boxed{5}$		
2	METHOD 1			
	$[1, 3, 2] \times [1, 2, -1]$	M1		For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c\ (= 17)$	A1		For correct vector *OR* LHS of equation
	$\theta = \sin^{-1} \dfrac{\lvert [1, 4, -1] \,.\, [-7, 3, -1] \rvert}{\sqrt{1^2 + 4^2 + 1^2}\sqrt{7^2 + 3^2 + 1^2}}$	M1√		For using correct vectors for line and plane f.t. from normal
		M1*		For using scalar product of line and plane vectors
		M1		For calculating both moduli in denominator
	$\theta = \sin^{-1} \dfrac{6}{\sqrt{18}\sqrt{59}} = 10.6°$	A1√ (*dep)		For scalar product. f.t. from their numerator
	(10.609...°, 0.18517...)	A1	7	For correct angle
	METHOD 2			
	$[1, 3, 2] \times [1, 2, -1]$	M1		For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ *OR* $7x - 3y + z = c$	A1		For correct vector *OR* LHS of equation
	$7x - 3y + z = 17$	M1√		For attempting to find RHS of equation f.t. from **n** or LHS of equation
	$d = \dfrac{\lvert 21 - 12 + 2 - 17 \rvert}{\sqrt{7^2 + 3^2 + 1^2}} = \dfrac{6}{\sqrt{59}}$	M1		For using distance formula from a point on the line, e.g. (3, 4, 2), to the plane
		A1√		For correct distance. f.t. from equation
	$\theta = \sin^{-1} \dfrac{\frac{6}{\sqrt{59}}}{\sqrt{1^2 + 4^2 + 1^2}} = 10.6°$	M1		For using trigonometry
	(10.609...°, 0.18517...)	A1		For correct angle
		$\boxed{7}$		
3 (i)	$\dfrac{dz}{dx} = 1 + \dfrac{dy}{dx}$	M1		For differentiating substitution (seen or implied)
	$\dfrac{dz}{dx} - 1 = \dfrac{z+3}{z-1} \Rightarrow \dfrac{dz}{dx} = \dfrac{2z+2}{z-1} = \dfrac{2(z+1)}{z-1}$	A1		For correct equation in z **AEF**
		A1	3	For correct simplification to **AG**
(ii)	$\displaystyle\int \frac{z-1}{z+1}\,dz = 2\int dx$	B1		For $\displaystyle\int \frac{z-1}{z+1}(dz)$ and $\displaystyle\int (1)\,(dx)$ seen or implied
	$\Rightarrow \displaystyle\int 1 - \frac{2}{z+1}\,dz$ *OR* $\displaystyle\int 1 - \frac{2}{u}\,du = 2x\ (+c)$	M1		For rearrangement of LHS into integrable form *OR* substitution e.g. $u = z + 1$ or $u = z - 1$
	$\Rightarrow$ $z - 2\ln(z+1)$ *OR* $z + 1 - 2\ln(z+1)$ $= 2x\ (+c)$	A1		For correct integration of LHS as f(z)
	$\Rightarrow -2\ln(x + y + 1) = x - y + c$	A1	4	For correct general solution **AEF**

$\boxed{7}$

4	**(i)**	$\cos^5\theta = \left(\dfrac{e^{i\theta}+e^{-i\theta}}{2}\right)^5$	B1		For $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ seen or implied z may be used for $e^{i\theta}$ throughout
		$\cos^5\theta = \frac{1}{32}\left(e^{i\theta}+e^{-i\theta}\right)^5$	M1		For expanding $\left(e^{i\theta}+e^{-i\theta}\right)^5$. At least 3 terms and 2 binomial coefficients required *OR* reasonable attempt at expansion in stages
		$\cos^5\theta = \frac{1}{32}\left(e^{5i\theta}+e^{-5i\theta}+5\left(e^{3i\theta}+e^{-3i\theta}\right)+10\left(e^{i\theta}+e^{-i\theta}\right)\right)$	A1		For correct binomial expansion
		$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$	M1		For grouping terms and using multiple angles
			A1	**5**	For answer obtained correctly **AG**
	(ii)	$\cos\theta = 16\cos^5\theta$	B1		For stating correct equation of degree 5 *OR* $1 = 16\cos^4\theta$ **AEF**
		$\Rightarrow \cos\theta = 0, \quad \cos\theta = \pm\frac{1}{2}$	M1		For obtaining at least one of the values of $\cos\theta$ from $\cos\theta = k\cos^5\theta$ *OR* from $1 = k\cos^4\theta$
		$\Rightarrow \theta = \frac{1}{2}\pi,\ \frac{1}{3}\pi,\ \frac{2}{3}\pi$	A1		A1 for any two correct values of θ
			A1	**4**	A1 for the 3rd value and no more in $0 \leqslant \theta \leqslant \pi$ Ignore values outside $0 \leqslant \theta \leqslant \pi$
			$\boxed{9}$		

5 (i)	METHOD 1 Lines meet where			
	$(x\ =)\ \ k+2\lambda\ =\ \ k+\mu$ $(y\ =)\ -1-5\lambda\ =\ -4-4\mu$ $(z\ =)\ \ 1-3\lambda\ =\ \ -2\mu$	M1 A1		For using parametric form to find where lines meet For at least 2 correct equations
		M1		For attempting to solve any 2 equations
	$\Rightarrow\ \ \lambda=-1,\quad \mu=-2$	A1		For correct values of λ and μ
		B1		For attempting a check in 3rd equation *OR* verifying point of intersection is on both lines
	$\Rightarrow\ \ (k-2,\ 4,\ 4)$	A1	6	For correct point of intersection (allow vector)
				SR For finding λ *OR* μ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1
	METHOD 2			
	$d=\dfrac{\lvert[0,3,1]\,\boldsymbol{.}\,[2,-5,-3]\times[1,-4,-2]\rvert}{\lvert\mathbf{b}\times\mathbf{c}\rvert}$			For using **a . b × c** with appropriate vectors (division by $\lvert\mathbf{b}\times\mathbf{c}\rvert$ is not essential)
	$d=c[0,3,1]\,\boldsymbol{.}\,[-2,1,-3]=0$	B1		and showing $d=0$ correctly
	$\Rightarrow$ lines intersect Lines meet where			
	$(x\ =)\ (k+)\ 2\lambda\ =\ (k+)\ \mu$ $(y\ =)\ -1-5\lambda\ =\ -4-4\mu$ $(z\ =)\ \ 1-3\lambda\ =\ \ -2\mu$	M1 A1		For using parametric form to find where lines meet For at least 2 correct equations
		M1		For attempting to solve any 2 equations
	$\Rightarrow\ \ \lambda=-1,\quad \mu=-2$	A1		For correct value of λ *OR* μ
	$\Rightarrow\ \ (k-2,\ 4,\ 4)$	A1		For correct point of intersection (allow vector)
	METHOD 3			
	e.g. $x-k=\dfrac{2(y+1)}{-5}=\dfrac{y+4}{-4}$	M1		For solving one pair of simultaneous equations
	$\Rightarrow\ y=4$	A1		For correct value of *x*, *y* or *z*
	$\dfrac{z-1}{-3}=\dfrac{y+1}{-5}$	M1		For solving for the third variable
	$x=k-2$ *OR* $z=4$	A1		For correct values of 2 of *x*, *y* and *z*
	$x-k=\dfrac{z}{-2}$ checks with $x=k-2,\ z=4$	B1		For attempting a check in 3rd equation
	$\Rightarrow\ \ (k-2,\ 4,\ 4)$	A1		For correct point of intersection (allow vector)
(ii)	METHOD 1			
	$\mathbf{n}=[2,-5,-3]\times[1,-4,-2]$	M1		For finding vector product of 2 directions
	$\mathbf{n}=c[-2,1,-3]$	A1		For correct normal **SR** Following Method 2 for **(i),** award M1 A1√ for **n**, f.t. from their **n**
	$(1,-1,1)$ *OR* $(1,-4,0)$ *OR* $(-1,4,4)$ $\Rightarrow 2x-y+3z=6$	M1 A1	4	For substituting a point in LHS For correct equation of plane **AEF cartesian**
	METHOD 2			
	$\mathbf{r}=[1,-1,1]+\lambda[2,-5,-3]+\mu[1,-4,-2]$	M1		For using vector equation of plane (*OR* $[1,-4,0]$ for **a)**
	$x\ =\ 1+2\lambda+\mu$ $y\ =\ -1-5\lambda-4\mu$ $z\ =\ 1-3\lambda-2\mu$	A1		For writing 3 linear equations
		M1		For eliminating λ and μ
	$\Rightarrow 2x-y+3z=6$	A1		For correct equation of plane **AEF cartesian**
		$\boxed{10}$		

6	**(i)**	When a, b have opposite signs,	M1		For considering sign of $a\lvert b\rvert$ *OR* $b\lvert a\rvert$ in general or in a specific case
		$a\lvert b\rvert = \pm ab$, $b\lvert a\rvert = \mp ba \Rightarrow a\lvert b\rvert \ne b\lvert a\rvert$	A1	**2**	For showing that $a\lvert b\rvert \ne b\lvert a\rvert$ Note that $\lvert x\rvert = \sqrt{x^2}$ may be used
	(ii)	$(a \circ b) \circ c = (a\lvert b\rvert) \circ c = a\lvert b\rvert\lvert c\rvert$ *OR* $a\lvert bc\rvert$	M1		For using 3 distinct elements and simplifying $(a \circ b) \circ c$ *OR* $a \circ (b \circ c)$
			A1		For obtaining correct answer
		$a \circ (b \circ c) = a \circ (b\lvert c\rvert) = a\lvert b\lvert c\rvert\rvert = a\lvert b\rvert\lvert c\rvert$ *OR* $a\lvert bc\rvert$	M1		For simplifying the other bracketed expression
			A1	**4**	For obtaining the same answer
	(iii)		B1*		For stating $e = \pm 1$ *OR* no identity
		EITHER $a \circ e = a\lvert e\rvert = a \Rightarrow e = \pm 1$	M1		For attempting algebraic justification of +1 and –1 for e
		OR $e \circ a = e\lvert a\rvert = a$ $\Rightarrow e = 1$ for $a > 0$, $e = -1$ for $a < 0$	A1		For deducing no (unique) identity
		Not a group	B1 (*dep)	**4**	For stating not a group
				$\boxed{\textbf{10}}$	

7 (i)		B1	**1**	Polar or cartesian values of ω and ω^2 may be used anywhere in this question For showing 3 points in approximately correct positions Allow ω and ω^2 interchanged, or unlabelled
(ii)	*EITHER* $1+\omega+\omega^2$ = sum of roots of cubic = 0 *OR* $\omega^3=1\Rightarrow(\omega-1)(\omega^2+\omega+1)=0$ $\Rightarrow 1+\omega+\omega^2=0$ (for $\omega\neq1$) *OR* sum of G.P. $1+\omega+\omega^2=\frac{1-\omega^3}{1-\omega}\ \left(=\frac{0}{1-\omega}\right)=0$ *OR*  shown on Argand diagram or explained in terms of vectors *OR* $1+\text{cis}\,\frac{2}{3}\pi+\text{cis}\,\frac{4}{3}\pi=1+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)+\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)=0$	M1 A1	 **2**	For result shown by any correct method **AG** Reference to vectors in part **(i)** diagram may be made
(iii) (a)	$(2+\omega)(2+\omega^2)=4+2(\omega+\omega^2)+\omega^3$ $=4-2+1=3$	M1 A1	 **2**	For using $1+\omega+\omega^2=0$ *OR* values of ω, ω^2 For correct answer
(b)	$\frac{1}{2+\omega}+\frac{1}{2+\omega^2}=\frac{2+(\omega+\omega^2)+2}{3}=1$	M1 A1√	 **2**	For combining fractions *OR* multiplying top and bottom of 2 fractions by complex conjugates For correct answer f.t. from **(a)**
(iv)	For the cubic $x^3+px^2+qx+r=0$ METHOD 1			
	$\sum\alpha=2+1=3\ \ (\Rightarrow p=-3)$	M1		For calculating two of $\sum\alpha,\ \sum\alpha\beta,\ \alpha\beta\gamma$
	$\sum\alpha\beta=\frac{2}{2+\omega}+\frac{2}{2+\omega^2}+\frac{1}{3}=\frac{7}{3}\ \ (=q)$	M1		For calculating all of $\sum\alpha,\ \sum\alpha\beta,\ \alpha\beta\gamma$ *OR* all of *p*, *q*, *r*
	$\alpha\beta\gamma=\frac{2}{3}\ \ \left(\Rightarrow\ \ r=-\frac{2}{3}\right)$	A1		For at least two of $\sum\alpha,\ \sum\alpha\beta,\ \alpha\beta\gamma$ correct (or values of *p*, *q*, *r*)
	$\Rightarrow\ \ 3x^3-9x^2+7x-2=0$	A1	**4**	For correct equation **CAO**
	METHOD 2 $(x-2)\left(x-\frac{1}{2+\omega}\right)\left(x-\frac{1}{2+\omega^2}\right)=0$			
	$x^3+\left(-2-\frac{1}{2+\omega}-\frac{1}{2+\omega^2}\right)x^2$ $+\left(\frac{1}{(2+\omega)(2+\omega^2)}+\frac{2}{2+\omega}+\frac{2}{2+\omega^2}\right)x$	M1		For multiplying out LHS in terms of ω or $\text{cis}\,\frac{1}{3}k\pi$
	$-\frac{2}{(2+\omega)(2+\omega^2)}=0$	M1		For simplifying, using parts **(ii)**, **(iii)** or values of ω
	$\Rightarrow\ \ x^3-3x^2+\frac{7}{3}x-\frac{2}{3}=0$	A1		For at least two of *p*, *q*, *r* correct
	$\Rightarrow\ \ 3x^3-9x^2+7x-2=0$	A1		For correct equation **CAO**
			$\boxed{11}$	

8 (i)	$m^2+1=0 \quad \Rightarrow \quad m=\pm \mathrm{i}$	M1		For stating and attempting to solve correct auxiliary equation
	$\Rightarrow$ C.F. $(y=)\ C\mathrm{e}^{\mathrm{i}x}+D\mathrm{e}^{-\mathrm{i}x}=A\cos x+B\sin x$	A1	**2**	For correct C.F. (must be in trig form) **SR** If some or all of the working is omitted, award full credit for correct answer
(ii)(a)	$y=p(\ln\sin x)\sin x+qx\cos x$	M1		For attempting to differentiate P.I. (product rule needed at least once)
	$\frac{\mathrm{d}y}{\mathrm{d}x}=p\frac{\cos x}{\sin x}\sin x+p(\ln\sin x)\cos x+q\cos x-qx\sin x$	A1		For correct (unsimplified) result **AEF**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-p\sin x-p(\ln\sin x)\sin x+\frac{p\cos^2 x}{\sin x}$ $-2q\sin x-qx\cos x$	A1		For correct (unsimplified) result **AEF**
	$-p\sin x+\frac{p\cos^2 x}{\sin x}-2q\sin x\equiv\frac{1}{\sin x}$	M1		For substituting their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and y into D.E.
		M1		For using $\sin^2 x+\cos^2 x=1$
	$\Rightarrow p-2(p+q)\sin^2 x\equiv 1$	A1	**6**	For simplifying to **AG** ($\equiv$ may be $=$)
(b)		M1		For attempting to find p and q by equating coefficients of constant and $\sin^2 x$ *AND/OR* giving value(s) to x (allow any value for x, including 0)
	$p=1,\ \ q=-1$	A1	**2**	For both values correct
(iii)	G.S. $y=A\cos x+B\sin x+(\ln\sin x)\sin x-x\cos x$	B1√		For correct G.S. f.t. from their C.F. and P.I. with 2 arbitrary constants in C.F. (allow given form of P.I. if p and q have not been found)
	$\operatorname{cosec} x$ undefined at $x=0,\ \pi,\ 2\pi$ *OR* $\sin x>0$ in $\ln\sin x$	M1		For considering domain of $\operatorname{cosec} x$ *OR* $\sin x\neq 0$ *OR* $\ln\sin x$ term
	$\Rightarrow\ 0<x<\pi$	A1	**3**	For stating correct range **CAO** **SR** Award B1 for correct answer with justification omitted or incorrect
		$\boxed{13}$		

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1 (i) (a)	$(n=)\ 3$	B1	**1**	For correct *n*
(b)	$(n=)\ 6$	B1	**1**	For correct *n*
(c)	$(n=)\ 4$	B1	**1**	For correct *n*
(ii)	$(n=)\ 4,\ 6$	B1		For *either* 4 *or* 6
		B1	**2**	For both 4 and 6 and no extras Ignore all $n \ldots 8$ **SR** B0 B0 if more than 3 values given, even if they include 4 or 6
			$\boxed{5}$	
2 (i)	$\frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}} \times \frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}+\mathrm{i}} = \frac{1}{2}+\frac{1}{2}\mathrm{i}\sqrt{3}$	M1		For multiplying top and bottom by complex conjugate
	OR $\frac{\sqrt{3}+\mathrm{i}}{\sqrt{3}-\mathrm{i}} = \frac{2\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}}{2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}}$			*OR* for changing top and bottom to polar form
	$= (1)\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1		For $(r=)$ 1 (may be implied)
		A1	**3**	For $(\theta =)\ \frac{1}{3}\pi$ **SR** Award maximum A1 A0 if $\mathrm{e}^{\mathrm{i}\theta}$ form is not seen
(ii)	$\left(\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}\right)^6 = \mathrm{e}^{2\pi\mathrm{i}} = 1 \Rightarrow \quad (n=)\ 6$	M1		For use of $\mathrm{e}^{2\pi\mathrm{i}}=1$, $\mathrm{e}^{\mathrm{i}\pi}=-1$, $\sin k\pi = 0$ *or* $\cos k\pi = \pm 1$ (may be implied)
		A1	**2**	For $(n=)$ 6 **SR** For $(n=)$ 3 only, award M1 A0
			$\boxed{5}$	
3 (i)	$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$	M1		For using direction vectors and attempt to find vector product
	$= [2, -1, -1]$	A1	**2**	For correct direction (allow multiples)
(ii)	$d = \frac{\lvert [5, 2, 1] \,.\, [2, -1, -1] \rvert}{\sqrt{6}}$	B1		For $(\mathbf{AB} =)\ [5, 2, 1]$ or any vector joining lines
		M1		For attempt at evaluating $\mathbf{AB} \,.\, \mathbf{n}$
		M1		For $\lvert\mathbf{n}\rvert$ in denominator
	$= \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6} = 2.8577$	A1	**4**	For correct distance
			$\boxed{6}$	

4	$m^2+4m+5\,(=0) \Rightarrow m=\dfrac{-4\pm\sqrt{16-20}}{2}$	M1		For attempt to solve correct auxiliary equation
	$=-2\pm \mathrm{i}$	A1		For correct roots
	CF $=\mathrm{e}^{-2x}(C\cos x+D\sin x)$	A1√		For correct CF (here or later). f.t. from m **AEtrig** but not forms including $\mathrm{e}^{\mathrm{i}x}$
	PI $=p\sin 2x+q\cos 2x$	B1		For stating a trial PI of the correct form
	$y'=2p\cos 2x-2q\sin 2x$ $y''=-4p\sin 2x-4q\cos 2x$	M1		For differentiating PI twice and substituting into the DE
	$\cos 2x\left(-4q+8p+5q\right)$ $+\sin 2x\left(-4p-8q+5p\right)\ =65\sin 2x$	A1		For correct equation
	$\left.\begin{array}{l}8p+q=0\\ p-8q=65\end{array}\right\}\quad p=1,\quad q=-8$	M1		For equating coefficients of $\cos 2x$ and $\sin 2x$ and attempting to solve for p and/or q
	PI $=\sin 2x-8\cos 2x$	A1		For correct p and q
	$\Rightarrow y=$ $\mathrm{e}^{-2x}(C\cos x+D\sin x)+\sin 2x-8\cos 2x$	B1√	9	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
		$\boxed{9}$		
5 (i)	$y=u-\dfrac{1}{x}\ \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}u}{\mathrm{d}x}+\dfrac{1}{x^2}$	M1 A1		For differentiating substitution For correct expression
	$x^3\left(\dfrac{\mathrm{d}u}{\mathrm{d}x}+\dfrac{1}{x^2}\right)=x\left(u-\dfrac{1}{x}\right)+x+1$	M1		For substituting y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ into DE
	$\Rightarrow x^2\dfrac{\mathrm{d}u}{\mathrm{d}x}=u$	A1	4	For obtaining correct equation **AG**
(ii)	METHOD 1 $\displaystyle\int\frac{1}{u}\,\mathrm{d}u=\int\frac{1}{x^2}\,\mathrm{d}x\ \Rightarrow\ \ln ku=-\frac{1}{x}$	M1 A1		For separating variables and attempt at integration For correct integration (k not required here)
	$ku=\mathrm{e}^{-1/x}\ \Rightarrow\ k\left(y+\dfrac{1}{x}\right)=\mathrm{e}^{-1/x}$	M1 M1		For any 2 of For all 3 of $\left.\right\}$ k seen, exponentiating, substituting for u
	$\Rightarrow\ y=A\mathrm{e}^{-1/x}-\dfrac{1}{x}$	A1	5	For correct solution **AEF** in form $y=\mathrm{f}(x)$
	METHOD 2 $\dfrac{\mathrm{d}u}{\mathrm{d}x}-\dfrac{1}{x^2}u=0\ \Rightarrow\ $ I.F. $\mathrm{e}^{\int -1/x^2\,\mathrm{d}x}=\mathrm{e}^{1/x}$	M1		For attempt to find I.F.
	$\Rightarrow\ \dfrac{\mathrm{d}}{\mathrm{d}x}\left(u\,\mathrm{e}^{1/x}\right)=0$	A1		For correct result
	$u\,\mathrm{e}^{1/x}=k\ \Rightarrow\ y+\dfrac{1}{x}=k\,\mathrm{e}^{-1/x}$	M1 M1		From $\boxed{u\times\text{I.F.}=}$, for k seen for substituting for u $\left.\right\}$ in either order
	$\Rightarrow\ y=k\,\mathrm{e}^{-1/x}-\dfrac{1}{x}$	A1		For correct solution **AEF** in form $y=\mathrm{f}(x)$
		$\boxed{9}$		

6 (i)	METHOD 1			
	Use 2 of $[-4, 2, 0], [0, 0, 3], [-4, 2, 3], [4, -2, 3]$ or multiples	M1		For finding vector product of 2 appropriate vectors in plane *ACGE*
	$\mathbf{n} = k\,[1, 2, 0]$	A1		For correct **n**
	Use $A[4, 0, 0], C[0, 2, 0], G[0, 2, 3]$ *OR* $E[4, 0, 3]$	M1		For substituting a point in the plane
	$\mathbf{r}.[1, 2, 0] = 4$	A1	**4**	For correct equation. **AEF** in this form
	METHOD 2			
	$\mathbf{r} = [4, 0, 0] + \lambda[-4, 2, 0] + \mu[0, 0, 3]$	M1		For writing plane in 2-parameter form
	$\Rightarrow x = 4 - 4\lambda,\ y = 2\lambda,\ z = 3\mu$	A1		For 3 correct equations
	$x + 2y = 4$	M1		For eliminating λ (and μ)
	$\Rightarrow \mathbf{r}.[1, 2, 0] = 4$	A1		For correct equation. **AEF** in this form
(ii)	$\theta = \cos^{-1}\dfrac{\lvert[3, 0, -4].[1, 2, 0]\rvert}{\sqrt{3^2+0^2+4^2}\sqrt{1^2+2^2+0^2}}$	B1√ M1 M1		For using correct vectors (allow multiples). f.t. from **n** For using scalar product For multiplying both moduli in denominator
	$\theta = \cos^{-1}\dfrac{3}{5\sqrt{5}} = 74.4°$ $(74.435...°, 1.299...)$	A1	**4**	For correct angle
(iii)	*AM*: $(\mathbf{r} =)\ [4, 0, 0] + t[-2, 2, 3]$	M1		For obtaining parametric expression for *AM*
	$(or\ \ [2, 2, 3] + t[-2, 2, 3])$	A1		For correct expression seen or implied
	$3(4-2t) - 4(3t) = 0$ $(or\ \ 3(2-2t) - 4(3+3t) = 0\ )$	M1		For finding intersection of *AM* with *ACGE*
	$t = \frac{2}{3}\ \ (or\ \ t = -\frac{1}{3})$ *OR* $\mathbf{w} = \left[\frac{8}{3}, \frac{4}{3}, 2\right]$	A1		For correct *t* *OR* position vector
	$AW : WM = 2 : 1$	A1	**5**	For correct ratio
		$\boxed{13}$		
7 (i) **(a)**	$x + y - a \in \mathrm{R}$	B1		For stating closure is satisfied
	$(x*y)*z = (x+y-a)*z = x+y+z-2a$	M1		For using 3 distinct elements bracketed both ways
	$x*(y*z) = x*(y+z-a) = x+y+z-2a$	A1		For obtaining the same result twice for associativity **SR** 3 distinct elements bracketed once, expanded, and symmetry noted scores M1 A1
	$x + e - a = x \ \Rightarrow\ e = a$	B1		For stating identity = *a*
	$x + x^{-1} - a = a \ \Rightarrow\ x^{-1} = 2a - x$	M1 A1	**6**	For attempting to obtain inverse of *x* For obtaining inverse $= 2a - x$ *OR* for showing that inverses exist, where $x + x^{-1} = 2a$
(b)	$x + y - a = y + x - a \Rightarrow$ commutative	B1	**1**	For stating commutativity is satisfied, with justification
(c)	*x* order 2 $\Rightarrow x*x = e \ \Rightarrow\ 2x - a = e$ $\Rightarrow\ 2x - a = a \ \Rightarrow\ x = a = e$ *OR* $x = x^{-1} \Rightarrow x = 2a - x \Rightarrow x = a = e$ $\Rightarrow$ no elements of order 2	M1 A1	**2**	For obtaining equation for an element of order 2 For solving and showing that the only solution is the identity (which has order 1) *OR* For proving that there are no self-inverse elements (other than the identity)

(ii)	e.g. $2+1-5=-2 \notin \mathrm{R}^{+}$	M1		For attempting to disprove closure
	⇒ not closed	A1		For stating closure is not necessarily satisfied ($0 < x+y \leqslant 5$ required)
	e.g. $2\times 5-11=-1 \notin \mathrm{R}^{+}$	M1		For attempting to find an element with no inverse
	⇒ no inverse	A1	**4**	For stating inverse is not necessarily satisfied ($x \geqslant 10$ required)
		$\boxed{13}$		
8 (i)	$\sin\theta = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}\right)$	B1		*z* may be used for $\mathrm{e}^{\mathrm{i}\theta}$ throughout For expression for sinθ seen or implied
		M1		For expanding $\left(\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}\right)^{6}$ At least 4 terms and 3 binomial coefficients required.
	$\sin^{6}\theta =$ $-\frac{1}{64}\left(\mathrm{e}^{6\mathrm{i}\theta}-6\mathrm{e}^{4\mathrm{i}\theta}+15\mathrm{e}^{2\mathrm{i}\theta}-20+15\mathrm{e}^{-2\mathrm{i}\theta}-6\mathrm{e}^{-4\mathrm{i}\theta}+\mathrm{e}^{-6\mathrm{i}\theta}\right)$	A1		For correct expansion. Allow $\frac{\pm(\mathrm{i})}{64}\left(\cdots\cdots\right)$
	$=-\frac{1}{64}\left(2\cos 6\theta-12\cos 4\theta+30\cos 2\theta-20\right)$	M1		For grouping terms and using multiple angles
	$\sin^{6}\theta=-\frac{1}{32}\left(\cos 6\theta-6\cos 4\theta+15\cos 2\theta-10\right)$	A1	**5**	For answer obtained correctly **AG**
(ii)	$\cos^{6}\theta =$ *OR* $\sin^{6}\left(\frac{1}{2}\pi-\theta\right)=$ $-\frac{1}{32}\left(\cos(3\pi-6\theta)-6\cos(2\pi-4\theta)+15\cos(\pi-2\theta)-10\right)$	M1		For substituting $\left(\frac{1}{2}\pi-\theta\right)$ for θ throughout
		A1		For correct unsimplified expression
	$\cos^{6}\theta=\frac{1}{32}\left(\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10\right)$	A1	**3**	For correct expression with $\cos n\theta$ terms **AEF**
(iii)	$\int_{0}^{\frac{1}{4}\pi}\frac{1}{32}\left(-2\cos 6\theta-30\cos 2\theta\right)\,\mathrm{d}\theta$	B1√		For correct integral. f.t. from $\sin^{6}\theta-\cos^{6}\theta$
	$=-\frac{1}{16}\left[\frac{1}{6}\sin 6\theta+\frac{15}{2}\sin 2\theta\right]_{0}^{\frac{1}{4}\pi}$	M1		For integrating $\cos n\theta, \sin n\theta$ *or* $\mathrm{e}^{\mathrm{i}n\theta}$
		A1√		For correct integration. f.t. from integrand
	$=-\frac{11}{24}$	A1	**4**	For correct answer **WWW**
		$\boxed{12}$		

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1		$\left(\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}\right)^{\frac{1}{3}}=\left(\cos\frac{1}{6}\pi+\mathrm{i}\sin\frac{1}{6}\pi\right)^{\frac{1}{3}}$	B1		For $\arg z=\frac{1}{6}\pi$ seen or implied
		$=\cos\frac{1}{18}\pi+\mathrm{i}\sin\frac{1}{18}\pi$,	M1		For dividing $\arg z$ by 3
		$\cos\frac{13}{18}\pi+\mathrm{i}\sin\frac{13}{18}\pi$,	A1		For any one correct root
		$\cos\frac{25}{18}\pi+\mathrm{i}\sin\frac{25}{18}\pi$	A1	**4**	For 2 other roots and no more in range 0 „ $\theta<2\pi$
			$\boxed{4}$		
2	**(i)**	$\frac{1}{5}\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$	B1	**1**	For stating correct inverse in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$
	(ii)	$r_1\mathrm{e}^{\mathrm{i}\theta}\times r_2\mathrm{e}^{\mathrm{i}\phi}=r_1r_2\,\mathrm{e}^{\mathrm{i}(\theta+\phi)}$	M1 A1	 **2**	For stating 2 distinct elements multiplied For showing product of correct form
	(iii)	$Z^2=\mathrm{e}^{2\mathrm{i}\gamma}$	B1		For $\mathrm{e}^{2\mathrm{i}\gamma}$ seen or implied
		$\Rightarrow\ \mathrm{e}^{2\mathrm{i}\gamma-2\pi\mathrm{i}}$	B1	**2**	For correct answer. aef
			$\boxed{5}$		
3	**(i)**	$[6-4\lambda,\ -7+8\lambda,\ -10+7\lambda]$ on p	B1		For point on l seen or implied
		$\Rightarrow 3(6-4\lambda)-4(-7+8\lambda)-2(-10+7\lambda)=8$	M1		For substituting into equation of p
		$\Rightarrow\ \lambda=1\ \Rightarrow\ (2,1,-3)$	A1	**3**	For correct point. Allow position vector
	(ii)	METHOD 1			
		$\mathbf{n}=[-4,8,7]\times[3,-4,-2]$	M1* M1 (*dep)		For direction of l and normal of p seen For attempting to find $\mathbf{n}_1\times\mathbf{n}_2$
		$\mathbf{n}=k[12,13,-8]$	A1		For correct vector
		$(2,1,-3)$ OR $(6,-7,-10)$	M1		For finding scalar product of their point on l with their attempt at **n**, or equivalent
		$\Rightarrow\ 12x+13y-8z=61$	A1	**5**	For correct equation, aef cartesian
		METHOD 2			
		$\mathbf{r}=[2,1,-3]$ *OR* $[6,-7,-10]$ $+\lambda[-4,8,7]+\mu[3,-4,-2]$	M1 A1√		For stating eqtn of plane in parametric form (may be implied by next stage), using $[2,1,-3]$ (ft from **(i)**) Or $[6,-7,-10]$, $\mathbf{n}_1$ and $\mathbf{n}_2$ (as above)
		$x=\ \ 2-4\lambda+3\mu$ $y=\ \ 1+8\lambda-4\mu$ $z=-3+7\lambda-2\mu$	M1 M1		For writing as 3 linear equations For attempting to eliminate λ and μ
		$\Rightarrow\ 12x+13y-8z=61$	A1		For correct equation aef cartesian
		METHOD 3			
		$3(6+3\mu)-4(-7-4\mu)-2(-10-2\mu)=8$	M1		For finding foot of perpendicular from point on l to p
		$\Rightarrow\ \mu=-2\ \Rightarrow\ (0,1,-6)$	A1		For correct point or position vector
		From 3 points $(2,1,-3)$, $(6,-7,-10)$, $(0,1,-6)$, **n** = vector product of 2 of $[2,0,3]$, $[6,-8,-4]$, $[-4,8,7]$	M1		Use vector product of 2 vectors in plane
		$\Rightarrow\ \mathbf{n}=k[12,13,-8]$			
		$(2,1,-3)$ OR $(6,-7,-10)$	M1		For finding scalar product of their point on l with their attempt at **n**, or equivalent
		$\Rightarrow\ 12x+13y-8z=61$	A1		For correct equation aef cartesian
			$\boxed{8}$		

4	**(i)**	IF $e^{\int \frac{1}{1-x^2}\,\mathrm{d}x} = e^{\frac{1}{2}\ln\frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$	M1 A1	**2**	For IF stated or implied. Allow $\pm\int$ and omission of dx For integration and simplification to **AG** (intermediate step must be seen)
	(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\right) = (1+x)^{\frac{1}{2}}$	M1*		For multiplying both sides by IF
		$y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$	M1 A1		For integrating RHS to $k(1+x)^n$ For correct equation (including + c)
		$(0,\,2) \Rightarrow\; 2 = \frac{2}{3} + c \;\Rightarrow\; c = \frac{4}{3}$	M1 (*dep) M1 (*dep)		In either order: For substituting $(0,\,2)$ into their GS (including $+c$) For dividing solution through by IF, including dividing c or their numerical value for c
		$y = \frac{2}{3}(1+x)(1-x)^{\frac{1}{2}} + \frac{4}{3}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$	A1	**6**	For correct solution aef (even unsimplified) in form $y = \mathrm{f}(x)$
			$\boxed{8}$		
5	**(i)**	$m^2 - 6m + 9\ (=0) \Rightarrow\; m = 3$	M1 A1		For attempting to solve correct auxiliary equation For correct m
		CF = $(A + Bx)e^{3x}$	A1	**3**	For correct CF
	(ii)	ke^{3x} and kxe^{3x} both appear in CF	B1	**1**	For correct statement
	(iii)	$y = kx^2e^{3x} \;\Rightarrow\; y' = 2kxe^{3x} + 3kx^2e^{3x}$	M1 A1		For differentiating kx^2e^{3x} twice For correct y' aef
		$\Rightarrow\; y'' = 2ke^{3x} + 12kxe^{3x} + 9kx^2e^{3x}$	A1		For correct y'' aef
		$\Rightarrow$ $ke^{3x}\left(2 + 12x + 9x^2 - 12x - 18x^2 + 9x^2\right) = e^{3x}$	M1		For substituting y'', y', y into DE
		$\Rightarrow\; k = \frac{1}{2}$	A1	**5**	For correct k
			$\boxed{9}$		

6 (i)	METHOD 1			
	$\mathbf{n}_1 = [1, 1, 0] \times [1, -5, -2]$	M1		For attempting to find vector product of the pair of direction vectors
	$= [-2, 2, -6] = k[1, -1, 3]$	A1		For correct $\mathbf{n}_1$
	Use $(2, 2, 1)$	M1		For substituting a point into equation
	$\Rightarrow \mathbf{r}.[-2, 2, -6] = -6 \Rightarrow \mathbf{r}.[1, -1, 3] = 3$	A1	**4**	For correct equation. aef in this form
	METHOD 2			
	$x = 2 + \lambda + \mu$ $y = 2 + \lambda - 5\mu$ $z = 1 \quad - 2\mu$	M1 M1		For writing as 3 linear equations For attempting to eliminate λ and μ
	$\Rightarrow x - y + 3z = 3$	A1		For correct cartesian equation
	$\Rightarrow \mathbf{r}.[1, -1, 3] = 3$	A1		For correct equation. aef in this form
(ii)	For $\mathbf{r} = \mathbf{a} + t\mathbf{b}$			
	METHOD 1			
	$\mathbf{b} = [1, -1, 3] \times [7, 17, -3]$	M1		For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$
	$= k[2, -1, -1]$	A1√		For a correct vector. ft from $\mathbf{n}_1$ in **(i)**
	e.g. x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1		For attempting to find a point on the line
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√		For a correct vector. ft from equation in **(i)** **SR** a correct vector may be stated without working
	Line is (e.g.) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	**5**	For stating equation of line ft from **a** and **b** **SR** for $\mathbf{a} = [2, 2, 1]$ stated award M0
	METHOD 2			
	Solve $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$ by eliminating one variable (e.g. z)	M1		In either order: For attempting to solve equations
	Use parameter for another variable (e.g. x) to find other variables in terms of t	M1		For attempting to find parametric solution
	(eg) $y = \frac{3}{2} - \frac{1}{2}t$, $z = \frac{3}{2} - \frac{1}{2}t$	A1√ A1√		For correct expression for one variable For correct expression for the other variable ft from equation in **(i)** for both
	Line is (eg) $\mathbf{r} = \left[0, \frac{3}{2}, \frac{3}{2}\right] + t[2, -1, -1]$	A1√		For stating equation of line. ft from parametric solutions
	METHOD 3			
	eg x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1		For attempting to find a point on the line
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ *OR* $[3, 0, 0]$ *OR* $[1, 1, 1]$	A1√		For a correct vector. ft from equation in **(i)** **SR** a correct vector may be stated without working **SR** for $\mathbf{a} = [2, 2, 1]$ stated award M0
	eg $[3, 0, 0] - [1, 1, 1]$	M1		For finding another point on the line and using it with the one already found to find **b**
	$\mathbf{b} = k[2, -1, -1]$	A1√		For a correct vector. ft from equation in **(i)**
	Line is (eg) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√		For stating equation of line. ft from **a** and **b**

6 (ii) contd	METHOD 4			
	A point on Π_1 is $[2+\lambda+\mu,\ 2+\lambda-5\mu,\ 1-2\mu]$	M1		For using parametric form for Π_1 and substituting into Π_2
	On $\Pi_2 \Rightarrow$ $[2+\lambda+\mu,\ 2+\lambda-5\mu,\ 1-2\mu]\,.\,[7,17,-3]=21$	A1		For correct unsimplified equation
	$\Rightarrow\ \lambda-3\mu=-1$	A1		For correct equation
	Line is (e.g.) $\mathbf{r}=[2,\,2,\,1]+(3\mu-1)[1,1,0]+\mu[1,\,-5,\,-2]$	M1		For substituting into Π_1 for λ *or* μ
	$\Rightarrow\ \mathbf{r}=[1,1,1]$ *or* $\left[\frac{7}{3},\frac{1}{3},\frac{1}{3}\right]+t\,[2,-1,-1]$	A1		For stating equation of line
			$\boxed{9}$	
7 (i)	$\cos 3\theta+\mathrm{i}\sin 3\theta=\ c^3+3\mathrm{i}\,c^2s-3cs^2-\mathrm{i}\,s^3$	M1		For using de Moivre with $n=3$
	$\Rightarrow \cos 3\theta=c^3-3cs^2$ and $\sin 3\theta=3c^2s-s^3$	A1		For both expressions in this form (seen or implied) **SR** For expressions found without de Moivre M0 A0
	$\Rightarrow \tan 3\theta=\dfrac{3c^2s-s^3}{c^3-3cs^2}$	M1		For expressing $\dfrac{\sin 3\theta}{\cos 3\theta}$ in terms of c and s
	$\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=\dfrac{\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}$	A1	4	For simplifying to **AG**
(ii) (a)	$\theta=\frac{1}{12}\pi \Rightarrow\ \tan 3\theta=1$ $\Rightarrow\ 1-3t^2=t(3-t^2)\ \Rightarrow$ $t^3-3t^2-3t+1=0$	B1	1	For both stages correct **AG**
(b)	$(t+1)(t^2-4t+1)=0$	M1 A1		For attempt to factorise cubic For correct factors
	$\Rightarrow\ (t=-1),\ \ t=2\pm\sqrt{3}$	A1		For correct roots of quadratic
	– sign for smaller root $\Rightarrow$ $\tan\frac{1}{12}\pi=2-\sqrt{3}$	A1	4	For choice of – sign and correct root **AG**
(iii)	$\mathrm{d}t=(1+t^2)\ \mathrm{d}\theta$	B1		For differentiation of substitution and use of $\sec^2\theta=1+\tan^2\theta$
	$\Rightarrow \displaystyle\int_0^{\frac{1}{12}\pi}\tan 3\theta\ \mathrm{d}\theta$	B1		For integral with correct θ limits seen
	$=\left[\frac{1}{3}\ln\left(\sec 3\theta\right)\right]_0^{\frac{1}{12}\pi}=\frac{1}{3}\ln\left(\sec\frac{1}{4}\pi\right)$	M1		For integrating to $k\ln\left(\sec 3\theta\right)$ OR $k\ln\left(\cos 3\theta\right)$
	$=\frac{1}{3}\ln\sqrt{2}=\frac{1}{6}\ln 2$	M1		For substituting limits and $\sec\frac{1}{4}\pi=\sqrt{2}$ OR $\cos\frac{1}{4}\pi=\frac{1}{\sqrt{2}}$ seen
		A1	5	For correct answer aef
			$\boxed{14}$	

8	**(i)**	$a^2 = (ap)^2 = apap \Rightarrow a = pap$	B1		For use of given properties to obtain **AG**
		$p^2 = (ap)^2 = apap \Rightarrow p = apa$	B1	**2**	For use of given properties to obtain **AG** **SR** allow working from **AG** to obtain relevant properties
	(ii)	$\left(p^2\right)^2 = p^4 = e \Rightarrow$ order p^2 = 2	B1		For correct order with no incorrect working seen
		$\left(a^2\right)^2 = \left(p^2\right)^2 = e \Rightarrow$ order a = 4	B1		For correct order with no incorrect working seen
		$(ap)^4 = a^4 = e \Rightarrow$ order ap = 4	B1		For correct order with no incorrect working seen
		$\left(ap^2\right)^2 = ap^2ap^2 = ap\,.\,a\,.\,p = a^2$ *OR* $ap^2 = a.a^2 = a^3 \Rightarrow$	M1		For relevant use of **(i)** or given properties
		$\left(ap^2\right)^2 = a^6 = a^2$ $\Rightarrow$ order ap^2 = 4	A1	**5**	For correct order with no incorrect working seen
	(iii)	METHOD 1 $p^2 = a^2,\ ap^2 = a^3$	M2		For use of the given properties to simplify p^2 and $a\,p^2$
		$\Rightarrow \{e, a, p^2, ap^2\} = \{e, a, a^2, a^3\}$	A1		For obtaining a^2 and a^3
		which is a cyclic group	A1	**4**	For justifying that the set is a group
		METHOD 2 $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$	M1 A1		For attempting closure with all 9 non-trivial products seen For all 16 products correct
		Completed table is a cyclic group	B2		For justifying that the set is a group
		METHOD 3 $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$	M1 A1		For attempting closure with all 9 non-trivial products seen For all 16 products correct
		Identity = e	B1		For stating identity
		Inverses exist since EITHER: e is in each row/column OR: p^2 is self-inverse; a, ap^2 form an inverse pair	B1		For justifying inverses ($e^{-1} = e$ may be assumed)

(iv)	METHOD 1 e.g. $\left.\begin{array}{l} a\,.\,ap = a^2p = p^3 \\ ap\,.\,a = p \end{array}\right\} \Rightarrow$ not commutative	M1	For attempting to find a non-commutative pair of elements, at least one involving *a* (may be embedded in a full or partial table)
		M1	For simplifying elements both ways round
		B1	For a correct pair of non-commutative elements
		A1 **4**	For stating *Q* non-commutative, with a clear argument
	METHOD 2 Assume commutativity, so (eg) $ap = pa$	M1	For setting up proof by contradiction
	(i) $\Rightarrow$ $p = ap.a \Rightarrow p = pa.a = pa^2 = pp^2 = p^3$	M1	For using **(i)** and/or given properties
	But p and p^3 are distinct	B1	For obtaining and stating a contradiction
	$\Rightarrow Q$ is non-commutative	A1	For stating *Q* non-commutative, with a clear argument
		$\boxed{\textbf{15}}$	

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1	METHOD 1			
	line segment between l_1 and l_2 = $\pm[4, -3, -9]$	B1		For correct vector
	$\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$	M1* A1		For finding vector product of direction vectors
	distance = $\frac{\lvert [4, -3, -9] . [-2, 0, 1] \rvert}{\left(\sqrt{2^2+0^2+1^2}\right)} = \frac{17}{\left(\sqrt{5}\right)}$	M1 (*dep)		For using numerator of distance formula
	$\neq 0$, so skew	A1	**5**	For correct scalar product and correct conclusion
	METHOD 2 lines would intersect where			
	$\left.\begin{array}{r} 1+s=-3+2t \\ -2-s=1+3t \\ -4+2s=5+4t \end{array}\right\} \Rightarrow \left\{\begin{array}{l} s-2t=-4 \\ s+3t=-3 \\ 2s-4t=9 \end{array}\right.$	B1		For correct parametric form for either line
		M1*		For 3 equations using 2 different parameters
		A1		
		M1 (*dep)		For attempting to solve to show (in)consistency
	$\Rightarrow$ contradiction, so skew	A1		For correct conclusion
		$\boxed{5}$		
2 (i)	$\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)$	M1		For using product of 2 distinct elements
	$= ac+5bd+(bc+ad)\sqrt{5} \in H$	A1	**2**	For correct expression
(ii)	$(e =)\ 1$ *OR* $1+0\sqrt{5}$	B1	**1**	For correct identity
(iii)	*EITHER* $\frac{1}{a+b\sqrt{5}} \times \frac{a-b\sqrt{5}}{a-b\sqrt{5}}$	M1		For correct inverse as $\left(a+b\sqrt{5}\right)^{-1}$ and multiplying top and bottom by $a-b\sqrt{5}$
	OR $\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)=1 \Rightarrow \left\{\begin{array}{l} ac+5bd=1 \\ bc+ad=0 \end{array}\right.$			*OR* for using definition and equating parts
	inverse $= \frac{a}{a^2-5b^2} - \frac{b}{a^2-5b^2}\sqrt{5}$	A1	**2**	For correct inverse. Allow as a single fraction
(iv)	5 is prime *OR* $\sqrt{5} \notin \mathbb{Q}$	B1	**1**	For a correct property (or equivalent)
		$\boxed{6}$		
3	Integrating factor = $e^{\int 2\mathrm{d}x} = e^{2x}$	B1		For correct IF
	$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left(ye^{2x}\right) = e^{-x}$	M1		For $\frac{\mathrm{d}}{\mathrm{d}x}(y.\text{their IF}) = e^{-3x}$. their IF
	$\Rightarrow ye^{2x} = -e^{-x}(+c)$	A1		For correct integration both sides
	$(0, 1) \Rightarrow c = 2$	M1		For substituting $(0, 1)$ into their GS and solving for *c*
		A1√		For correct *c* f.t. from their GS
	$\Rightarrow y = -e^{-3x} + 2e^{-2x}$	A1	**6**	For correct solution
		$\boxed{6}$		
4 (i)	$(z =)\ 2, -2,\ 2i, -2i$	M1		For at least 2 roots of the form $k\{1,\ i\}$ **AEF**
		A1	**2**	For correct values

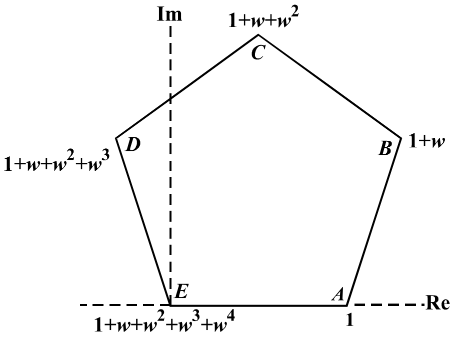
(ii)	$\frac{w}{1-w} = 2, -2, 2\mathrm{i}, -2\mathrm{i}$	M1	For $\frac{w}{1-w}$ = any one solution from **(i)**
	$w = \frac{z}{1+z}$	M1	For attempting to solve for w, using any solution or in general
	$w = \frac{2}{3}, 2$	B1 A1	For any one of the 4 solutions For both real solutions
	$w = \frac{4}{5} \pm \frac{2}{5}\mathrm{i}$	A1 **5**	For both complex solutions **SR** Allow B1√ and one A1√ from $k \neq 2$
		$\boxed{7}$	
5 (i)	$\mathbf{AB} = k\left[\frac{2}{3}\sqrt{3}, 0, -\frac{2}{3}\sqrt{6}\right]$,	B1	For any one edge vector of ΔABC
	$\mathbf{BC} = k\left[-\sqrt{3}, 1, 0\right]$, $\mathbf{CA} = k\left[\frac{1}{3}\sqrt{3}, -1, \frac{2}{3}\sqrt{6}\right]$	B1	For any other edge vector of ΔABC
	$\mathbf{n} = k_1\left[\frac{2}{3}\sqrt{6}, \frac{2}{3}\sqrt{18}, \frac{2}{3}\sqrt{3}\right] = k_2\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right]$	M1	For attempting to find vector product of any two edges
		M1	For substituting A, B or C into $\mathbf{r.n}$
	substitute A, B or C $\Rightarrow$ $x+\sqrt{3}y+\frac{1}{2}\sqrt{2}z = \frac{2}{3}\sqrt{3}$	A1 **5**	For correct equation **AG**
			SR For verification only allow M1, then A1 for 2 points and A1 for the third point
(ii)	Symmetry	B1*	For quoting symmetry or reflection
	in plane OAB or Oxz or $y = 0$	B1 (*dep)**2**	For correct plane Allow "in y coordinates" or "in y axis" **SR** For symmetry implied by reference to opposite signs in y coordinates of C and D, award B1 only
(iii)	$\cos\theta = \frac{\left\lvert\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right].\left[1, -\sqrt{3}, \frac{1}{2}\sqrt{2}\right]\right\rvert}{\sqrt{1+3+\frac{1}{2}}\sqrt{1+3+\frac{1}{2}}}$	M1 A1	For using scalar product of normal vectors For correct scalar product
	$= \frac{\left\lvert 1-3+\frac{1}{2}\right\rvert}{\frac{9}{2}} = \frac{\frac{3}{2}}{\frac{9}{2}} = \frac{1}{3}$	M1 A1 **4**	For product of both moduli in denominator For correct answer. Allow $-\frac{1}{3}$
		$\boxed{11}$	
6 (i)	$\left(m^2+16=0\Rightarrow\right)\ m = \pm 4\mathrm{i}$	M1	For attempt to solve correct auxiliary equation (may be implied by correct CF)
	CF = $A\cos 4x + B\sin 4x$	A1 **2**	For correct CF (**AEtrig** but not $A\mathrm{e}^{4\mathrm{i}x}+B\mathrm{e}^{-4\mathrm{i}x}$ only)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = p\sin 4x + 4px\cos 4x$	M1 A1	For differentiating PI twice, using product rule For correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8p\cos 4x - 16px\sin 4x$	A1√	For unsimplified $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. f.t. from $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$\Rightarrow 8p\cos 4x = 8\cos 4x$	M1	For substituting into DE
	$\Rightarrow p = 1$	A1	For correct p
	$\Rightarrow (y=) A\cos 4x + B\sin 4x + x\sin 4x$	B1√ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI

(iii)	$(0, 2) \Rightarrow A = 2$	B1√		For correct A. f.t. from their GS
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4A\sin 4x + 4B\cos 4x + \sin 4x + 4x\cos 4x$	M1		For differentiating their GS
	$x = 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow B = 0$	M1		For substituting values for x and $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find B
	$\Rightarrow y = 2\cos 4x + x\sin 4x$	A1	4	For stating correct solution **CAO** including $\boxed{y=}$
		$\boxed{12}$		
7 (i)	$\cos 6\theta = 0 \Rightarrow 6\theta = k \times \frac{1}{2}\pi$	M1		For multiples of $\frac{1}{2}\pi$ seen or implied
	$\Rightarrow \theta = \frac{1}{12}\pi\{1, 3, 5, 7, 9, 11\}$	A1		A1 for any 3 correct
		A1	3	A1 for the rest, and no extras in $0 < \theta < \pi$
(ii)	METHOD 1			
	$\mathrm{Re}(c + \mathrm{i}\,s)^6 = \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	M1		For expanding $(c + \mathrm{i}\,s)^6$ at least 4 terms and 2 binomial coefficients needed
		A1		For 4 correct terms
	$\cos 6\theta = c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$	M1		For using $s^2 = 1 - c^2$
	$\Rightarrow \cos 6\theta = 32c^6 - 48c^4 + 18c^2 - 1$	A1		For correct expression for $\cos 6\theta$
	$\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$	A1	5	For correct result **AG** (may be written down from correct $\cos 6\theta$)
	METHOD 2			
	$\mathrm{Re}(c + \mathrm{i}\,s)^3 = \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$	M1		For expanding $(c + \mathrm{i}\,s)^3$ at least 2 terms and 1 binomial coefficient needed
		A1		For 2 correct terms
	$\Rightarrow \cos 6\theta = \cos 2\theta\left(\cos^2 2\theta - 3\sin^2 2\theta\right)$	M1		For replacing θ by 2θ
	$\Rightarrow \cos 6\theta = \left(2\cos^2\theta - 1\right)\left(4\left(2\cos^2\theta - 1\right)^2 - 3\right)$	A1		For correct expression in $\cos\theta$ (unsimplified)
	$\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$	A1		For correct result **AG**
(iii)	METHOD 1			
	$\cos 6\theta = 0$	M1		For putting $\cos 6\theta = 0$
	$\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $16c^4 - 16c^2 + 1 = 0$ and $2c^2 - 1 = 0$	A1		For association of roots with quartic and quadratic
	But $\theta = \frac{1}{4}\pi, \frac{3}{4}\pi$ satisfy $2c^2 - 1 = 0$	B1		For correct association of roots with quadratic
	EITHER Product of 4 roots *OR* $c = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{3}}$	M1		For using product of 4 roots *OR* for solving quartic
	$\Rightarrow \cos\frac{1}{12}\pi\cos\frac{5}{12}\pi\cos\frac{7}{12}\pi\cos\frac{11}{12}\pi = \frac{1}{16}$	A1	5	For correct value (may follow A0 and B0)

	METHOD 2			
	$\cos 6\theta = 0$	M1		For putting $\cos 6\theta = 0$
	$\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $32c^6 - 48c^4 + 18c^2 - 1 = 0$	A1		For association of roots with sextic
	Product of 6 roots $\Rightarrow$	M1		For using product of 6 roots
	$\cos\frac{1}{12}\pi \,.\, \frac{1}{\sqrt{2}} . \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \,.\, \frac{-1}{\sqrt{2}} . \cos\frac{11}{12}\pi = -\frac{1}{32}$	B1		For using $\cos\left\{\frac{3}{12}\pi, \frac{9}{12}\pi\right\} = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$
	$\cos\frac{1}{12}\pi \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \cos\frac{11}{12}\pi = \frac{1}{16}$	A1		For correct value
		$\boxed{13}$		
8 (i)	$\mathrm{g}(x) = \dfrac{1}{2 - 2\,.\,\dfrac{1}{2-2x}} = \dfrac{2-2x}{2-4x} = \dfrac{1-x}{1-2x}$	M1 A1		For use of $\mathrm{f\,f}(x)$ For correct expression **AG**
	$\mathrm{gg}(x) = \dfrac{1 - \dfrac{1-x}{1-2x}}{1 - 2 . \dfrac{1-x}{1-2x}} = \dfrac{-x}{-1} = x$	M1 A1	 **4**	For use of $\mathrm{gg}(x)$ For correct expression **AG**
(ii)	Order of f = 4 order of g = 2	B1 B1	 **2**	For correct order For correct order
(iii)	METHOD 1			
	$y = \dfrac{1}{2-2x} \Rightarrow x = \dfrac{2y-1}{2y}$	M1		For attempt to find inverse
	$\Rightarrow \mathrm{f}^{-1}(x) = \mathrm{h}(x) = \dfrac{2x-1}{2x}$ *OR* $1 - \dfrac{1}{2x}$	A1	**2**	For correct expression
	METHOD 2			
	$\mathrm{f}^{-1} = \mathrm{f}^3 = \mathrm{f\,g}$ or $\mathrm{g\,f}$	M1		For use of $\mathrm{f\,g}(x)$ or $\mathrm{g\,f}(x)$
	$\mathrm{f\,g}(x) = \mathrm{h}(x) = \dfrac{1}{2 - 2\left(\dfrac{1-x}{1-2x}\right)} = \dfrac{1-2x}{-2x}$	A1		For correct expression
(iv)	$\begin{array}{c\|cccc} & e & f & g & h \\ \hline e & e & f & g & h \\ f & f & g & h & e \\ g & g & h & e & f \\ h & h & e & f & g \end{array}$	M1 A1 A1 A1	 **4**	For correct row 1 and column 1 For e, f, g, h in a latin square For correct diagonal e - g - e - g For correct table
		$\boxed{12}$		

1	Direction of $l_1 = k[7, 0, -10]$ Direction of $l_2 = k[1, 3, -1]$	B1	For both directions
	EITHER $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$ *OR* $\begin{cases} [x, y, z] . [7, 0, -10] = 0 \Rightarrow 7x - 10z = 0 \\ [x, y, z] . [1, 3, -1] = 0 \Rightarrow x + 3y - z = 0 \end{cases}$	M1	For finding vector product of directions of l_1 and l_2 *OR* for using 2 scalar products and obtaining equations
	$\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	For correct **n**
	METHOD 1 Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$ *OR* $\pm[-4, 3, 1]$ *OR* $\pm[3, 3, -9]$ *OR* $\pm[-3, 6, 0]$	B1	For a correct vector
	$d = \dfrac{\lvert(\mathbf{a} - \mathbf{b}) . \mathbf{n}\rvert}{\lvert\mathbf{n}\rvert} = \dfrac{36}{\sqrt{150}}$	M1*	For finding $(\mathbf{a} - \mathbf{b}) . \mathbf{n}$
		M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
	$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1 **7**	For correct distance **AEF**
	METHOD 2 Planes containing l_1 and l_2 perp. to **n**	M1*	For finding planes and $p_1 - p_2$ seen
	are $\mathbf{r} . [10, -1, 7] = p_1 = 70$, $\mathbf{r} . [10, -1, 7] = p_2 = 34$	B1	For $p_1 = 70k$ and $p_2 = 34k$
	$\Rightarrow d = \dfrac{\lvert 70 - 34 \rvert}{\sqrt{150}} = \dfrac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ in denominator *OR* for using $\hat{\mathbf{n}}$
		A1	For correct distance **AEF**
	METHOD 3 $\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ *OR* $[7 + 7\lambda, 0, -10\lambda]$ $\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ *OR* $[3 + \mu, 3 + 3\mu, 1 - \mu]$	B1	For correct points on l_1 and l_2 using different parameters
	$\begin{array}{rc\|c\|c\|c\|c\|} 7\lambda + 10\alpha - \mu & = & 4 & -3 & 3 & -4 \\ -\alpha - 3\mu & = & 6 & 6 & 3 & 3 \\ -10\lambda + 7\alpha + \mu & = & -10 & 0 & -9 & 1 \end{array}$ $\Rightarrow \alpha = -\frac{6}{25}$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α
	$\lvert\mathbf{n}\rvert = \sqrt{150}$	M1 (*dep)	For $\lvert\mathbf{n}\rvert$ seen multiplying α
	$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	For correct distance **AEF**
		$\boxed{7}$	

2	**(i)**	$ar = r^5a \Rightarrow rar = r^6a$	M1		Pre-multiply $ar = r^5a$ by r
		$r^6 = e \Rightarrow rar = a$	A1	**2**	Use $r^6 = e$ and obtain answer **AG**
	(ii)	METHOD 1			
		For n = 1, $rar = a$ *OR* For n = 0, $r^0ar^0 = a$	B1		For stating true for n = 1 *OR* for n = 0
		Assume $r^kar^k = a$			
		EITHER Assumption $\Rightarrow$ $r^{k+1}ar^{k+1} = rar = a$	M1		For attempt to prove true for k + 1
		OR $r^{k+1}ar^{k+1} = r.r^kar^k.r = rar = a$			
		OR $r^{k+1}ar^{k+1} = r^k.rar.r^k = r^kar^k = a$	A1		For obtaining correct form
		Hence true for all $n \in \mathbb{Z}^+$	A1	**4**	For statement of induction conclusion
		METHOD 2			
		$r^2ar^2 = r.rar.r = rar = a$, similarly for	M1		For attempt to prove for n = 2, 3
		$r^3ar^3 = a$			
		$r^4ar^4 = r.r^3ar^3.r = rar = a$,	A1		For proving true for n = 2, 3, 4, 5
		similarly for $r^5ar^5 = a$			
		$r^6ar^6 = eae = a$	B1		For showing true for n = 6
		For n > 6, $r^n = r^{n \bmod 6}$, hence true for all $n \in \mathbb{Z}^+$	A1		For using n mod 6 and correct conclusion
		METHOD 3			
		$r^nar^n = r^{n-1}.rar.r^{n-1}$	M1		Starting from n, for attempt to prove true for $n-1$
		OR $r^nar^n = r^n.r^5a.r^{n-1} = r^{n+5}ar^{n-1}$			
		$= r^{n-1}ar^{n-1}$	A1		For proving true for $n-1$
		$= r^{n-2}ar^{n-2} = \ldots$	A1		For continuation from $n-2$ downwards
		$= rar = a$	B1		For final use of $rar = a$
					SR can be done in reverse
		METHOD 4			
		$ar = r^5a \Rightarrow ar^2 = r^5ar = r^{10}a$ etc.	M1		For attempt to derive $ar^n = r^{5n}a$
		$\Rightarrow ar^n = r^{5n}a$	A1		For correct equation **SR** may be stated without proof
		$\Rightarrow$ $r^nar^n = r^{6n}a$	B1		For pre-multiplication by r^n
		$= ea = a$	A1		For obtaining a ($r^6 = e$ may be implied)
				$\boxed{6}$	

3					Allow $\operatorname{cis}\frac{k}{5}\pi$ and $e^{\frac{k}{5}\pi i}$ throughout
	(i)	$w^2=\cos\frac{4}{5}\pi+i\sin\frac{4}{5}\pi$	B1		For correct value
		$w^3=\cos\frac{6}{5}\pi+i\sin\frac{6}{5}\pi$	B1		For correct value
		$w^*=\cos\frac{2}{5}\pi-i\sin\frac{2}{5}\pi$	B1		For w^* seen or implied
		$=\cos\frac{8}{5}\pi+i\sin\frac{8}{5}\pi$	B1	4	For correct value **SR** For exponential form with i missing, award B0 first time, allow others
	(ii)		B1*		For $1+w$ in approximately correct position
			B1 (*dep)		For $AB \approx BC \approx CD$
			B1 (*dep)		For BC, CD equally inclined to Im axis
			B1	4	For E at the origin
					Allow points joined by arcs, or not joined Labels not essential
	(iii)	$z^5-1=0$ *OR* $z^5+z^4+z^3+z^2+z=0$	B1	1	For correct equation **AEF (**in any variable) Allow factorised forms using w, exp or trig
			$\boxed{9}$		
4	**(i)**	$y=xz\Rightarrow\frac{dy}{dx}=z+x\frac{dz}{dx}$	B1		For correct differentiation of substitution
		$\Rightarrow xz+x^2\frac{dz}{dx}-xz=x\cos z\Rightarrow x\frac{dz}{dx}=\cos z$	M1 A1		For substituting into DE For DE in variables separable form
		$\Rightarrow\int\sec z\ dz=\int\frac{1}{x}dx$	M1		For attempt at integration to ln form on LHS
		$\Rightarrow\ln(\sec z+\tan z)=\ln kx$ *OR* $\ln\tan\left(\frac{1}{2}z+\frac{1}{4}\pi\right)=\ln kx$	A1		For correct integration (k not required here)
		$\Rightarrow\sec\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)=kx$ *OR* $\tan\left(\frac{y}{2x}+\frac{1}{4}\pi\right)=kx$	A1	6	For correct solution **AEF** including $\text{RHS}=e^{(\ln x)+c}$
	(ii)	$(4,\pi)\Rightarrow\sec\frac{1}{4}\pi+\tan\frac{1}{4}\pi=4k$ *OR* $\tan\left(\frac{1}{8}\pi+\frac{1}{4}\pi\right)=4k$	M1		For substituting $(4,\pi)$ into their solution (with k)
		$\Rightarrow\sec\left(\frac{y}{x}\right)+\tan\left(\frac{y}{x}\right)=\frac{1}{4}\left(1+\sqrt{2}\right)x$ *OR* $\tan\left(\frac{y}{2x}+\frac{1}{4}\pi\right)=\left(\frac{1}{4}\tan\frac{3}{8}\pi\right)x$ *or* $\frac{1}{4}\left(1+\sqrt{2}\right)x$	A1	2	For correct solution **AEF** Allow decimal equivalent $0.60355x$ Allow $e^{\ln x}$ for x
			$\boxed{8}$		

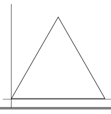
5	**(i)**	$C+\mathrm{i}\,S=1+\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}+\frac{1}{4}\mathrm{e}^{2\mathrm{i}\theta}+\frac{1}{8}\mathrm{e}^{3\mathrm{i}\theta}+\ldots$	M1		For using $\cos n\theta+\mathrm{i}\sin n\theta=\mathrm{e}^{\mathrm{i}\,n\theta}$ at least once for $n\geqslant 2$
			A1		For correct series
		$=\dfrac{1}{1-\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}}=\dfrac{2}{2-\mathrm{e}^{\mathrm{i}\theta}}$	M1		For using sum of infinite GP
			A1	**4**	For correct expression **AG** **SR** For omission of 1st stage award up to M0 A0 M1 A1 **OEW**
	(ii)	$C+\mathrm{i}\,S=\dfrac{2\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}{\left(2-\mathrm{e}^{\mathrm{i}\theta}\right)\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}$	M1		For multiplying top and bottom by complex conjugate
		$=\dfrac{4-2\mathrm{e}^{-\mathrm{i}\theta}}{4-2\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)+1}=\dfrac{4-2\cos\theta+2\mathrm{i}\sin\theta}{4-4\cos\theta+1}$	M1		For reverting to cosθ and sinθ and equating Re *OR* Im parts
		$\Rightarrow C=\dfrac{4-2\cos\theta}{5-4\cos\theta},\quad S=\dfrac{2\sin\theta}{5-4\cos\theta}$	A1		For correct expression for C **AG**
			A1	**4**	For correct expression for S
			$\boxed{8}$		
6	**(i)**	Aux. equation $m^2+2m+17(=0)$	M1		For attempting to solve correct auxiliary equation
		$\Rightarrow m=-1\pm 4\mathrm{i}$	A1		For correct roots
		CF $(y=)\ \mathrm{e}^{-x}\left(A\cos 4x+B\sin 4x\right)$	A1√		For correct CF (allow $A\genfrac{}{}{0pt}{}{\cos}{\sin}(4x+\varepsilon)$) (trig terms required, not $\mathrm{e}^{\pm 4\mathrm{i}\,x}$) f.t. from their m with 2 arbitrary constants
		PI $(y=)\ px+q\ \Rightarrow\ 2p+17(px+q)=17x+36$	M1		For stating and substituting PI of correct form
		$\Rightarrow\ \ p=1$	A1		For correct value of p
		and $q=2$	A1		For correct value of q
		GS $y=\mathrm{e}^{-x}\left(A\cos 4x+B\sin 4x\right)+x+2$	B1√	**7**	For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI. Requires $\boxed{y=}$.
	(ii)	$x\gg 0\Rightarrow \mathrm{e}^{-x}\to 0$ *OR* very small	B1		For correct statement. Allow graph
		$\Rightarrow y=x+2$ approximately	B1√	**2**	For correct equation Allow ≈ , → and in words Allow relevant f.t. from linear part of GS
			$\boxed{9}$		

7	(i)	$(1, 3, 5)$ and $(5, 2, 5) \Rightarrow \pm[4, -1, 0]$ in Π	M1		For finding a vector in Π
		$\mathbf{n} = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2]$	M1		For finding vector product of direction vectors of l and a line in Π
			A1		For correct **n**
		$\Rightarrow \mathbf{r}.[1, 4, 2] = 23$	A1	**4**	For correct equation. Allow multiples
	(ii)	METHOD 1			
		Perpendicular to Π through $(-7, -3, 0)$ meets Π	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		where $(-7+k)+4(-3+4k)+2(2k)=23$	M1		For substituting parametric line coords into Π
		$\Rightarrow\ k=2 \quad \Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1 A1	**4**	For normalising the **n** used in this part For correct distance **AEF**
		METHOD 2			
		Π is $x+4y+2z=23$	M1		For attempt to use formula for perpendicular distance
		$\Rightarrow d = \dfrac{\lvert(-7)+4(-3)+2(0)-23\rvert}{\sqrt{1^2+4^2+2^2}} = 2\sqrt{21} \approx 9.165$	M1		For substituting a point on l into plane equation
			M1 A1		For normalising the **n** used in this part For correct distance **AEF**
		METHOD 3			
		$\mathbf{m} = [1, 3, 5] - [-7, -3, 0] = (\pm)[8, 6, 5]$	M1		For finding a vector from l to Π
		OR $= [5, 2, 5] - [-7, -3, 0] = (\pm)[12, 5, 5]$			
		$\Rightarrow d = \dfrac{\mathbf{m}\,.\,[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{42}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1 M1 A1		For finding $\mathbf{m}\,.\,\mathbf{n}$ For normalising the **n** used in this part For correct distance **AEF**
		METHOD 4			As Method 1, using parametric form of Π
		$[-7, -3, 0] + k[1, 4, 2] = [1, 3, 5] + s[2, -2, 3] + t[4, -1, 0]$	M1		For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
		$\left.\begin{aligned} k-2s-4t &= 8 \\ 4k+2s+t &= 6 \\ 2k-3s &= 5 \end{aligned}\right\} \Rightarrow\ k=2 \quad \left(s=-\tfrac{1}{3},\ t=-\tfrac{4}{3}\right)$	M1		For setting up and solving 3 equations
		$\Rightarrow d = 2\sqrt{1^2+4^2+2^2} = 2\sqrt{21} \approx 9.165$	M1 A1		For normalising the **n** used in this part For correct distance **AEF**
		METHOD 5			
		$d_1 = \dfrac{23}{\sqrt{1^2+4^2+2^2}} = \dfrac{23}{\sqrt{21}}$	M1		For attempt to find distance from O to Π *OR* from O to parallel plane containing l
		$d_2 = \dfrac{[-7, -3, 0]\,.\,[1, 4, 2]}{\sqrt{1^2+4^2+2^2}} = \dfrac{-19}{\sqrt{21}}$	M1		For normalising the **n** used in this part
		$\Rightarrow\ d_1 - d_2 = d = \dfrac{23-(-19)}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1 A1		For finding $d_1 - d_2$ For correct distance **AEF**
	(iii)	$(-7, -3, 0) + k\,(1, 4, 2)$	M1		State or imply coordinates of a point on the reflected line
		Use $k=4$	M1		State or imply 2 × distance from **(ii)** Allow $k = \pm4$ *OR* $\pm4\sqrt{21}$ f.t. from **(ii)**
		$\mathbf{b} = [2, -2, 3]$	B1		For stating correct direction
		$\mathbf{a} = [-3, 13, 8]$	A1	**4**	For correct point seen in equation $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
		$\mathbf{r} = [-3, 13, 8] + t[2, -2, 3]$			**AEF** in this form

$\boxed{12}$

8	**(i)**	$\{A, D\}$ *OR* $\{A, E\}$ *OR* $\{A, F\}$	B1	**1**	For stating any one subgroup
	(ii)	A is the identity 5 is not a factor of 6 *OR* elements can be only of order 1, 2, 3, 6	B1 B1	**2**	For identifying A as the identity For reference to factors of 6
	(iii)	$BE = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = D$, $\quad EB = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = F$ D *or* $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, F *or* $\begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \in M$ $\Rightarrow$ closure property satisfied	M1 A1 A1 A1	**4**	For finding BE and EB AND using $\omega^3 = 1$ For correct BE (D *or* matrix) For correct EB (F *or* matrix) For justifying closure
	(iv)	$B^{-1} = \frac{1}{1}\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = C$ $E^{-1} = \frac{1}{-1}\begin{pmatrix} 0 & -\omega^2 \\ -\omega & 0 \end{pmatrix} = E$	M1 A1 A1	**3**	For correct method of finding either inverse For correct $B^{-1} = C$ Allow $\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$ For correct $E^{-1} = E$ Allow $\begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$
	(v)	METHOD 1 M is not commutative e.g. from $BE \neq EB$ in part **(iii)** N is commutative (as × mod 9 is commutative) $\Rightarrow M$ and N not isomorphic	B1 B1 B1#	**3**	For justification of M being not commutative For statement that N is commutative For correct conclusion
		METHOD 2 Elements of M have orders 1, 3, 3, 2, 2, 2 Elements of N have orders 1, 6, 3, 2, 3, 6 Different orders *OR* self-inverse elements $\Rightarrow M$ and N not isomorphic	B1* B1 (*dep) B1#		For all orders of one group correct For sufficient orders of the other group correct For correct conclusion **SR** Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic
		METHOD 3 M has no generator since there is no element of order 6 N has 2 *OR* 5 as a generator $\Rightarrow M$ and N not isomorphic	B1 B1 B1#		For all orders of M shown correctly For stating that N has generator 2 *OR* 5 For correct conclusion
		METHOD 4 $\begin{array}{c\|cccccc} M & A & B & C & D & E & F \\ \hline A & A & B & C & D & E & F \\ B & B & C & A & F & D & E \\ C & C & A & B & E & F & D \\ D & D & E & F & A & B & C \\ E & E & F & D & C & A & B \\ F & F & D & E & B & C & A \end{array}$ $\begin{array}{c\|cccccc} N & 1 & 2 & 4 & 8 & 7 & 5 \\ \hline 1 & 1 & 2 & 4 & 8 & 7 & 5 \\ 2 & 2 & 4 & 8 & 7 & 5 & 1 \\ 4 & 4 & 8 & 7 & 5 & 1 & 2 \\ 8 & 8 & 7 & 5 & 1 & 2 & 4 \\ 7 & 7 & 5 & 1 & 2 & 4 & 8 \\ 5 & 5 & 1 & 2 & 4 & 8 & 7 \end{array}$ $\Rightarrow M$ and N not isomorphic	B1* B1 (*dep) B1#		For stating correctly all 6 squared elements of one group For stating correctly sufficient squared elements of the other group For correct conclusion
					# In all Methods, the last B1 is dependent on at least one preceding B1
				$\boxed{13}$	

1	**(i)**	Integrating factor. $e^{\int x\,dx}=e^{\frac{1}{2}x^2}$	B1		For correct IF
		$\Rightarrow \frac{d}{dx}\left(y\,e^{\frac{1}{2}x^2}\right)=x\,e^{x^2}$	M1		For $\frac{d}{dx}\left(y.\text{their IF}\right)=x\,e^{\frac{1}{2}x^2}.\text{their IF}$
		$\Rightarrow y\,e^{\frac{1}{2}x^2}=\frac{1}{2}e^{x^2}(+c)$	A1		For correct integration both sides
		$\Rightarrow\ y=e^{-\frac{1}{2}x^2}\left(\frac{1}{2}e^{x^2}+c\right)=\frac{1}{2}e^{\frac{1}{2}x^2}+c\,e^{-\frac{1}{2}x^2}$	A1	**4**	For correct solution **AEF** as $y=f(x)$
	(ii)	$(0,\ 1)\Rightarrow c=\frac{1}{2}$	M1		For substituting (0, 1) into their GS, solving for c and obtaining a solution of the DE
		$\Rightarrow\ y=\frac{1}{2}\left(e^{\frac{1}{2}x^2}+e^{-\frac{1}{2}x^2}\right)$	A1	**2**	For correct solution **AEF** Allow $y=\cosh\left(\frac{1}{2}x^2\right)$
			$\boxed{6}$		
2	**(i)**	$\mathbf{n}=[2,1,-3]\times[-1,2,4]$	M1		For using × of direction vectors
		$=[10,-5,5]=k[2,-1,1]$	A1		For correct **n**
		$(1,3,4)\Rightarrow 2x-y+z=3$	A1	**3**	For substituting $(1,3,4)$ and obtaining **AG** (Verification only M0)
	(ii)	METHOD 1 distance = $\frac{21-3}{\lvert\mathbf{n}\rvert}$ *OR* $\frac{\lvert[1,3,4].[2,-1,1]-21\rvert}{\lvert\mathbf{n}\rvert}$	M1		For $21-3$ *OR* $[1,3,4].[2,-1,1]-21$ *OR* $\lvert([1,3,4]-[a,b,c]).[2,-1,1]\rvert$ soi
		OR $\frac{\lvert([1,3,4]-[a,b,c]).[2,-1,1]\rvert}{\lvert\mathbf{n}\rvert}$ where (a,b,c) is on q	B1		For $\lvert\mathbf{n}\rvert=\sqrt{6}$ soi
		$=\frac{18}{\sqrt{6}}=3\sqrt{6}$	A1	**3**	For correct distance **AEF**
		METHOD 2 $[1+2t,3-t,4+t]$ on q	M1		For forming and solving an equation in t
		$\Rightarrow\ 2(1+2t)-(3-t)+(4+t)=21\ \Rightarrow\ t=3$	B1		For $\lvert\mathbf{n}\rvert=\sqrt{6}$ soi
		$\Rightarrow$ distance $=3\lvert\mathbf{n}\rvert=3\sqrt{6}$	A1		For correct distance **AEF**
		METHOD 3 As Method 2 to $t=3\ \Rightarrow\ (7,0,7)$ on q	M1*		For finding point where normal meets q
		distance from $(1,3,4)$	M1 (*dep)		For finding distance from $(1,3,4)$
		$=\sqrt{(7-1)^2+(0-3)^2+(7-4)^2}=\sqrt{54}=3\sqrt{6}$	A1		For correct distance **AEF**
			$\boxed{6}$		
3	**(i)**	$\sin\theta=\frac{1}{2i}\left(e^{i\theta}-e^{-i\theta}\right)$	B1		z or $e^{i\theta}$ may be used throughout For correct expression for $\sin\theta$ soi
		$\sin^4\theta=\frac{1}{16}\left(z^4-4z^2+6-4z^{-2}+z^{-4}\right)$	M1		For expanding $\left(e^{i\theta}-e^{-i\theta}\right)^4$ (with at least 3 terms and 1 binomial coefficient)
		$\Rightarrow\ \sin^4\theta=\frac{1}{16}\left(2\cos 4\theta-8\cos 2\theta+6\right)$	M1		For grouping terms and using multiple angles
		$\Rightarrow\ \sin^4\theta=\frac{1}{8}\left(\cos 4\theta-4\cos 2\theta+3\right)$	A1	**4**	For answer obtained correctly **AG**
	(ii)	$\int_0^{\frac{1}{6}\pi}\sin^4\theta\,d\theta=\frac{1}{8}\left[\frac{1}{4}\sin 4\theta-2\sin 2\theta+3\theta\right]_0^{\frac{1}{6}\pi}$	M1		For integrating **(i)** to $A\sin 4\theta+B\sin 2\theta+C\theta$
			A1		For correct integration
		$=\frac{1}{8}\left(\frac{1}{8}\sqrt{3}-\sqrt{3}+\frac{1}{2}\pi\right)=\frac{1}{64}\left(4\pi-7\sqrt{3}\right)$	M1		For completing integration and substituting limits
			A1	**4**	For correct answer **AEF**(exact)
			$\boxed{8}$		

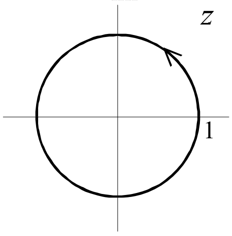
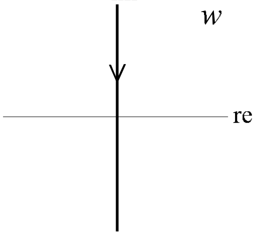
4	**(i)**	*EITHER* $1+\omega+\omega^2$ = sum of roots of ($z^3-1=0$) = 0	M1 A1	**2**	For result shown by any correct method **AG**
		OR $\omega^3=1\Rightarrow(\omega-1)(\omega^2+\omega+1)=0$ $\Rightarrow 1+\omega+\omega^2=0$ (for $\omega\neq 1$)			
		OR sum of G.P. $1+\omega+\omega^2=\dfrac{1-\omega^3}{1-\omega}\left(=\dfrac{0}{1-\omega}\right)=0$			
		OR  shown on Argand diagram or explained in terms of vectors			
		OR $1+\text{cis}\,\frac{2}{3}\pi+\text{cis}\,\frac{4}{3}\pi=1+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)+\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)=0$			
	(ii)	Multiplication by $\omega\Rightarrow$ rotation through $\frac{2}{3}\pi$ ↺	B1		For correct interpretation of × by ω (allow 120° and omission of, or error in, ↺)
		$z_1-z_3=\overrightarrow{CA}$, $z_3-z_2=\overrightarrow{BC}$	B1		For identification of vectors soi (ignore direction errors)
		$\overrightarrow{BC}$ rotates through $\frac{2}{3}\pi$ to direction of $\overrightarrow{CA}$	M1		For linking *BC* and *CA* by rotation of $\frac{2}{3}\pi$ *OR* ω
		Δ *ABC* has *BC* = *CA*, hence result	A1	**4**	For stating equal magnitudes ⇒ **AG**
	(iii)	**(ii)** $\Rightarrow z_1+\omega z_2-(1+\omega)z_3=0$	M1		For using $1+\omega+\omega^2=0$ in **(ii)**
		$1+\omega+\omega^2=0\Rightarrow z_1+\omega z_2+\omega^2 z_3=0$	A1	**2**	For obtaining **AG**
				$\boxed{8}$	
5	**(i)**	Aux. equation $3m^2+5m-2\ (=0)$	M1		For correct auxiliary equation seen and solution attempted
		$\Rightarrow m=\frac{1}{3},-2$	A1		For correct roots
		CF $(y=)\ A\text{e}^{\frac{1}{3}x}+B\text{e}^{-2x}$	A1√		For correct CF f.t. from *m* with 2 arbitrary constants
		PI $(y=)\ px+q\Rightarrow 5p-2(px+q)=-2x+13$	M1		For stating and substituting PI of correct form
		$\Rightarrow p=1,\quad q=-4$	A1 A1		For correct value of *p*, and of *q*
		GS $(y=)\ A\text{e}^{\frac{1}{3}x}+B\text{e}^{-2x}+x-4$	B1√	**7**	For GS f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI
	(ii)	$\left(0,-\frac{7}{2}\right)\Rightarrow A+B=\frac{1}{2}$	M1		For substituting $\left(0,-\frac{7}{2}\right)$ in their GS and obtaining an equation in *A* and *B*
		$y'=\frac{1}{3}A\text{e}^{\frac{1}{3}x}-2B\text{e}^{-2x}+1,\ (0,0)\Rightarrow A-6B=-3$	M1		For finding y', substituting $(0,0)$ and obtaining an equation in *A* and *B*
			M1		For solving their 2 equations in *A* and *B*
		$\Rightarrow A=0,\ B=\frac{1}{2}$	A1		For correct *A* and *B* **CAO**
		$\Rightarrow (y=)\ \frac{1}{2}\text{e}^{-2x}+x-4$	B1√	**5**	For correct solution f.t. with their *A* and *B* in their GS
	(iii)	*x* large $\Rightarrow (y=)\ x-4$	B1√	**1**	For correct equation or function (allow ≈ and →) **WWW** f.t. from **(ii)** if valid
				$\boxed{13}$	

<table>
<tr><td>6</td><td>(i)</td><td>$a^4 = r^6 = e \Rightarrow a$ has order 4, a^2 has order 2</td><td>M1</td><td></td><td>For considering powers of <i>a</i></td></tr>
<tr><td></td><td></td><td>$\left(a^3\right)^4 = a^{12} = e \Rightarrow a^3$ has order 4</td><td>A1
A1</td><td></td><td>For order of any one of $a,\ a^2, a^3$ correct
For all correct</td></tr>
<tr><td></td><td></td><td>$\left(r^2\right)^3 = e \Rightarrow r^2$ has order 3</td><td>B1</td><td>4</td><td>For order of r^2 correct</td></tr>
<tr><td></td><td>(ii)</td><td><i>G</i> order 4
<table>
<tr><td>Order of element</td><td>1</td><td>2</td><td>(4)</td></tr>
<tr><td>Number of elements</td><td>1</td><td>3</td><td>(0)</td></tr>
</table></td><td>M1</td><td></td><td>For top line in either table
Allow inclusion of 4 and 6 respectively
(and other orders if 0 appears below)</td></tr>
<tr><td></td><td></td><td><i>H</i> order 6
<table>
<tr><td>Order of element</td><td>1</td><td>2</td><td>3</td><td>(6)</td></tr>
<tr><td>Number of elements</td><td>1</td><td>3</td><td>2</td><td>(0)</td></tr>
</table></td><td>A1
A1</td><td></td><td>For order 4 table
For order 6 table</td></tr>
<tr><td></td><td></td><td><i>G</i> and <i>H</i> are the only non-cyclic groups of order which divides 12</td><td>B1</td><td></td><td>For stating that only <i>G</i> and <i>H</i> need be considered AEF</td></tr>
<tr><td></td><td></td><td><i>Q</i> has 1 element of order 2, <i>G</i> and <i>H</i> have 3,
so no non-cyclic subgroups in <i>Q</i></td><td>B1</td><td>5</td><td>For argument completed by elements of order 2 AG
SR Allow equivalent arguments for B1 B1</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\boxed{9}$</td><td></td></tr>
<tr><td>7</td><td>(i)</td><td>$[1, 1, -2] \times [1, -1, 3] = (\pm)[1, -5, -2]$</td><td>M1
A1</td><td></td><td>For using × of direction vectors
For correct direction</td></tr>
<tr><td></td><td></td><td>$[1, -1, 3] \times [1, 5, -12] = (\pm)[-3, 15, 6]$</td><td>M1
A1</td><td></td><td>For using × of direction vectors
For correct direction</td></tr>
<tr><td></td><td></td><td>$[-3, 15, 6] = k\,[1, -5, -2] \Rightarrow$ parallel</td><td>A1</td><td>5</td><td>For argument completed AG
($k = -3$ not essential)</td></tr>
<tr><td></td><td>(ii)</td><td>Line of intersection is parallel to <i>l</i> and <i>m</i></td><td>B1</td><td>1</td><td>For correct statement</td></tr>
<tr><td></td><td>(iii)</td><td>METHOD 1
$\left.\begin{array}{l} x+y-2z=5 \\ x-y+3z=6 \end{array}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{11}{2}, -\frac{1}{2}, 0\right)$ on l</td><td>M1
A1</td><td></td><td>For attempt to find points on 2 lines
For a correct point on one line</td></tr>
<tr><td></td><td></td><td>$\left.\begin{array}{r} x-y+3z=6 \\ x+5y-12z=12 \end{array}\right\}$ e.g. $z=0 \Rightarrow (7,\ 1,\ 0)$ on m</td><td>A1</td><td></td><td>For a correct point on another line</td></tr>
<tr><td></td><td></td><td>$\left.\begin{array}{r} x+y-2z=5 \\ x+5y-12z=12 \end{array}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{13}{4},\ \frac{7}{4}, 0\right)$ on l_3</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Different points ⇒ no common line of intersection</td><td>A1</td><td>4</td><td>For correct answer</td></tr>
<tr><td></td><td></td><td>METHOD 2
$\left.\begin{array}{l} x+y-2z=5 \\ x-y+3z=6 \end{array}\right\}$ e.g. $\Rightarrow z = 11-2x,\ y = 27-5x$</td><td>M1

A1</td><td></td><td>For finding (e.g.) <i>y</i> and <i>z</i> in terms of <i>x</i>
<i>OR</i> eliminating one variable
For correct expressions <i>OR</i> equations</td></tr>
<tr><td></td><td></td><td>LHS of eqn 3 =
$x + (135-25x) - (132-24x) = 3 \ne 12$</td><td>A1</td><td></td><td>For obtaining a contradiction from 3rd equation</td></tr>
<tr><td></td><td></td><td>⇒ no common line of intersection</td><td>A1</td><td></td><td>For correct answer</td></tr>
<tr><td></td><td></td><td>METHOD 3
LHS $\Pi_3 = 3\Pi_1 - 2\Pi_2$</td><td>M2</td><td></td><td>For attempt to link 3 equations</td></tr>
<tr><td></td><td></td><td>RHS $3\times5 - 2\times6 = 3 \ne 12$</td><td>A1</td><td></td><td>For obtaining a contradiction</td></tr>
<tr><td></td><td></td><td>⇒ no common line of intersection</td><td>A1</td><td></td><td>For correct answer</td></tr>
<tr><td></td><td></td><td>SR Variations on all methods may gain full credit</td><td></td><td></td><td>SR f.t. may be allowed from relevant working</td></tr>
<tr><td></td><td></td><td></td><td></td><td>$\boxed{10}$</td><td></td></tr>
</table>

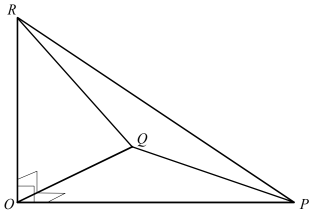
8	**(i)**	$((a,b)*(c,d))*(e,f)=(ac,\ ad+b)*(e,f)$	M1		For 3 distinct elements bracketed and attempt to expand
		$=(ace,\ acf+ad+b)$	A1		For correct expression
		$(a,b)*((c,d)*(e,f))=(a,b)*(ce,\ cf+d)$			
		$=(ace,\ acf+ad+b)$	A1	**3**	For correct expression again
	(ii)	$(a,\ b)*(1,1)=(a,\ a+b)$, $(1,1)*(a,\ b)=(a,\ b+1)$	M1		For combining both ways round
		$a+b=b+1\ \Rightarrow\ a=1$	M1		For equating components (allow from incorrect pairs)
		$\Rightarrow\ (1,\ b)\ \forall\ b$	A1	**3**	For correct elements **AEF**
	(iii)	$(mp,\ mq+n)$ *OR* $(pm,\ pn+q)=(1,\ 0)$	M1		For either element on LHS
		$\Rightarrow\ (p,\ q)=\left(\frac{1}{m},-\frac{n}{m}\right)$	A1	**2**	For correct inverse
	(iv)	$(a,b)*(a,b)=(a^2,\ ab+b)=(1,\ 0)$ *OR* $(a,\ b)=\left(\frac{1}{a},-\frac{b}{a}\right)\ \Rightarrow\ a^2=1,\ ab=-b$	M1		For attempt to find self-inverses
		$\Rightarrow$ self-inverse elements $(1,\ 0)$ and $(-1,\ b)\ \forall\ b$	B1 A1	**3**	For $(1,\ 0)$. For $(-1,\ b)$ **AEF**
	(v)	$(0,\ y)$ has no inverse for any $y\Rightarrow$ not a group	B1	**1**	For stating any one element with no inverse. Allow $x\neq 0$ required, provided reference to inverse is made “Some elements have no inverse” B0
				$\boxed{12}$	

1 (i)	$\theta = \sin^{-1}\dfrac{\lvert[5,\,6,\,-7]\,.\,[1,\,2,\,-1]\rvert}{\sqrt{5^2+6^2+(-7)^2}\,\sqrt{1^2+2^2+(-1)^2}}$	M1*	For using scalar product of line and plane vectors
		M1 (*dep)	For both moduli seen
	$\theta = \sin^{-1}\dfrac{24}{\sqrt{110}\sqrt{6}} = 69.1^\circ\ (69.099...^\circ,\ 1.206)$	A1	For correct scalar product
		A1 **4**	For correct angle
	$\phi = \sin^{-1}\dfrac{\lvert[5,\,6,\,-7]\times[1,\,2,\,-1]\rvert}{\sqrt{5^2+6^2+(-7)^2}\,\sqrt{1^2+2^2+(-1)^2}}$	**SR** M1*	For vector product of line and plane vectors AND finding modulus of result
		M1 (*dep)	For moduli of line and plane vectors seen
	$\phi = \sin^{-1}\dfrac{\sqrt{84}}{\sqrt{110}\sqrt{6}} = 20.9^\circ \Rightarrow \theta = 69.1^\circ$	A1	For correct modulus $\sqrt{84}$
		A1	For correct angle
(ii)	METHOD 1		
	$d = \dfrac{\lvert 1+12+3-40\rvert}{\sqrt{1^2+2^2+(-1)^2}} = \dfrac{24}{\sqrt{6}} = 4\sqrt{6} \approx 9.80$	M1	For use of correct formula
		A1 **2**	For correct distance
	METHOD 2		
	$(1+\lambda)+2(6+2\lambda)-(-3-\lambda)=40$	M1	For substituting parametric form into plane
	$\Rightarrow\ \lambda = 4\ \Rightarrow\ d = 4\sqrt{6}$	A1	For correct distance
	OR distance from $(1,\,6,\,-3)$ to $(5,\,14,\,-7)$		
	$=\sqrt{4^2+8^2+(-4)^2} = \sqrt{96}$		
	METHOD 3		
	Plane through $(1,\,6,\,-3)$ parallel to p is	M1	For finding parallel plane through $(1,\,6,\,-3)$
	$x+2y-z=16\ \Rightarrow\ d = \dfrac{40-16}{\sqrt{6}} = \dfrac{24}{\sqrt{6}}$	A1	For correct distance
	METHOD 4		
	e.g. $(0,\,0,\,-40)$ on p $\Rightarrow$ vector to $(1,\,6,\,-3)$ = $\pm(1,\,6,\,37)$	M1	For using any point on p to find vector and scalar product seen e.g. $[1,\,6,\,37]\,.\,[1,\,2,\,-1]$
	$d = \dfrac{\lvert[1,\,6,\,37]\,.\,[1,\,2,\,-1]\rvert}{\sqrt{6}} = \dfrac{24}{\sqrt{6}}$	A1	For correct distance
	METHOD 5		
	l meets p where $(1+5t)+2(6+6t)-(-3-7t)=40$ $\Rightarrow\ t=1\ \Rightarrow\ d = \lvert[5,\,6,\,-7]\rvert\sin\theta$	M1	For finding t where l meets p and linking d with triangle
	$\Rightarrow\ d = \sqrt{110}\,\dfrac{24}{\sqrt{110}\sqrt{6}} = \dfrac{24}{\sqrt{6}}$	A1	For correct distance
		$\boxed{6}$	
2 (i)	METHOD 1		
	EITHER $\dfrac{1+e^{i\theta}}{1-e^{i\theta}} = \dfrac{e^{-\frac{1}{2}i\theta}+e^{\frac{1}{2}i\theta}}{e^{-\frac{1}{2}i\theta}-e^{\frac{1}{2}i\theta}}$	M1	*EITHER* For changing LHS terms to $e^{\pm\frac{1}{2}i\theta}$ *OR in reverse* For using $\cot\frac{1}{2}\theta = \dfrac{\cos\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}$
	$= \dfrac{2\cos\frac{1}{2}\theta}{-2i\sin\frac{1}{2}\theta} = i\cot\frac{1}{2}\theta$	M1	For either of $\begin{matrix}\cos\\ \sin\end{matrix}\frac{1}{2}\theta = \dfrac{e^{\frac{1}{2}i\theta}\pm e^{-\frac{1}{2}i\theta}}{(2)\,(i)}$ soi
	OR in reverse with similar working	A1 **3**	For fully correct proof to **AG** **SR** If factors of 2 or i are not clearly seen, award M1 M1 A0

2	**(i)**	METHOD 2			
		EITHER $\frac{1+e^{i\theta}}{1-e^{i\theta}}\times\frac{1-e^{-i\theta}}{1-e^{-i\theta}}=\frac{e^{i\theta}-e^{-i\theta}}{2-\left(e^{i\theta}+e^{-i\theta}\right)}$ *OR* $\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}\times\frac{1-\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}$	M1		For multiplying top and bottom by complex conjugate in exp or trig form
		$=\frac{2i\sin\theta}{2-2\cos\theta}=\frac{2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\sin^2\frac{1}{2}\theta}=i\cot\frac{1}{2}\theta$	M1 A1		For using both double angle formulae correctly For fully correct proof to **AG**
		METHOD 3			
		$\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}=\frac{2\cos^2\frac{1}{2}\theta+2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\sin^2\frac{1}{2}\theta-2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}$	M1		For using both double angle formulae correctly
		$=\frac{2\cos\frac{1}{2}\theta\left(\cos\frac{1}{2}\theta+i\sin\frac{1}{2}\theta\right)}{2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta\right)}$	M1		For appropriate factorisation
		$=i\cot\frac{1}{2}\theta\frac{\left(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta\right)}{\left(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta\right)}=i\cot\frac{1}{2}\theta$	A1		For fully correct proof to **AG**
		METHOD 4			
		$\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}=\frac{1+\frac{1-t^2}{1+t^2}+i\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}-i\frac{2t}{1+t^2}}$	M1		For substituting both *t* formulae correctly
		$=\frac{2+2it}{2t^2-2it}=\frac{1}{t}\frac{1+it}{t-i}=\frac{i}{t}\frac{t-i}{t-i}=i\cot\frac{1}{2}\theta$	M1 A1		For appropriate factorisation For fully correct proof to **AG**
		METHOD 5			
		$\frac{1+e^{i\theta}}{1-e^{i\theta}}\times\frac{1+e^{i\theta}}{1+e^{i\theta}}=\frac{1+2e^{i\theta}+e^{2i\theta}}{1-e^{2i\theta}}$ $=\frac{2+e^{i\theta}+e^{-i\theta}}{e^{-i\theta}-e^{i\theta}}$	M1		For multiplying top and bottom by $1+e^{i\theta}$ and attempting to divide by $e^{i\theta}$ *OR* multiplying top and bottom by $1+e^{-i\theta}$
		$=\frac{2(1+\cos\theta)}{-2i\sin\theta}=\frac{2\cos^2\frac{1}{2}\theta}{-2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}=\frac{\cos\frac{1}{2}\theta}{-i\sin\frac{1}{2}\theta}$	M1		For using both double angle formulae correctly
		$=i\cot\frac{1}{2}\theta$	A1	3	For fully correct proof to **AG**
	(ii)	 	M1 A1 B1	 3	For a circle centre *O* For indication of radius = 1 and anticlockwise arrow shown For locus of *w* shown as imaginary axis described downwards
				$\boxed{6}$	

3	**(i)**	METHOD 1 $m+4\,(=0) \Rightarrow$ CF $(y=)Ae^{-4x}$	M1 A1	 **2**	For correct auxiliary equation (soi) For correct CF
		METHOD 2 Separating variables on $\frac{\mathrm{d}y}{\mathrm{d}x}+4y=0$ $\Rightarrow \ln y = -4x$	M1		For integration to this stage
		$\Rightarrow$ CF $(y=)Ae^{-4x}$	A1		For correct CF
	(ii)	PI $(y=)\ p\cos 3x+q\sin 3x$	B1		For stating PI of correct form
		$y' = -3p\sin 3x+3q\cos 3x$	M1		For substituting y and y' into DE
		$\Rightarrow\ (-3p+4q)\sin 3x+(4p+3q)\cos 3x = 5\cos 3x$	A1		For correct equation
		$\Rightarrow \left.\begin{array}{r}-3p+4q=0\\ 4p+3q=5\end{array}\right\} \Rightarrow p=\frac{4}{5},\ q=\frac{3}{5}$	M1 A1 A1		For equating coeffs and solving For correct value of p, and of q
		GS $(y=)\ Ae^{-4x}+\frac{4}{5}\cos 3x+\frac{3}{5}\sin 3x$	B1√	**7**	For GS f.t. from their CF+PI with 1 arbitrary constant in CF and none in PI
		SR Integrating factor method may be used, followed by 2-stage integration by parts or *C*+i*S* method Marks for **(i)** are awarded only if CF is clearly identified			
	(iii)	$e^{-4x} \to 0$, $\frac{4}{5}\cos 3x+\frac{3}{5}\sin 3x = \substack{\sin\\ \cos}(3x+\alpha)$	M1		For considering either term
		$\Rightarrow\ -1 \leqslant y \leqslant 1$ *OR* $-1 \lessapprox y \lessapprox 1$	A1√	**2**	For correct range (allow <) **CWO** f.t. as $-\sqrt{p^2+q^2} \leqslant y \leqslant \sqrt{p^2+q^2}$ from **(ii)**
			$\boxed{11}$		
4	**(i)**	$abc=(ab)c=(ba)c=b(ac)=$ $b(ca)=(bc)a=(cb)a=cba$ Minimum working: $abc=bac=bca=cba$ *OR* $abc=acb=cab=cba$ *OR* $abc=bac=bca=cba$	M1 A1	 **2**	For using commutativity correctly For correct proof (use of associativity may be implied)
	(ii)	$\{e, a\},\{e, b\},\{e, c\},\{e, bc\},\{e, ca\},\{e, ab\},\{e, abc\}$	B1 B1	 **2**	For any 5 subgroups For the other 2 subgroups and none incorrect
	(iii)	$\{e, a, b, ab\}, \{e, a, c, ca\}, \{e, b, c, bc\}$	B1		For any 3 subgroups
		$\{e, a, bc, abc\}, \{e, b, ca, abc\}, \{e, c, ab, abc\}$	B1		For 1 more subgroup
		$\{e, bc, ca, ab\}$	B1	**3**	For 1 more subgroup (5 in total) and none incorrect
	(iv)	All elements $(\neq e)$ have order 2 *OR* all are self-inverse *OR* no element of *G* has order 4 *OR* no order 4 subgroup has a generator *or* is cyclic *OR* subgroups are of the form $\{e, a, b, ab\}$ (the Klein group)	B1*		For appropriate reference to order of elements in *G*
		$\Rightarrow$ all order 4 subgroups are isomorphic	B1 (*dep)	**2**	For correct conclusion
			$\boxed{9}$		

5	**(i)**	$\frac{\mathrm{d}y}{\mathrm{d}x}=k\,u^{k-1}\frac{\mathrm{d}u}{\mathrm{d}x}$	M1 A1		For using chain rule For correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
		$\Rightarrow x\,k\,u^{k-1}\frac{\mathrm{d}u}{\mathrm{d}x}+3u^{k}=x^{2}u^{2k}$	M1		For substituting for y and $\frac{\mathrm{d}y}{\mathrm{d}x}$
		$\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}+\frac{3}{kx}u=\frac{1}{k}xu^{k+1}$	A1	**4**	For correct equation **AG**
	(ii)	$k=-1$	B1	**1**	For correct k
	(iii)	$\frac{\mathrm{d}u}{\mathrm{d}x}-\frac{3}{x}u=-x \Rightarrow \text{IF} \;\; \mathrm{e}^{-\int \frac{3}{x}\mathrm{d}x}=\mathrm{e}^{-3\ln x}=\frac{1}{x^{3}}$	B1√		For correct IF f.t. for IF = $x^{\frac{3}{k}}$ using k or their numerical value for k
		$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left(u.\frac{1}{x^{3}}\right)=-\frac{1}{x^{2}}$	M1		For $\frac{\mathrm{d}}{\mathrm{d}x}(u.\text{their IF})=-x.\text{their IF}$
		$\Rightarrow u.\frac{1}{x^{3}}=\frac{1}{x}(+c) \;\; \Rightarrow \;\; y=\frac{1}{cx^{3}+x^{2}}$	A1 A1	 **4**	For correct integration both sides For correct solution for y
			$\boxed{9}$		
6	**(a)**	Closure $(ax+b)+(cx+d)=(a+c)x+(b+d)$	B1		For obtaining correct sum from 2 distinct elements
		$\in P$	B1		For stating result is in P *OR* is of the correct form **SR** award this mark if any of the closure result, the identity or the inverse element is stated to be in P *OR* of the correct form
		Identity $0x+0$	B1		For stating identity (allow 0)
		Inverse $-ax-b$	B1	**4**	For stating inverse
	(b) (i)	Order 9	B1*	**1**	For correct order
	(ii)	$x+2$	B1	**1**	For correct inverse element
	(iii)	$(ax+b)+(ax+b)+(ax+b)=3ax+3b$	M1		For considering sums of $ax+b$ and obtaining $3ax+3b$
		$=0x+0$ $\Rightarrow ax+b$ has order 3 $\forall\, a, b$ (except $a=b=0$)	A1		For equating to $0x+0$ *OR* 0 and obtaining order 3 **SR** For order 3 stated only *OR* found from incomplete consideration of numerical cases award B1
		Cyclic group of order 9 has element(s) of order 9	M1 (*dep)		For reference to element(s) of order 9
		$\Rightarrow (Q,+(\mathrm{mod}\,3))$ is not cyclic	A1	**4**	For correct conclusion
			$\boxed{10}$		

7 **(i)**		B1		For sketch of tetrahedron labelled in some way At least one right angle at *O* must be indicated or clearly implied
		M1		For using $\Delta = \frac{1}{2}$ base × height
	$\Delta\ OPQ = \frac{1}{2}pq$, $\Delta\ OQR = \frac{1}{2}qr$, $\Delta\ ORP = \frac{1}{2}rp$	A1	**3**	For all areas correct **CAO**
(ii)	$\frac{1}{2}\left\lvert \overrightarrow{RP} \times \overrightarrow{RQ} \right\rvert = \frac{1}{2}\lvert \overrightarrow{RP} \rvert \lvert \overrightarrow{RQ} \rvert \sin R = \Delta\ PQR$	B1	**1**	For correct justification
(iii)	LHS = $\left(\frac{1}{2}pq\right)^2 + \left(\frac{1}{2}qr\right)^2 + \left(\frac{1}{2}rp\right)^2$	B1		For correct expression
	$\Delta\ PQR\ =\ \frac{1}{2}\lvert(p\mathbf{i} - q\mathbf{j}) \times (p\mathbf{i} - r\mathbf{k})\rvert$ $OR\quad \frac{1}{2}\lvert(p\mathbf{i} - r\mathbf{k}) \times (q\mathbf{j} - r\mathbf{k})\rvert$ $OR\quad \frac{1}{2}\lvert(p\mathbf{i} - q\mathbf{j}) \times (q\mathbf{j} - r\mathbf{k})\rvert$	B1		For ΔPQR in vector form
	$\Delta\ PQR = \frac{1}{2}\lvert qr\mathbf{i} + pr\mathbf{j} + pq\mathbf{k}\rvert$	M1		For finding vector product of their attempt at ΔPQR
		A1		For correct expression
	$\text{RHS} = \frac{1}{4}\left((pq)^2 + (qr)^2 + (rp)^2\right)$	M1		For using $\lvert a\mathbf{i} + b\mathbf{j} + c\mathbf{k}\rvert = \sqrt{a^2 + b^2 + c^2}$
		A1	**6**	For completing proof of **AG WWW**
			$\boxed{10}$	

8	(i)	$\text{Re}(c+\mathrm{i}\,s)^4 = \cos 4\theta = c^4 - 6c^2s^2 + s^4$	M1*		For expanding $(c+\mathrm{i}\,s)^4$: at least 2 terms and 1 binomial coefficient needed
			A1		For 3 correct terms
		$\cos 4\theta = c^4 - 6c^2(1-c^2) + (1-c^2)^2$	M1 (*dep)		For using $s^2 = 1 - c^2$
		$\Rightarrow \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	A1	4	For correct expression for $\cos 4\theta$ **CAO**
	(ii)	$\cos 4\theta \cos 2\theta = \left(8c^4 - 8c^2 + 1\right)\left(2c^2 - 1\right)$			For multiplying by $\left(2c^2 - 1\right)$
		$= 16\cos^6\theta - 24\cos^4\theta + 10\cos^2\theta - 1$	B1	1	to obtain **AG** **WWW**
	(iii)	$16c^6 - 24c^4 + 10c^2 - 2 = 0$	M1		For factorising sextic
		$\Rightarrow \left(c^2 - 1\right)\left(8c^4 - 4c^2 + 1\right) = 0$			with $(c-1)$, $(c+1)$ *or* $(c^2 - 1)$
		For quartic, $b^2 - 4ac = 16 - 32 < 0$	A1		For justifying no other roots **CWO**
		$\Rightarrow$ $c = \pm 1$ only $\Rightarrow$ $\theta = n\pi$	A1	3	For obtaining $\theta = n\pi$ **AG**
					Note that M1 A0 A1 is possible
				SR	For verifying $\theta = n\pi$ by substituting $c = \pm 1$ into $16c^6 - 24c^4 + 10c^2 - 2 = 0$ B1
	(iv)	$16c^6 - 24c^4 + 10c^2 = 0$			
		$\Rightarrow c^2\left(8c^4 - 12c^2 + 5\right) = 0$	M1		For factorising sextic with c^2
		For quartic, $b^2 - 4ac = 144 - 160 < 0$	A1		For justifying no other roots **CWO**
		$\Rightarrow$ $\cos\theta = 0$ only	A1	3	For correct condition obtained **AG**
					Note that M1 A0 A1 is possible
				SR	For verifying $\cos\theta = 0$ by substituting $c = 0$ into $16c^6 - 24c^4 + 10c^2 = 0$ B1
				SR	For verifying $\theta = \frac{1}{2}\pi$ and $\theta = -\frac{1}{2}\pi$ satisfy $\cos 4\theta \cos 2\theta = -1$ B1
				$\boxed{11}$	

Question			Answer	Marks	Guidance
1	(i)		$(y = xu \Rightarrow)\ \frac{\mathrm{d}y}{\mathrm{d}x} = x\frac{\mathrm{d}u}{\mathrm{d}x} + u$	B1	For a correct statement
			$x\frac{\mathrm{d}u}{\mathrm{d}x} + u = \frac{2+u^2}{u}$	M1	For using the substitution to eliminate y (If B0, then y must be eliminated from LHS, but $\frac{d(uv)}{dx}$ sufficient)
			$\Rightarrow\ x\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{u}$	A1	For correct equation **AG**
				[3]	
1	(ii)		$\int u\ \mathrm{d}u = \int \frac{2}{x}\ \mathrm{d}x$	M1	For separating variables and writing/attempting integrals
			$\Rightarrow\ \frac{1}{2}u^2 = 2\ln((k)x)\ \ OR\ \ \frac{1}{2}u^2 = 2\ln x\ (+c)$	A1	For correct integration both sides (k or c not required here)
			$\Rightarrow\ \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln(kx)\ \ OR\ \ \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln x + c$	M1	For substituting for u into integrated terms with constant (on either side)
			$\Rightarrow\ y^2 = 4x^2\ln(kx)\ \ OR\ \ y^2 = 4x^2\ln x + Cx^2$	A1	For correct solution **AEF** $y^2 = \mathrm{f}(x)$ Do not penalise "c" being used for different constants e.g. $2\ln x + c = 2\ln(cx)$
				[4]	
2	(i)		$\left(z^n - \mathrm{e}^{\mathrm{i}\theta}\right)\left(z^n - \mathrm{e}^{-\mathrm{i}\theta}\right) \equiv z^{2n} - 2z^n\left(\frac{\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}}{2}\right) + 1$ $\equiv z^{2n} - (2\cos\theta)z^n + 1$	B1	For multiplying out to **AG** with evidence of $\cos\theta = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}\right)$ (Can be implied by $2\cos\theta = \left(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}\right)$)
				[1]	

Question			Answer	Marks	Guidance
2	(ii)		METHOD 1		
			$2\cos\theta = 1 \Rightarrow \theta = \frac{1}{3}\pi$	M1	For using **(i)** to find θ
			$\Rightarrow z^4 - z^2 + 1 \equiv \left(z^2 - e^{\frac{1}{3}\pi i}\right)\left(z^2 - e^{-\frac{1}{3}\pi i}\right)$	A1	For correct quadratic factors (Or $\frac{5\pi}{3}i$ in place of $-\frac{\pi}{3}i$)
			$\equiv \left(z + e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{1}{6}\pi i}\right)\left(z + e^{-\frac{1}{6}\pi i}\right)\left(z - e^{-\frac{1}{6}\pi i}\right)$	M1	For factorising $\left(z^2 - a^2\right)$
				A1	For correct linear factors
			$\equiv \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	M1	For adjusting arguments (must attempt correct range **and** "(z – root)")
				A1	For correct factors **CAO**
					Correct answer www gets 6
				[6]	
			METHOD 2		
			$z^4 - z^2 + 1 = 0 \Rightarrow z^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{3}\,i = e^{\frac{1}{3}\pi i},\ e^{-\frac{1}{3}\pi i}$	M1 A1	For solving quadratic For correct roots in exp form
			$\Rightarrow z = \pm e^{\frac{1}{6}\pi i},\ \pm e^{-\frac{1}{6}\pi i}$	M1 A1	For attempt to find 4 roots For correct roots $\pm e^{i\alpha}$
			$= e^{\frac{1}{6}\pi i},\ e^{\frac{7}{6}\pi i},\ e^{\frac{5}{6}\pi i},\ e^{\frac{11}{6}\pi i}$	M1	For adjusting arguments
			$\Rightarrow \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	A1	For correct factors **CAO**
3	(i)		METHOD 1		
			$(yx)(yx)^{-1} = e \Rightarrow x(yx)^{-1} = y^{-1}$	M1	For starting point and appropriate multiplication
			$\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$	A1	For correct result **AG**
				[2]	
			METHOD 2		
			Compare $(yx)(yx)^{-1} = e$ with $yxx^{-1}y^{-1} = e$	M1	For appropriate comparison
			$\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$	A1	For correct result **AG** For A1, proof cannot be written in the form 'LHS = RHS → ... → e = e'

Question			Answer	Marks	Guidance
3	(ii)		$x^n y^n = (xy)^n = x(yx)^{n-1} y$ $\Rightarrow x^{-1}x^n y^n y^{-1} = x^{-1}x(yx)^{n-1} y y^{-1}$ $\Rightarrow x^{n-1}y^{n-1} = (yx)^{n-1}$	M1 M1 A1 **[3]**	For using associativity or an inverse with respect to LHS, RHS or initial equality **www beforehand** For using $(xy)^n = x(yx)^{n-1} y$ **oe** For correct result **AG** **SR** for numerical *n* used, allow M1 M1 only
3	(iii)		METHOD 1 All steps in **(ii)** are reversible ⇒ result follows METHOD 2 Show working for **(ii)** in reverse ⇒ result follows	B1*dep B1*dep **[2]** B1* B1*dep	For correct reason. Dep on correct part(ii) For correct conclusion For correct working For correct conclusion

Question			Answer	Marks	Guidance
4	(i)				Coordinates or vectors allowed throughout
			METHOD 1 (*M*, then distance)		
			$M=(1+2t,1+3t,-1+2t)$	B1	For correct parametric form soi
			$\mathbf{AM}=(\pm)[2t-6,3t-2,2t-8]$	B1 FT	For correct vector. FT from M
			$\mathbf{AM}$ perp. $l\Rightarrow\ 2(2t-6)+3(3t-2)+2(2t-8)=0$	M1 A1	For using perpendicular condition For correct equation
			$\Rightarrow\ t=2\ ,\ M=(5,7,3)$	A1	For correct coordinates
			$AM=\sqrt{2^2+4^2+4^2}=6$	M1 A1 **[7]**	For using distance formula For correct distance
			METHOD 2(a) (distance, then *M*)		
			$(C=(1,1,-1))\ \mathbf{AC}=\pm[6,2,8]$	B1	For correct vector
			$\mathbf{n}=\mathbf{AC}\times[2,3,2]=k[-20,4,14]$	M1	For finding $\mathbf{AC}\times$ direction of l
			$d=\dfrac{\lvert\mathbf{n}\rvert}{\lvert[2,3,2]\rvert}=\dfrac{\sqrt{612}}{\sqrt{17}}=6$	A1 FT A1	For correct $\lvert\mathbf{n}\rvert$. FT from $\mathbf{n}$ For correct distance
			$CM=\sqrt{\left(6^2+2^2+8^2\right)-6^2}=2\sqrt{17}$	M1	For a correct method for finding position of M
			$\lvert[2,3,2]\rvert=\sqrt{17}\ \Rightarrow\ t=2\ ,\ M=(5,7,3)$	B1 A1	For $\lvert[2,3,2]\rvert=\sqrt{17}$ soi
			METHOD 2(b)		
			$(C=(1,1,-1))\ \mathbf{AC}=\pm[6,2,8]$	B1	For correct vector
			$\cos\theta=\dfrac{AC\cdot(2,3,2)}{\lvert AC\rvert\lvert(2,3,2)\rvert}$, $\theta=36.0(39..)$ or $\sin\theta=\dfrac{153}{\sqrt{442}}$	M1,A1	
			$\lvert AM\rvert=\lvert AC\rvert\sin\theta=6$	M1,A1	
			$M=(5,7,3)$	M1,A1	As above

Question			Answer	Marks	Guidance
4	(ii)		$\mathbf{AM} = [-2, 4, -4]$ *or* $\mathbf{MA} = [2, -4, 4]$ $\Rightarrow B = (7, 3, 7) + \frac{3}{4}(-2, 4, -4) = \left(7 - \frac{3}{2}, 3 + 3, 7 - 3\right)$	M1	For using $A + k_1 \overrightarrow{AM}$ *or* $M + k_2 \overrightarrow{MA}$ *or* ratio theorem *or* equivalent
			OR		
			$B = (5, 7, 3) + \frac{1}{4}(2, -4, 4) = \left(5 + \frac{1}{2}, 7 - 1, 3 + 1\right)$	M1	For $B = (7, 3, 7) + \frac{3}{4}$ x'*their* (-2,4,-4) **oe**
			OR		(**or** M1 for quadratic in parameter for line AM, followed by M1 for attempt to use **correct** value of parameter to find B)
			$B = \frac{3}{4}(5, 7, 3) + \frac{1}{4}(7, 3, 7) = \left(\frac{15}{4} + \frac{7}{4}, \frac{21}{4} + \frac{3}{4}, \frac{9}{4} + \frac{7}{4}\right)$		
			$B = \left(\frac{11}{2}, 6, 4\right)$	A1	For correct coordinates
				[3]	
5	(i)		$\left(2m^2 + 3m - 2 = 0\right) \Rightarrow m = \frac{1}{2}, -2$	M1	For attempt to solve correct auxiliary equation
			CF = $A\mathrm{e}^{\frac{1}{2}x} + B\mathrm{e}^{-2x}$	A1	For correct CF
				[2]	
5	(ii)		$\frac{\mathrm{d}y}{\mathrm{d}x} = p\,\mathrm{e}^{-2x} - 2px\,\mathrm{e}^{-2x}$	M1	For differentiating PI twice, using product rule
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -4p\,\mathrm{e}^{-2x} + 4px\mathrm{e}^{-2x}$	A1	For correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$
			$\Rightarrow \left(-8p + 3p + 8px - 6px - 2px\right)\mathrm{e}^{-2x} = 5\mathrm{e}^{-2x}$	M1	For substituting into DE
			$\Rightarrow p = -1$	A1	For correct p
				[4]	

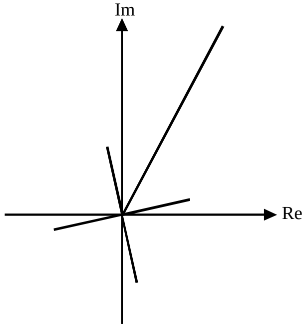
Question			Answer	Marks	Guidance
5	(iii)		GS $(y=)\,A\mathrm{e}^{\frac{1}{2}x}+B\mathrm{e}^{-2x}-x\mathrm{e}^{-2x}$	B1 FT	For GS soi. FT from CF (2 constants) and p
			$(0,0)\Rightarrow A+B=0$	B1 FT	For correct equation. FT from GS of form $A\mathrm{e}^{\alpha x}+B\mathrm{e}^{\beta x}-Cx\mathrm{e}^{-2x}$
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}A\mathrm{e}^{\frac{1}{2}x}-2B\mathrm{e}^{-2x}-\mathrm{e}^{-2x}+2x\mathrm{e}^{-2x}$		
			$\left(0,\frac{\mathrm{d}y}{\mathrm{d}x}=4\right)\Rightarrow\frac{1}{2}A-2B=5$	M1	For differentiating GS and substituting values, using GS of form $A\mathrm{e}^{\alpha x}+B\mathrm{e}^{\beta x}-Cx\mathrm{e}^{-2x}$
			$\Rightarrow A=2,\ B=-2$	M1	For solving for A and B(can be gained from incorrect GS)
			$\Rightarrow\ \ y=2\mathrm{e}^{\frac{1}{2}x}-2\mathrm{e}^{-2x}-x\mathrm{e}^{-2x}$	A1	For correct solution, including $\boxed{y=}$
				[5]	
6	(i)		METHOD 1		
			$\mathbf{n}=[2,-1,-1]\times[2,-3,-5]=[2,8,-4]$	M1	For finding vector product of 2 vectors in Π(or 2 scalar products = 0, with attempt to solve)
			$\mathbf{n}=k[1,4,-2]$	A1	For correct n
			Π is $\mathbf{r}.\mathbf{n}=[1,6,7].\mathbf{n}$	M1	For attempt to find equation of Π, including cartesian equation
			$\Rightarrow\ \mathbf{r}.[1,4,-2]=11$	A1	For correct equation (allow multiples)
			METHOD 2		
			$y-z=-1+2\mu$	M1	for finding λ or μ in terms of two from x,y,z.
			$\mu=\frac{y-z+1}{2}$		
			$\lambda=7-z-5\frac{y-z+1}{2}$	M1	For both λ & μ
			$x=11+2z-4y$	A1	AEF
			$r.(1,4,-2)=11$	A1 **[4]**	

Question			Answer	Marks	Guidance
6	(ii)		$[7+3t, 4, 1-t]\,.\,\mathbf{n}=11 \Rightarrow t=-2$	M1	For attempt to find *t*, (or to find λ and μ by equating original equations)
			$\Rightarrow [1, 4, 3]$	A1 **[2]**	For correct position vector *OR* point
6	(iii)		METHOD 1 $\mathbf{c}=[1, 4, -2]\times[2, -1, -1]$	M1	For using given vector product (or 2 correct 'scalar products = 0')
				M1	For calculating given vector product (or 2 correct scalar products = 0,with attempt to solve) (or M1 for using vector product of c with n or (2,-1,-1) in an equation, followed by M1 for calculating vector product and attempting to solve)
			$\mathbf{c}=k[2, 1, 3]$	A1 **[3]**	For correct c
			METHOD 2 $\mathbf{c}=[2, -3, -5]+s[2, -1, -1]$	M1	For c = linear combination of $[2, -3, -5]$ and $[2, -1, -1]$
			$\mathbf{c}\,.\,[2, -1, -1]=0 \Rightarrow$ $2(2+2s)-1(-3-s)-1(-5-s)=0$	M1	For an equation in s from $\mathbf{c}\,.\,[2, -1, -1]=0$
			$\Rightarrow s=-2 \Rightarrow \mathbf{c}=k[2, 1, 3]$	A1	For correct c

Question			Answer	Marks	Guidance
7	(i)		$\begin{pmatrix}1&0\\n&1\end{pmatrix}\begin{pmatrix}1&0\\m&1\end{pmatrix}=\begin{pmatrix}1&0\\n+m&1\end{pmatrix}=\begin{pmatrix}1&0\\m+n&1\end{pmatrix}$ $=\begin{pmatrix}1&0\\m&1\end{pmatrix}\begin{pmatrix}1&0\\n&1\end{pmatrix}\Rightarrow$ commutative	M1 A1 **[2]**	For multiplying 2 distinct matrices of the correct form both ways, or generalised form at least one way, For stating or implying that addition is commutative **and** correct conclusion **SR** Use of numerical matrices must be generalised for any credit
7	(ii)		$(I=)\begin{pmatrix}1&0\\0&1\end{pmatrix}$ EITHER $\begin{pmatrix}1&0\\2&1\end{pmatrix}^{-1}=\begin{pmatrix}1&0\\-2&1\end{pmatrix}=\begin{pmatrix}1&0\\4&1\end{pmatrix}$ OR $\begin{pmatrix}1&0\\2&1\end{pmatrix}\begin{pmatrix}1&0\\n&1\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\Rightarrow 2+n=0\Rightarrow\begin{pmatrix}1&0\\4&1\end{pmatrix}$	B1 M1 A1 **[3]**	For correct identity For using inverse property For correct inverse
7	(iii)		$\begin{pmatrix}1&0\\3&1\end{pmatrix}$ has order 2 4 is not a factor of 6	B1 B1 **[2]**	For correct order For correct reason (Award B0 for “Lagrange” only). Must be explicit about the ‘6’
7	(iv)		$\begin{pmatrix}1&0\\1&1\end{pmatrix}$ *OR* $\begin{pmatrix}1&0\\5&1\end{pmatrix}$ has order 6, (or > 3) OR $(M,\times)$ is cyclic, *G* is non-cyclic (having no element of order 6) OR $(M,\times)$ is commutative *G* is not commutative (being the non-cyclic group) ⇒ groups are not isomorphic	B1* B1*dep **[2]**	For stating (that there is) an element of *M* with order 6 Award B1* for a relevant statement about *M* and *G* For correct conclusion and no false statements attached to conclusion

Question			Answer	Marks	Guidance
8	(i)		$\cos 5\theta + \mathrm{i}\sin 5\theta =$ $c^5 + 5\mathrm{i}c^4 s - 10c^3 s^2 - 10\mathrm{i}c^2 s^3 + 5cs^4 + \mathrm{i}s^5$ $\Rightarrow \tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta} = \dfrac{5c^4 s - 10c^2 s^3 + s^5}{c^5 - 10c^3 s^2 + 5cs^4}$ Division of numerator & denominator by c^5. $\Rightarrow \tan 5\theta = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$	B1 M1 M1 A1 **[4]**	For explicit use of de Moivre with $n = 5$ For correct expressions for $\sin 5\theta$ and $\cos 5\theta$ For $\dfrac{\sin 5\theta}{\cos 5\theta}$ in terms of c and s For simplifying to **AG, www** with explicit mention of division by c^5
8	(ii)		$5\theta = \{1, 5, 9, 13, 17\}\frac{1}{4}\pi$ $\theta = \{1, 5, 9, 13, 17\}\frac{1}{20}\pi$	M1 A1 A1 **[3]**	For at least 2 of given values and no extras. For at least 3 values of θ and no extras in range For all 5 values and no extras outside range
8	(iii)		$\tan 5\theta = 1 \Rightarrow t^5 - 5t^4 - 10t^3 + 10t^2 + 5t - 1 = 0$ $\Rightarrow (t-1)\left(t^4 - 4t^3 - 14t^2 - 4t + 1\right) = 0$ $\tan\alpha = 1$ *OR* $\alpha = \frac{1}{4}\pi$ is not included in roots of the quartic $\Rightarrow t = \tan\alpha$ for $\alpha = \{1, 9, 13, 17\}\frac{1}{20}\pi$	M1* A1 B1 M1*dep A1 **[5]**	For $\tan 5\theta = 1$ and equation in t For correct factors For solution rejected (may be implied by $\frac{5}{20}\pi$ not appearing in set of solutions) For 2 correct values of t For all 4 values and no more in range

Question			**Answer**	**Marks**	**Guidance**	
1			METHOD 1			
			$\mathbf{b}=[1,-3,4]\times[3,1,2]=[-10,10,10]$	M1	For attempt to find vector product of directions	
			$=k[-1,1,1]$	M1	Correct calculation of vector product	Allow 1 error
				A1	For correct **b** .	
			$\Rightarrow\ \mathbf{r}=[1,4,2]+t[-1,1,1]$	B1 FT	For correct equation. FT from **b**	
				[4]		
			METHOD 2			
			$[x,y,z]\,.\,[1,-3,4]=0\ \Rightarrow\ x-3y+4z=0$		For an equation from l_2 perpendicular to normal of plane	
			$[x,y,z]\,.\,[3,1,2]=0\ \Rightarrow\ 3x+y+2z=0$	M1	and an equation from l_2 perpendicular to l_1	
			Solving $\Rightarrow [x,y,z]=\mathbf{b}=k[-1,1,1]$	M1		
				A1		
			$\Rightarrow\ \mathbf{r}=[1,4,2]+t[-1,1,1]$	B1FT	For correct equation. FT. from **b**	Must show "**r** ="
2	(i)		$z^4=4\left(\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}\right)=4\,\mathrm{cis}\,\frac{1}{3}\pi$	B1	For $\arg(z^4)=\frac{1}{3}\pi$ soi	
				M1	For dividing $\arg(z^4)$ by 4	
			$z=\sqrt{2}\,\mathrm{cis}\left(k\frac{\pi}{12}\right),\quad k=1,7,13,19$	A1	For any 2 correct values of k	
				A1	For all 4 values of k *and no extras.* Ignore values outside range	For second A1, must be in correct form.
				B1	For modulus of all stated roots = $\sqrt{2}$	Don't accept 1.41.. or $\sqrt[4]{4}$
					SR For $\arg(z^4)=\frac{1}{6}\pi$ award B0 M1 A1 FT for all $\mathrm{cis}\left(k\frac{\pi}{24}\right)$, $k=1,13,25,37$, A0 B0/B1	
				[5]		

Question			Answer	Marks	Guidance	
2	(ii)			B1	For roots forming a square, centre O, on equal-scale axes.	Must be roots distinct from z^4 Penalise once use of points not lines
				B1	For z^4 and only one root in first quadrant with arguments in ratio approximately 3:1	
				B1	For $\lvert z^4\rvert : \lvert z\rvert \approx 4 : \sqrt{2}$ (allow (2,4):1)	For all four roots
				[3]		
3			Integrating factor = $e^{\int \cot x\,dx} = e^{\ln\sin x} = \sin x$	M1	For IF = $e^{\pm\ln\sin x}$ *OR* $e^{\pm\ln\cos x}$	
				A1	For simplified IF	
			$\Rightarrow \frac{d}{dx}(y\sin x) = 2x\sin x$	M1	For $\frac{d}{dx}(y.\text{their IF}) = 2x.\text{their IF}$	
			$\Rightarrow y\sin x = -2x\cos x + \int 2\cos x\,dx$	M1*	For attempt to integrate RHS using parts for $\int x\left\{\begin{matrix}\sin x\\ \cos x\end{matrix}\right. dx$	(Must use u = (2)x)
				A1	For correct RHS 1st stage	
			$\Rightarrow y\sin x = -2x\cos x + 2\sin x\ (+c)$	A1	oe	
			$\left(\frac{1}{6}\pi, 2\right) \Rightarrow c = \frac{1}{6}\pi\sqrt{3}$	M1dep *	For substituting $\left(\frac{1}{6}\pi, 2\right)$ into their GS (with c)	c = 0.907
				A1 FT	For correctly finding c (FT from GS)	
			$\Rightarrow y = -2x\cot x + 2 + \frac{1}{6}\pi\sqrt{3}\,\mathrm{cosec}\,x$	A1	For correct solution **AEF** of standard notation $y = \mathrm{f}(x)$	
				[9]		

Question			Answer	Marks	Guidance	
4	(i)		$\begin{array}{c\|cccc} H & e & r & r^2 & r^3 \\ \hline e & e & r & r^2 & r^3 \\ r & r & r^2 & r^3 & e \\ r^2 & r^2 & r^3 & e & r \\ r^3 & r^3 & e & r & r^2 \end{array}$ $\begin{array}{c\|cccc} K & e & p & q & pq \\ \hline e & e & p & q & pq \\ p & p & e & pq & q \\ q & q & pq & e & p \\ pq & pq & q & p & e \end{array}$	B2 B2 **[4]**	For correct table for H For correct table for K **SR** In both tables allow B1 for 1 or 2 errors	
4	(ii)		Identity = b	B1 **[1]**	For correct identity	
4	(iii)		G is isomorphic to H	B1	For H identified as isomorphic to G (may be implied by table)	
			$\begin{array}{c\|c\|c} G & H & H \\ \hline a & r^2 & r^2 \\ b & e & e \\ c & r & r^3 \\ d & r^3 & r \end{array}$	B1 B1 B1 **[4]**	For $a \leftrightarrow r^2$ at least once For $c, d \leftrightarrow r, r^3$ either way For $c, d \leftrightarrow r, r^3$ both ways **and** b corresponds to e explicit. Award fourth **B1** only for completely correct answer. If none of last 3 marks gained, then **SC1** for order of all elements of G and H	
5	(i)		METHOD 1		z may be used for $\mathrm{e}^{\mathrm{i}\theta}$ throughout	
			$\sin^3\theta\cos^2\theta = \left(\frac{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}}\right)^3\left(\frac{\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}}{2}\right)^2$	B1	For $\left(\frac{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}}\right)$ *OR* $\left(\frac{\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}}{2}\right)$ soi	
			$= -\frac{1}{32\mathrm{i}}\left(z^3-3z+3z^{-1}-z^{-3}\right)\left(z^2+2+z^{-2}\right)$	M1	For expanding brackets (binomial theorem or otherwise)	
				M1 B1	For full expansion with 12 terms. For $-\frac{1}{32\mathrm{i}}$	two brackets expanded soi by alternate method
			$= -\frac{1}{32\mathrm{i}}\left(\left(z^5-z^{-5}\right)-\left(z^3-z^{-3}\right)-2\left(z-z^{-1}\right)\right)$	M1	For grouping terms	Can be seen at any stage
			$= -\frac{1}{16}\left(\frac{z^5-z^{-5}}{2\mathrm{i}}-\frac{z^3-z^{-3}}{2\mathrm{i}}-2\frac{z-z^{-1}}{2\mathrm{i}}\right)$		This step, oe, is needed for the final mark	oe includes replacing z^5-z^{-5} with 2isin5θ etc
			$= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	A1 **[6]**	For simplification to **AG** WWW	

Question			Answer	Marks	Guidance	
			METHOD 2 $\sin^3\theta\cos^2\theta = \sin^3\theta - \sin^5\theta$ $2i\sin\theta = z - \frac{1}{z}$	B1		
			$-8i\sin^3\theta = z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}$	M1	For RHS	
			$= (z^3 - \frac{1}{z^3}) - (3z - \frac{3}{z})$ $= 2i\sin 3\theta - 6i\sin\theta$		*	
			$32i\sin^5\theta = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= (z^5 - \frac{1}{z^5}) - (5z^3 - \frac{5}{z^3}) + (10z - \frac{10}{z})$	M1	For grouping terms	
			$= 2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta$	B1	For RHS of this line and line * above	
			$\sin^3\theta\cos^2\theta$ $= -\frac{1}{32i}(4(2is3\theta - 6is\theta) + (2is5\theta - 10is3\theta + 20is\theta))$	B1	For $-\frac{1}{32i}$	
			$= -\frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 4\sin 3\theta + 10\sin\theta - 12\sin\theta)$ $= -\frac{1}{16}(\sin 5\theta - \sin 3\theta - 2\sin\theta)$	A1	For **ag** www	
5	(ii)		$\sin^3\theta\cos^2\theta = 0 \Rightarrow \sin\theta = 0 \;\; OR \;\; \cos\theta = 0$	M1	For either equation Accept also Sinθ = +/-1	Can be implied by the A mark plus at least $\sin^3\theta = 0$ or similar.
			$\Rightarrow \;\; \theta = r\pi \;\; OR \;\; \theta = (2r+1)\frac{1}{2}\pi$	A1	For either solution, **AEF** including a list of the first few	At least 2 in list (and no wrong solution)
			$\Rightarrow \theta = \frac{n\pi}{2}$	A1 **[3]**	For both of above solutions leading to general solution in form of **AG** where $k = 2$	

Question			Answer	Marks	Guidance	
6	(i)		METHOD 1			
			$m^2+4m=0 \Rightarrow m=0,-4$	M1	For attempt to solve correct auxiliary equation	
			CF = $A+Be^{-4x}$	A1	For correct CF	
			PI $y=pe^{2x} \Rightarrow 4p+8p=12$	B1	For PI of correct form seen	Beware poor use of pxe^{2x}
				M1	For differentiating PI and substituting	Scores maximum
			$\Rightarrow p=1$	A1	For correct *p*	of **M1 A1 B0 M1**
			GS $y=A+Be^{-4x}+e^{2x}$	B1 FT	For using GS = CF + PI with 2 arbitrary constants in GS and none in PI	**A0 B0**
				[6]		
			METHOD 2			
			Integrating $\Rightarrow \frac{dy}{dx}+4y=6e^{2x}+c$	M1	For attempt to integrate equation	
				B1	For $+c$ included	
			IF $e^{4x} \Rightarrow \frac{d}{dx}\left(ye^{4x}\right)=6e^{6x}+ce^{4x}$	B1√	For correct IF. f.t. from their DE	
				M1	For multiplying through by their IF and attempting to integrate	
			$\Rightarrow ye^{4x}=e^{6x}+\frac{1}{4}ce^{4x}+B$	A1	For correct integration both sides, including $+B$	
			$\Rightarrow y=e^{2x}+A+Be^{-4x}$	A1	For correct solution	Must include "y ="
6	(ii)		$\frac{dy}{dx}=-4Be^{-4x}+2e^{2x}$	M1	For differentiating "their GS" with 2 arbitrary constants and substituting values to obtain an equation	If "their CF" is $(A+Bx)e^{-4x}$
			$\left(0, \frac{dy}{dx}=6\right) \Rightarrow -4B+2=6 \Rightarrow B=-1$	A1	For correct *B*	can score max of **M1 A0 B1 A0**
			$(y\approx e^{2x} \Rightarrow) A=0$	B1	For correct *A* and consistent with" their GS"	
			$\Rightarrow y=-e^{-4x}+e^{2x}$	A1	For correct equation **www**	
				[4]		
7	(i)		$\mathbf{m}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v}) \Rightarrow$	M1	For using vector triangle, or equivalent, for *M*	$\overrightarrow{UM}=\overrightarrow{UV}+\overrightarrow{VM}$ $=(\mathbf{v}-\mathbf{u})+\frac{1}{2}(\mathbf{w}-\mathbf{v})$
			$\overrightarrow{UM}=\mathbf{v}+\frac{1}{2}(\mathbf{w}-\mathbf{v})-\mathbf{u}=\frac{1}{2}(\mathbf{v}+\mathbf{w}-2\mathbf{u})$	A1	For correct expression **AG**	
					SR Allow use of ratio theorem	Minimum $-\mathbf{u}+\frac{1}{2}(\mathbf{v}+\mathbf{w})$
				[2]		

Question			Answer	Marks	Guidance	
7	(ii)		METHOD 1 (first 3 marks)			
			$\overrightarrow{UM}$ is $\mathbf{r}=\mathbf{u}+\frac{1}{2}t\left(\mathbf{v}+\mathbf{w}-2\mathbf{u}\right)$	M1*	For equation of *UM*	
				M1*	For attempt to find a suitable value of *t*	
			$t=\frac{2}{3} \Rightarrow \mathbf{u}+\frac{1}{3}\left(\mathbf{v}+\mathbf{w}-\mathbf{2u}\right)=\frac{1}{3}\left(\mathbf{u}+\mathbf{v}+\mathbf{w}\right)$	A1	For $t=\frac{2}{3}$ and *G* obtained **AG**	
			METHOD 2 (first 3 marks)			
			$\overrightarrow{UG}=\frac{1}{3}\left(\mathbf{u}+\mathbf{v}+\mathbf{w}\right)-\mathbf{u}=\frac{1}{3}\left(\mathbf{v}+\mathbf{w}-\mathbf{2u}\right)$	M1* M1*	For finding directions of *UG* or *MG* For comparing with *UM*	
			OR			
			$\overrightarrow{MG}=\frac{1}{3}\left(\mathbf{u}+\mathbf{v}+\mathbf{w}\right)-\frac{1}{2}\left(\mathbf{v}+\mathbf{w}\right)=-\frac{1}{6}\left(\mathbf{v}+\mathbf{w}-2\mathbf{u}\right)$			
			⇒ *U*, *G*, *M* collinear	A1	For showing *G* lies on *UM* **AG**	
			By symmetry of $\overrightarrow{OG}$ in **u**, **v**, **w**	B1	For use of symmetry, or by repeating method for UM twice more.	
			G also lies on *VN*, *WP* ⇒ *UM*, *VN*, *WP* intersect at *G*	B1dep *	For complete reasoning to **AG**	
				[5]		
7	(iii)		Line is $\mathbf{r}=\frac{1}{3}\left(\mathbf{u}+\mathbf{v}+\mathbf{w}\right)+t\left(\mathbf{u}-\mathbf{v}\right)\times\left(\mathbf{u}-\mathbf{w}\right)$ (etc)	B1	For *r* = $\frac{1}{3}\left(\mathbf{u}+\mathbf{v}+\mathbf{w}\right)+t\times$"any vector"	
				B1	For a correct **n**, using any 2 of $\pm\left(\mathbf{u}-\mathbf{v}\right)$, $\pm\left(\mathbf{v}-\mathbf{w}\right)$, $\pm\left(\mathbf{w}-\mathbf{u}\right)$	Allow $\overrightarrow{UV}\times\overrightarrow{VW}$ or similar
				[2]		

Question			Answer	Marks	Guidance	
7	(iv)		METHOD 1 $\mathbf{n} = [1, 0, -1] \times [0, 1, -1]$ (etc) $= k[1, 1, 1]$	M1*	For attempt to find **n**	May see use of $\dfrac{\lvert p.n - d\rvert}{\lvert n\rvert}$
			UVW is $\mathbf{r}\,.\,\mathbf{n} = [1, 0, 0]\,.\,[1, 1, 1] = 1$	M1dep*	For substituting a point	
			$\Rightarrow\ d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	
				[3]		
			METHOD 2 UVW is $x + y + z = 1$ (from given **u**, **v**, **w**)	M2	For attempt to find cartesian equation	
			$\Rightarrow\ d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	
			METHOD 3 $\overrightarrow{OG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$	M1*	For stating or implying $\lvert\overrightarrow{OG}\rvert$ is d	
			$\Rightarrow\ OG = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}}$	M1dep*	For finding magnitude	
			$\Rightarrow\ d = \dfrac{1}{\sqrt{3}}$	A1	For correct d	

Question			Answer	Marks	Guidance	
8	(i)		For R, $\cos^2\theta+\sin^2\theta=1$ ⇒ad-bc =1 (⇒ $R\subset M$)	B1	For showing $R\subset M$	
			$R(\theta)R(\phi)=R(\theta+\phi)$ and hence closed, since	M1	For multiplying 2 distinct elements	
			$\cos\theta\cos\phi-\sin\theta\sin\phi=\cos(\theta+\phi)$ and $\pm(\cos\theta\sin\phi+\sin\theta\cos\phi)=\pm\sin(\theta+\phi)$	A1	For obtaining $R(\theta)R(\phi)\in R$	Must demonstrate use of compound angles or explain rotations.
			Identity $\theta=0\Rightarrow\begin{pmatrix}1&0\\0&1\end{pmatrix}\in R$	B1	For identity element related to θ = 0	
			Inverse $R(-\theta)=\begin{pmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{pmatrix}$	B1	For inverse element …	
			$=\begin{pmatrix}\cos(-\theta)&-\sin(-\theta)\\\sin(-\theta)&\cos(-\theta)\end{pmatrix}$	B1	...converted to form of elements of R	
				[6]		
			SR For use of $\left(a,b\in R\Rightarrow ab^{-1}\in R\right)\Leftrightarrow R$ is a subgroup of M			
			For R, $\cos^2\theta+\sin^2\theta=1$ ⇒ $R\subset M$	B1	For showing $R\subset M$	
			$R(\theta)R(\phi)^{-1}=\begin{pmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{pmatrix}\begin{pmatrix}\cos(-\phi)&-\sin(-\phi)\\\sin(-\phi)&\cos(-\phi)\end{pmatrix}$	B1	For considering $R(\theta)R(\phi)^{-1}$	
				B1	For correct inverse	
				M1	For multiplying elements	
			$=\begin{pmatrix}\cos(\theta-\phi)&-\sin(\theta-\phi)\\\sin(\theta-\phi)&\cos(\theta-\phi)\end{pmatrix}\in R$	A1	For correct product	
			Set is non-empty	B1	Can be implied by identity element related to θ = 0	

Question			Answer	Marks	Guidance	
8	(ii)		For $\theta = \frac{1}{3}k\pi$ elements are	B1	For $\theta = \frac{1}{3}\pi$ soi	Allow degrees instead of radians.
			$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix},$	M1	For using "their θ" in $\begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix}$ for at least 2 values of *k*, or lists all 6 values of θ	
				A1	For identity and one other element other than (-I)	
			$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$	A1	For 2 more elements	
				A1	For all 6 elements correct	
				[5]		

Question			Answer	Marks	Guidance	
1	(i)		$\cos\theta = \dfrac{\left\|\begin{pmatrix}1\\2\\5\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\3\end{pmatrix}\right\|}{\sqrt{1^2+2^2+5^2}\sqrt{2^2+(-1)^2+3^2}} = \dfrac{15}{\sqrt{30}\sqrt{14}}$	M1 A1	Accept unsimplified	
			$\theta = 0.750$ or $43.0°$	A1 **[3]**	If zero, then **sc1** for $n_1.n_2 = 15$ seen	
1	(ii)		$\begin{pmatrix}1\\2\\5\end{pmatrix}\times\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}11\\7\\-5\end{pmatrix}$	M1 A1		M1 requires evidence of method for cross product or at least 2 correct values calculated
			(eg) $x=0 \Rightarrow 2y+5z=12, -y+3z=5 \Rightarrow y=1, z=2$	M1		or any valid point e.g.(-11/7, 0, 19/7) (22/5, 19/5, 0)
			$\mathbf{r} = \begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	A1 **[4]**	oe vector form	Must have full equation including '**r** ='
			Alternative: Find one point	M1		
			Find a second point and vector between points	M1		
			multiple of $\begin{pmatrix}11\\7\\-5\end{pmatrix}$	A1		
			$\mathbf{r} = \begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	A1		

Question			Answer	Marks	Guidance	
			Alternative: Solve simultaneously	M1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	
			Point found	A1		
			Direction found	A1		
			$\mathbf{r}=\begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	A1		
2	**(i)**		identity 0 + 0i order 25	B1 B1 **[2]**	Or '0'	
2	**(ii)**		$3+\mathrm{i}$	B1 **[1]**		
2	**(iii)**		$5(a+b\mathrm{i})=5a+5b\mathrm{i}=0+0\mathrm{i}$	M1	Shows 5 times any element equals e	
			every non-zero element has order 5 or 25	M1	Attempt to show that order ≠ 2,3,4	Must consider all(non-zero) elements
			So order is 5	A1	Argument is convincing, exhaustive and conclusive.	
				[3]		
3			$\frac{\mathrm{d}y}{\mathrm{d}x}-3\frac{y}{x}=x^3\mathrm{e}^{2x}$	M1	Divide by *x*	
			$I=\exp\left(\int-\frac{3}{x}\mathrm{d}x\right)=\mathrm{e}^{-3\ln x}$	M1		
			$=x^{-3}$	A1		
			$x^{-3}\frac{\mathrm{d}y}{\mathrm{d}x}-3x^{-4}y=\mathrm{e}^{2x}$	M1	Multiply and recognise derivative	
			$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-3}y\right)=\mathrm{e}^{2x}$	M1	Integrate	
			$x^{-3}y=\frac{1}{2}\mathrm{e}^{2x}+A$	A1		
			$x=1,y=0\Rightarrow A=-\frac{1}{2}\mathrm{e}^2$	M1	Use condition	
			$y=\frac{1}{2}x^3\left(\mathrm{e}^{2x}-\mathrm{e}^2\right)$	A1		
				[8]		

Question			Answer	Marks	Guidance	
4	(i)		$\begin{pmatrix}2\\3\\-1\end{pmatrix}\times\begin{pmatrix}4\\-1\\-1\end{pmatrix}=\begin{pmatrix}-4\\-2\\-14\end{pmatrix}=-2\begin{pmatrix}2\\1\\7\end{pmatrix}$	M1 A1	Or any multiple	
			$\begin{pmatrix}3\\0\\1\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix}$	B1	Or negative	Or use of n.(a$_1$ + pb$_1$ +kn) = n.(a$_2$ + qb$_2$) **B1** followed by attempt to calculate magnitude of kn **M1**
			$\text{shortest distance}=\dfrac{\left\lvert\begin{pmatrix}2\\-2\\0\end{pmatrix}\bullet\begin{pmatrix}2\\1\\7\end{pmatrix}\right\rvert}{\sqrt{2^2+1^2+7^2}}=\dfrac{2}{\sqrt{54}}$ oe	M1 A1 **[5]**	Component of their vector in their direction	
4	(ii)		$2x+y+7z=\ldots$ $\ldots 11$	B1ft B1 dep **[2]**	ft from 4(i) only if 1st M1 mark gained If zero, then **sc 1** for any correct **vector** equation.	
5	(i)		$1, e^{\frac{2}{5}\pi i}, e^{\frac{4}{5}\pi i}, e^{\frac{6}{5}\pi i}, e^{\frac{8}{5}\pi i}$ oe polar form	M1 A1 **[2]**	Attempt roots	e.g. gives roots in an incorrect form.

Question			Answer	Marks	Guidance	
5	(ii)		$z^5=(z+1)^5=z^5+5z^4+10z^3+10z^2+5z+1$	M1		
			$\Leftrightarrow 5z^4+10z^3+10z^2+5z+1=0$	A1		
			so $z+1=z\mathrm{e}^{\frac{2k}{5}\pi\mathrm{i}}$, $k=0,1,2,3,4$	M1		
			$k=0$ no solution	B1	soi	
			$1=z\left(\mathrm{e}^{\frac{2k}{5}\pi\mathrm{i}}-1\right)$			
			$z=\dfrac{1}{\mathrm{e}^{\frac{2k}{5}\pi\mathrm{i}}-1}$, $k=1,2,3,4$	A1 **[5]**	If B0, then give A1 ft for correct solution plus $k=0$	
6	(i)		PI: $y=ax\cos 2x+bx\sin 2x$			
			$\dfrac{dy}{dx}=a\cos 2x-2ax\sin 2x+b\sin 2x+2bx\cos 2x$	B1	For correct $\dfrac{dy}{dx}$ or better	
			$\dfrac{d^2y}{dx^2}=-4a\sin 2x-4ax\cos 2x+4b\cos 2x-4bx\sin 2x$			
			in DE: $-4a\sin 2x-4ax\cos 2x+4b\cos 2x-4bx\sin 2x$ $+4(ax\cos 2x+bx\sin 2x)$	M1	Differentiate twice and substitute	
			compare coefficients: $-4a=1,\ 4b=0$	M1		
			$\Rightarrow a=-\frac{1}{4},b=0$	A1		
			AE: $\lambda^2+4=0$	M1	For correct auxiliary equation and attempt to solve	
			$\lambda=\pm 2\mathrm{i}$ CF: $A\cos 2x+B\sin 2x$	A1	oe form	
			GS: $y=\left(A-\frac{1}{4}x\right)\cos 2x+B\sin 2x$	A1ft **[7]**	Must be real and contain 2 unknowns	

Question			Answer	Marks	Guidance	
6	**(ii)**		oscillations unbounded	B1 B1 **[2]**	oe (accept sketch) dep consistent with 6(i) oe (accept sketch) dep consistent with 6(i) If zero, then **sc1** for recognition that xcos2x term becomes dominant	
6	**(iii)**		If $k \neq 2$ then PI $y = \alpha\cos kx + \beta\sin kx$ So bounded oscillations	B1 B1 **[2]**	oe (accept sketch)	
7	**(i)**	**(a)**	$\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{2\mathrm{i}\theta} + \ldots + \mathrm{e}^{10\mathrm{i}\theta} = \dfrac{\mathrm{e}^{\mathrm{i}\theta}\left(\left(\mathrm{e}^{\mathrm{i}\theta}\right)^{10} - 1\right)}{\mathrm{e}^{\mathrm{i}\theta} - 1}$ $= \dfrac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta} - 1\right)}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta} - \mathrm{e}^{-\frac{1}{2}\mathrm{i}\theta}}$ $= \dfrac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta} - 1\right)}{2\mathrm{i}\sin\left(\frac{1}{2}\theta\right)}$	M1 A1 M1 A1 **[4]**	Sum of a GP AG	
7	**(i)**	**(b)**	$\theta = 2n\pi \Rightarrow \text{sum} = 10$	B1 **[1]**		

Question			Answer	Marks	Guidance	
7	(ii)		$\cos\theta+\cos 2\theta+\ldots+\cos 10\theta=\mathrm{Re}\left(\frac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta}-1\right)}{2\mathrm{i}\sin\left(\frac{1}{2}\theta\right)}\right)$	M1	Take real parts	
			$=\frac{\mathrm{Re}\left(-\mathrm{i}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta}-1\right)\right)}{2\sin\left(\frac{1}{2}\theta\right)}=\frac{\mathrm{Re}\left(-\mathrm{i}\,\mathrm{e}^{\frac{21}{2}\mathrm{i}\theta}+\mathrm{i}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\right)}{2\sin\left(\frac{1}{2}\theta\right)}$	M1	Manipulate expression	Must at least make genuine progress in sorting real part of numerator, or in converting numerator to trig terms.
			$=\frac{\sin\left(\frac{21}{2}\theta\right)-\sin\left(\frac{1}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)}$			
			$=\frac{\sin\left(\frac{21}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)}-\frac{1}{2}$	A1	AG	
				[3]		
7	(iii)		$\cos\frac{1}{11}\pi+\cos\frac{2}{11}\pi+\ldots+\cos\frac{10}{11}\pi=\frac{\sin\left(\frac{21}{22}\pi\right)}{2\sin\left(\frac{1}{22}\pi\right)}-\frac{1}{2}$	M1		For second M1, must convince that solution is exact and not simply from calculator.
			But $\sin\frac{21}{22}\pi=\sin\left(\pi-\frac{21}{22}\pi\right)=\sin\frac{1}{22}\pi$	M1		
			So $\text{RHS}=\frac{1}{2}-\frac{1}{2}=0$, so $\frac{1}{11}\pi$ is a root	A1	AG	
			Using $\sin\left(2\pi+x\right)=\sin x$ gives $2\pi+\frac{1}{2}\theta=\frac{21}{2}\theta\Rightarrow\theta=\frac{1}{5}\pi$	A1		
				[4]		

8	**(i)**		$wa^2 = waa = a^3wa = a^3a^3w$ $= a^4a^2w = ea^2w$ $= a^2w$ Either result $\Rightarrow wa^3 = a^3wa^2$ $= a^3a^2w$ $= eaw = aw$	M1 B1 A1 M1 M1 A1 **[6]**	Use $wa = a^3w$ to simplify Use $a^4 = e$ (oe) in either proof Complete argument AG AG	
8	**(ii)**		$(aw)^2 = (aw)(aw)$ $= awwa^3 = aea^3 = a^4 = e$ so order 2 $(a^2w)(a^2w) = a^2wwa^2 = a^2ea^2 = a^4 = e$ so order 2 $(a^3w)(a^3w) = a^3wwa = a^3ea = a^4 = e$ so order 2	M1 M1 A1 A1 **[4]**	for squaring any of elements for attempt to simplify to e for at least two squared elements shown equal to e for complete argument	
8	**(iii)**		$\{e, a^2, w, a^2w\}$ $\{e, a^2, aw, a^3w\}$ a^2, w, aw, a^2w, a^3w all of order 2 so not cyclic as no element of order 4 in either	B1 B1 M1 A1 **[4]**	 Consider orders Or considers form {e, x, y, xy} where x, y order 2 Dep on both groups correct	Condone equivalents Condone 'no generator' or 'Klein (V) group' in place of 'no element of order 4'

Question			Answer	Marks	Guidance	
1	(i)		vectors in plane: two of $\begin{pmatrix}-4\\4\\1\end{pmatrix}, \begin{pmatrix}0\\6\\4\end{pmatrix}=2\begin{pmatrix}0\\3\\2\end{pmatrix}, \begin{pmatrix}4\\2\\3\end{pmatrix}$	M1	Differences between two pairs	Any multiple
			$\mathbf{r}=\begin{pmatrix}1\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\3\\2\end{pmatrix}+\mu\begin{pmatrix}4\\2\\3\end{pmatrix}$	A1	Aef of parametric equation	Must have “**r = ...**”
				[2]		
1	(ii)		$\begin{pmatrix}0\\3\\2\end{pmatrix}\times\begin{pmatrix}4\\2\\3\end{pmatrix}=\begin{pmatrix}5\\8\\-12\end{pmatrix}$	M1 A1	Calculate vector product or multiple	M1 can be awarded where vector product has method shown or only one term wrong
			$\left(\mathbf{r}-\begin{pmatrix}1\\6\\2\end{pmatrix}\right).\begin{pmatrix}5\\8\\-12\end{pmatrix}=0$	M1		Or Cartesian form = *d* with attempt to compute *d*
			$5x+8y-12z=29$	A1	Aef of cartesian equation, isw.	
				[4]		
			Alternate method			
					EITHER	
				M1	*x*, *y*, *z* in parametric form	
				A1	both parameters in terms of e.g. *x*, *y*	
				M1A1	substitute into parametric form of *z*	
					OR	
				M1	*x*, *y*, *z* in parametric form	
				A1	2 equations in *x*, *y*, *z* and one parameter	
				M1 A1	eliminate parameter	

Question			Answer	Marks	Guidance	
2	(i)		$\begin{array}{c\|cccc} & 1 & 3 & 5 & 7 \\ \hline 1 & 1 & 3 & 5 & 7 \\ 3 & 3 & 1 & 7 & 5 \\ 5 & 5 & 7 & 1 & 3 \\ 7 & 7 & 5 & 3 & 1 \end{array}$	B2	–1 each error	
			From table clearly closed	B1		Must be clear they are referring to tabulated results
			1 is identity	B1		
			$3^{-1} \equiv 3, 5^{-1} \equiv 5, 7^{-1} \equiv 7 \pmod 8$	B1		Or “1 appears in every row”
				[5]	Superfluous fact/s gets –1	
2	(ii)		1 has order 1 and 3, 5, 7 all have order 2	B1 **[1]**		
2	(iii)		{1, 3}, {1, 5}, {1, 7} (and {1})	B1 **[1]**	All correct, no extras	Allow {1} included or omitted
2	(iv)		in *H* $3^2 \equiv 9 \pmod{10}$ so 3 not order 2	M1	Shows and states that 3 or that 7 is not order 2 (or **is** order 4)	
			no element of order > 2 in *G* so not isomorphic	A1 **[2]**	Completely correct reasoning **Or**, if zero, then **SC1** for merely stating comparable orders and then saying that “orders don’t correspond, so not isomorphic" **Or** table for H with saying “not all elements self inverse, so not isomorphic”	

Question			Answer	Marks	Guidance	
3			$u = y^3 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1		Or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}u^{-\frac{2}{3}} \frac{\mathrm{d}u}{\mathrm{d}x}$
			in DE gives $x\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = \frac{\cos x}{x}$	A1		
			$\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{2}{x}u = \frac{\cos x}{x^2}$	B1	Divide	Both sides
			$I = \exp\left(\int \frac{2}{x}\mathrm{d}x\right) = \mathrm{e}^{2\ln x}$	M1	Correctly integrates	Must have form $\frac{\mathrm{d}u}{\mathrm{d}x} + \mathrm{f}(x)u = \mathrm{g}(x)$
			$= x^2$	A1		Can be implied by subsequent work
			$x^2 \frac{\mathrm{d}u}{\mathrm{d}x} + 2xu = \cos x$			
			$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 u\right) = \cos x$			
			$x^2 u = \sin x \quad (+A)$	M1	Integrate	
			$u = \frac{\sin x + A}{x^2}$	A1	Or gives GS in implicit form	Must include constant at this stage
			$y = \left(\frac{\sin x + A}{x^2}\right)^{\frac{1}{3}}$	A1		
				[8]		

Question			Answer	Marks	Guidance	
4	(i)		Sketch	B1		Must have axes, A shown 3 across and either scale (or co-ordinates) with B in rough position, or angle and distance on argand diagram. No inconsistencies
			$OA=\lvert 3\rvert=3,\ OB=\left\lvert 3e^{\frac{1}{3}\pi i}\right\rvert=3$			
			and $\angle BOA=\frac{1}{3}\pi$	M1	Can be seen on diagram	Alt. Attempts AB or second angle
			hence ΔOAB equilateral	A1 **[3]**		
4	(ii)		$3e^{-\frac{1}{3}\pi i}$	M1A1 **[2]**	Or $3e^{\frac{5}{3}\pi i}$. Isw M1 for evidence they are considering BA, or for $\frac{3}{2}-\frac{3}{2}\sqrt{3}\,i$	For full marks can use CiS form, or clear polar co-ordinates, in radians. Not C-iS
4	(iii)		$\left(3-3e^{\frac{1}{3}\pi i}\right)^5=3^5\,e^{-\frac{5}{3}\pi i}$	M1	For mod^5 and arg × 5	“Hence” so must use ‘their $3e^{-\frac{1}{3}\pi i}$’
			$=243\left(\cos\frac{5}{3}\pi-i\sin\frac{5}{3}\pi\right)$	A1ft	aef	
			$=\frac{243}{2}+\frac{243}{2}\sqrt{3}\,i$	B1 **[3]**		Condone use of “121.5”.

Question			Answer	Marks	Guidance	
5			AE: $\lambda^2 + 2\lambda + 5 = 0$	M1		
			$\lambda = -1 \pm 2\text{i}$	A1		
			CF: $\text{e}^{-x}\left(A\cos 2x + B\sin 2x\right)$	A1ft		Or $A\text{e}^{-x}\cos(2x+\alpha)$ Must be in real form
			PI: $y = a\text{e}^{-x}$	B1		If PI $y = ax\text{e}^{-x}$,then max of M1,A1,A1, B0,M1,A0,A0 (since cannot be consistent) M1, M1, A1. Must have a constant in "their PI"
			$a\text{e}^{-x} - 2a\text{e}^{-x} + 5a\text{e}^{-x} = \text{e}^{-x}$ $4a = 1$	M1	Differentiate & substitute	
			$a = \frac{1}{4}$	A1		
			GS: $y = \text{e}^{-x}\left(\frac{1}{4} + A\cos 2x + B\sin 2x\right)$	A1ft		Must have "*y* ="
			$\frac{\text{d}y}{\text{d}x} = -\text{e}^{-x}\left(\frac{1}{4} + A\cos 2x + B\sin 2x\right)$ $+\text{e}^{-x}\left(-2A\sin 2x + 2B\cos 2x\right)$	M1*	Differentiate their GS of form $y = \text{e}^{-x}\left(P + A\cos nx + B\sin nx\right)$ where *P* is constant or linear term, *n* not 0 or 1	Allow one error
			$x = 0, \frac{\text{d}y}{\text{d}x} = 0 \Rightarrow -\left(\frac{1}{4} + A\right) + 2B = 0$ $x = 0, y = 0 \Rightarrow \frac{1}{4} + A = 0$	*M1	Use conditions	But M0 if leads to solution of *y* = 0
			$A = -\frac{1}{4},\ B = 0$	A1ft	From their GS	
			$y = \frac{1}{4}\text{e}^{-x}\left(1 - \cos 2x\right)$	A1		Must have '*y* =' and be in real form
				[11]		
6	(i)		$x = 2t+1,\ y = 5t-1,\ z = t+2$	B1	Parameterise	or B1 for *y* and *z* correctly in terms of *x* e.g. 2*y* = 5*x* – 7, 2*z* = *x* + 3 Then M1 for full simultaneous equations method.
			$\left(2t+1\right) + 2\left(5t-1\right) - 2\left(t+2\right) = 5$		Substitute into plane	
			$\Rightarrow 10t = 10 \Rightarrow t = 1$	M1	Solve	
			Intersect at (3, 4, 3)	A1	cao	Accept vector form
				[3]		

Question			Answer	Marks	Guidance	
6	**(ii)**		$\cos\left(\frac{1}{2}\pi-\theta\right)=\dfrac{\left\lvert\begin{pmatrix}2\\5\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-2\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}2\\5\\1\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}1\\2\\-2\end{pmatrix}\right\rvert}=\dfrac{10}{3\sqrt{30}}$	M1A1		Attempt to find angle or its complement
			$\theta=0.654$	A1 **[3]**	or 37.5°	
6	**(iii)**		If P is point of intersection and Q is required point, $\overrightarrow{PQ}=\lambda\begin{pmatrix}2\\5\\1\end{pmatrix}$ so $\sin\theta=\dfrac{2}{PQ}=\dfrac{2}{\lvert\lambda\rvert\sqrt{30}}$	M1*	or $\overrightarrow{PQ}.\hat{\mathbf{n}}=\pm2$ where $\mathbf{n}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Use $\overrightarrow{PQ}$ with right angled triangle or consider component of $\overrightarrow{PQ}$ in direction of normal vector.
			$\dfrac{10}{3\sqrt{30}}=\dfrac{2}{\lvert\lambda\rvert\sqrt{30}}$	M1		Valid method to set up equation in λ alone.
			$\lambda=\pm\dfrac{3}{5}$	A1		(May work from general point on original equation)
			points have position vectors $\begin{pmatrix}3\\4\\3\end{pmatrix}\pm\dfrac{3}{5}\begin{pmatrix}2\\5\\1\end{pmatrix}$	*M1	Dep on 1st M1	
			points at (1.8, 1, 2.4) and (4.2, 7, 3.6)	A1	cao	
			Alternative: Distance $=\dfrac{\lvert 2t+1+2(5t-1)-2(t+2)-5\rvert}{\sqrt{1^2+2^2+2^2}}=2$	M1* A1		Zero if formula used without substitution in of parametric form.
				*M1	Solve	
			$\Rightarrow t=0.4$ or 1.6	A1	At least one value found	
			(1.8, 1, 2.4) and (4.2, 7, 3.6)	A1 **[5]**		

Question			Answer	Marks	Guidance	
7	**(i)**		$(ab)^6 = abab...ab = a^6b^6$ as commutative	M1	Must give reason	Some demonstration that they understand commutativity
			$= \left(a^2\right)^3 \left(b^3\right)^2 = e^3e^2 = e$	A1	Using orders of *a* and *b*	
			So *ab* has order 1, 2, 3, or 6 ($b \ne a \Rightarrow ab \ne a^2 \Rightarrow ab \ne e$ so *ab* not order 1)			Condone absence of this line
			$(ab)^2 = a^2b^2 = eb^2 = b^2$ and *b* not order 2, so *ab* not order 2 $(ab)^3 = a^3b^3 = aa^2e = aee = a \ne e$, so *ab* not order 3	M1	Consider other cases	Insufficient to merely have simplified **all** $(ab)^n$
			(So must be order 6)	A1 **[4]**	AG Complete argument	
7	**(ii)**		*ac* has order 18	B1		Or *abc* or generator
			18 is LCM of 2 and 9 , (so we can use a similar argument to part (i))	M1	or explicit consideration of other cases	
			So as *G* has an element of order 18 it must be cyclic.	A1 **[3]**	AG Complete argument	
8	**(i)**		$\cos 5\theta + \mathrm{i}\sin 5\theta = \left(\cos\theta + \mathrm{i}\sin\theta\right)^5$	B1	Or $\cos 5\theta = re\{\left(\cos\theta + \mathrm{i}\sin\theta\right)^5\}$	
			$= c^5 + 5\mathrm{i}c^4s - 10c^3s^2 - 10\mathrm{i}c^2s^3 + 5cs^4 + \mathrm{i}s^5$	M1		No more than 1 error, can be unsimplified
			$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$	M1	Take real parts	
			$= c^5 - 10c^3\left(1-c^2\right) + 5c\left(1-c^2\right)^2$ $= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$	M1		
			$\cos 5\theta = 16c^5 - 20c^3 + 5c$	A1 **[5]**	AG	

Question			**Answer**	**Marks**	**Guidance**	
8	**(ii)**		Multiplying by x gives $16x^5 - 20x^3 + 5x = 0$			Hence, so no marks for using quadratic at this stage.
			letting $x = \cos\alpha$ gives $\cos 5\alpha = 0$	M1		
			hence $5\alpha = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi$	A1		
			$\alpha = \frac{1}{10}\pi, \frac{3}{10}\pi, \frac{5}{10}\pi, \frac{7}{10}\pi, \frac{9}{10}\pi$			
			$\cos\frac{5}{10}\pi = 0$ which is not a root	A1		
			so roots $x = \cos\frac{1}{10}\pi, \cos\frac{3}{10}\pi, \cos\frac{7}{10}\pi, \cos\frac{9}{10}\pi$	A1		
				[4]		
8	**(iii)**		$16x^4 - 20x^2 + 5 = 0 \Leftrightarrow x^2 = \dfrac{20 \pm \sqrt{80}}{32}$	B1		Can be gained if seen in (ii)
			cos decreases between 0 and π so $\cos\frac{1}{10}\pi$ is greatest root	M1		
			so $\cos\frac{1}{10}\pi = \sqrt{\dfrac{20+\sqrt{80}}{32}} = \sqrt{\dfrac{5+\sqrt{5}}{8}}$	A1	Dep on full marks in (ii)	
				[3]		

Question			Answer	Marks	Guidance	
1	**(i)**		$\begin{pmatrix}2\\1\\-1\end{pmatrix}\times\begin{pmatrix}3\\5\\2\end{pmatrix}=\begin{pmatrix}7\\-7\\7\end{pmatrix}=7\begin{pmatrix}1\\-1\\1\end{pmatrix}$	M1 A1		M1 requires evidence of method for cross product or at least 2 correct values calculated
			(eg) $z=0\Rightarrow 2x+y=4, 3x+5y=13\Rightarrow x=1, y=2$	M1		or any valid point e.g.(0, 3, -1), (3, 0, 2)
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1	oe vector form	Must have full equation including '**r** ='
			Alternative: Find one point	M1		
			Find a second point and vector between points	M1		
			multiple of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
			Alternative: Solve simultaneously	M1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	
			Point **and** direction found	M1 A1		
			$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
				[4]		

Question			Answer	Marks	Guidance	
1	(ii)		$\frac{\lvert 2\times 2+5--2-4\rvert}{\sqrt{2^2+1^2+1^2}}=\frac{7}{\sqrt{6}}$	M1 A1	Condone lack of absolute signs for M1 oe surd form. isw	2.86 with no workings scores M1
			Alternative: find parameter for perpendicular meets plane and use to find distance	M1 **[2]**	For complete method with calculation errors	look for λ = -7/6
2			$u=y^2\Rightarrow\frac{du}{dx}=2y\frac{dy}{dx}$	M1	Correctly finds	Or $\frac{dy}{dx}=\frac{1}{2}u^{-\frac{1}{2}}\frac{du}{dx}$
			so DE $\Rightarrow 2y\frac{dy}{dx}-4y^2=2\mathrm{e}^x$	M1	or for complete unsimplified substitution	
			$\Rightarrow\frac{du}{dx}-4u=2\mathrm{e}^x$	A1		Can be implied by next A1
			$I=\exp\int -4\,\mathrm{d}x=\mathrm{e}^{-4x}$	A1ft		Must have form $\frac{\mathrm{d}u}{\mathrm{d}x}+f(x)u=g(x)$ for this mark and any further marks Can be implied by subsequent work
			$\mathrm{e}^{-4x}\frac{du}{dx}-4\mathrm{e}^{-4x}u=2\mathrm{e}^{-3x}$	M1*	Multiples through by IF of form e^{kx}, simplifying RHS	
			$u\mathrm{e}^{-4x}=-\frac{2}{3}\mathrm{e}^{-3x}(+A)$	*M1dep*	Integrates	
			$u=-\frac{2}{3}\mathrm{e}^x+A\mathrm{e}^{4x}$	M1dep *	Rearranges to make u or y^2 the subject	No more than 1 numerical error at this step
			$y=\sqrt{-\frac{2}{3}\mathrm{e}^x+A\mathrm{e}^{4x}}$	A1	Cao	ignore use of '±'
			Alternative from 4th mark to 6th mark			
			CF: (u=…) Ae^{4x}	A1		
			PI: $u=ke^x$, $\frac{du}{dx}=ke^x$	M1*	PI chosen & differentiated correctly	
			$ke^x-4ke^x=2e^x,\quad k=-\frac{2}{3}$	M1 dep*	Substitutes and solves	
				[8]		

Question			Answer	Marks	Guidance	
3	(i)		$z^6=1 \Rightarrow z=e^{2k\pi i/6}$	M1		
			$k=0,1,2,3,4,5$	A1	Oe exactly 6 roots	accept roots 1, -1 given as integers.
			Diagram	B1	6 roots in right quadrant,	
				B1	correct angles and moduli	as evidenced by labels, circles, or accurate diagram, or by co-ordinates
				[4]		
3	(ii)		$(1+i)^6=\left(\sqrt{2}\,e^{\frac{1}{4}\pi i}\right)^6$	M1	Attempts modulus-argument form, getting at least 1 correct	
			$8e^{\frac{6}{4}\pi i}$	M1	for (mod)^6 and arg x 6	
			$=-8i$	A1	**ag**	complete argument including start line
			Alternative:			
			$(1+i)^6=1+6i+15i^2+20i^3+15i^4+6i^5+i^6$	M1		
			$=1+6i-15-20i+15+6i\ \ -1$	M1	no more than 1 term wrong	**Sc 2** for only lines 2 & 3correct
			$=-8i$	A1	**ag**	
			Alternative: $(1+i)^2=2i$	M1		
			$(1+i)^6=(2i)^3$	M1		
			$=-8i$	A1	**ag**	
				[3]		

Question			Answer	Marks	Guidance	
3	**(iii)**		$z^6=-8\mathrm{i} \Rightarrow z=(1+\mathrm{i})\mathrm{e}^{2k\pi \mathrm{i}/6}$	M1		
			$=\sqrt{2}e^{i\frac{\pi}{4}}\mathrm{e}^{2k\pi \mathrm{i}/6}$	M1		
			$\sqrt{2}\,\mathrm{e}^{\mathrm{i}\pi(1/4+k/3)},\ k=0,1,2,3,4,5$	A1	or equivalent *k*	
			Alternative: $z^6=8\mathrm{e}^{\mathrm{i}\pi(\frac{3}{2}+2k)}$	M1		
			$\sqrt{2}\,\mathrm{e}^{\mathrm{i}\pi(1/4+k/3)},\ k=0,1,2,3,4,5$	M1 A1 **[3]**		or equivalent: e.g. $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\pi(-1/12+k/3)}$ accept unsimplified modulus

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td rowspan="4"><table><tr><td>element</td><td>(1)</td><td>3</td><td>7</td><td>9</td><td>11</td><td>13</td><td>17</td><td>19</td></tr><tr><td>inverse</td><td>(1)</td><td>7</td><td>3</td><td>9</td><td>11</td><td>17</td><td>13</td><td>19</td></tr></table></td><td>B1</td><td>2 or more</td><td>Ignore 1</td></tr>
<tr><td></td><td></td><td></td><td>B1</td><td>4 or more</td><td></td></tr>
<tr><td></td><td></td><td></td><td>B1</td><td>all 7 correct</td><td></td></tr>
<tr><td></td><td></td><td></td><td>[3]</td><td></td><td></td></tr>
<tr><td>4</td><td>(ii)</td><td></td><td>(1 has order 1)
9,11,19 have order 2</td><td>M1</td><td>Correctly identifies order of all elements</td><td>Allow one error</td></tr>
<tr><td></td><td></td><td></td><td>$3^2 = 9 \Rightarrow 3^4 = 1$ so order 4
similarly 7,13,17 order 4</td><td>B1</td><td>justifies order for at least 1 element of order 4</td><td>must show workings towards a^4 for demonstration that these elements are order 4`</td></tr>
<tr><td></td><td></td><td></td><td>no element of order 8 so not cyclic</td><td>A1</td><td>www</td><td>condone “no generator” in place of ”no element or order 8”</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td><td></td></tr>
<tr><td>4</td><td>(iii)</td><td></td><td></td><td>M1</td><td>For two sets which both contain “1” and all (4) elements’ inverses</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>B1</td><td>One subgroup of order 4</td><td></td></tr>
<tr><td></td><td></td><td></td><td>{1,13, 9, 17} and {1, 3, 9, 7}</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>M1</td><td>for correspondence of “their” elements of same order</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$1 \leftrightarrow 1,\ 9 \leftrightarrow 9,\ 3 \leftrightarrow 13,\ 7 \leftrightarrow 17$</td><td>A1</td><td>or $3 \leftrightarrow 17,\ 7 \leftrightarrow 13$</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[5]</td><td></td><td></td></tr>
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Question			Answer	Marks	Guidance	
5			AE: $\lambda^2+5\lambda+6=0$ $\lambda=-2,-3$	B1		
			CF: $A\mathrm{e}^{-2x}+B\mathrm{e}^{-3x}$	B1ft		
			PI: $y=a\mathrm{e}^{-x}$	B1ft		
			$a\mathrm{e}^{-x}-5a\mathrm{e}^{-x}+6a\mathrm{e}^{-x}=\mathrm{e}^{-x}$	M1	Differentiate and substitute	
			$2a=1$ $a=\frac{1}{2}$	A1		
			GS: $(y=)\frac{1}{2}\mathrm{e}^{-x}+A\mathrm{e}^{-2x}+B\mathrm{e}^{-3x}$	A1ft		ft must be of form " $k\mathrm{e}^{-x}$ plus a standard CF form" with 2 arbitrary constants
			$x=0, y=0\Rightarrow\frac{1}{2}+A+B=0$	M1	Use condition on GS	Must have 2 arbitrary constants
			$y'=-\frac{1}{2}\mathrm{e}^{-x}-2A\mathrm{e}^{-2x}-3B\mathrm{e}^{-3x}$	M1*	Differentiate their GS of form $y=k\mathrm{e}^{-x}+A\mathrm{e}^{mx}+B\mathrm{e}^{nx}$ where k, *m, n* are non-zero constants and *m, n* not 1	
			$x=0, y'=0\Rightarrow-\frac{1}{2}-2A-3B=0$			
			$A=-1,\ B=\frac{1}{2}$	M1dep*	Use condition and attempt to find A, B	
			$y=\frac{1}{2}\mathrm{e}^{-x}-\mathrm{e}^{-2x}+\frac{1}{2}\mathrm{e}^{-3x}$	A1	www	Must have 'y ='
				[10]		

Question			Answer	Marks	Guidance	
6	(i)		$l \parallel \begin{pmatrix}2\\3\\5\end{pmatrix}$ $\Pi \perp \begin{pmatrix}4\\-1\\-1\end{pmatrix}$ so $\begin{pmatrix}2\\3\\5\end{pmatrix} \bullet \begin{pmatrix}4\\-1\\-1\end{pmatrix} = 0 \Rightarrow l \parallel \Pi$	M1	dot product of correct vectors = 0	
			(1, –2, 7) on l but $4\times1--2-7=-1\neq8$ so not in Π	M1	substitute point on line into Π and calculate d	
			hence l not in Π	A1 **[3]**	Full argument includes key components	Argument can be about a general point on line
6	(ii)		$(\mathbf{r} =)\begin{pmatrix}1\\-2\\7\end{pmatrix} + \lambda\begin{pmatrix}4\\-1\\-1\end{pmatrix}$	B1		
			closest point where meets Π $4(1+4\lambda)-(-2-\lambda)-(7-\lambda)=8$	M1	parametric form of (x, y, z) substituted into plane	
			$\Rightarrow \lambda = \frac{1}{2}$	A1ft		
			$\Rightarrow \mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix}$	A1 **[4]**		
6	(iii)		$\mathbf{r} = \begin{pmatrix}3\\-\frac{5}{2}\\\frac{13}{2}\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\5\end{pmatrix}$	B1ft **[1]**	oe	must have "**r** ="

Question			Answer	Marks	Guidance	
7	(i)		$2\mathrm{i}\sin\theta = \mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}$	B1	any equivalent form	If use *z*, must define it
			$2\mathrm{i}\sin n\theta = \mathrm{e}^{\mathrm{i}n\theta} - \mathrm{e}^{-\mathrm{i}n\theta}$			
			$\left(2\mathrm{i}\sin\theta\right)^5 = \left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)^5$			
			$= \mathrm{e}^{\mathrm{i}5\theta} - 5\mathrm{e}^{\mathrm{i}3\theta} + 10\mathrm{e}^{\mathrm{i}\theta} - 10\mathrm{e}^{-\mathrm{i}\theta} + 5\mathrm{e}^{-\mathrm{i}3\theta} - \mathrm{e}^{-\mathrm{i}5\theta}$	M1*	binomial expansion	can be unsimplified
			$32i\sin^5\theta = (e^{5i\theta} - e^{-5i\theta}) - 5(e^{3i\theta} - e^{-3i\theta}) + 10(e^{i\theta} - e^{-i\theta})$	M1dep*	grouping terms	Award **B1 then sc M1A1** for candidates who omit this stage from otherwise complete argument
			$= 2\mathrm{i}\sin 5\theta - 5\left(2\mathrm{i}\sin 3\theta\right) + 10\left(2\mathrm{i}\sin\theta\right)$			
			$\sin^5\theta = \tfrac{1}{16}\left(\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right)$	A1	AG	must convince on the $\frac{1}{16}$ and on the elimination of *i*
				[4]		
7	(ii)		$16\sin^5\theta - 10\sin\theta = \sin 5\theta - 5\sin 3\theta$	M1*	Attempts to eliminate sin5θ and sin3θ	
			$16\sin^5\theta - 6\sin\theta = 0$	A1		Or 16sin^5 θ =6sin θ
			$\sin\theta = 0, \pm\sqrt[4]{\frac{3}{8}}$	M1dep*	must have 3 values for sin θ	
			$\theta = 0, \pm 0.899$	A1 **[4]**		

Question			Answer	Marks	Guidance	
8	**(i)**		$\begin{pmatrix}1&0\\0&1\end{pmatrix}$ is identity $\begin{pmatrix}a&-b\\b&a\end{pmatrix}^{-1}=\frac{1}{a^2+b^2}\begin{pmatrix}a&b\\-b&a\end{pmatrix}\in G$ $\begin{pmatrix}a&-b\\b&a\end{pmatrix}\begin{pmatrix}c&-d\\d&c\end{pmatrix}=\begin{pmatrix}ac-bd&-bc-ad\\bc+ad&ac-bd\end{pmatrix}$ and $(ac-bd)^2+(bc+ad)^2=a^2c^2+b^2d^2+b^2c^2+a^2d^2$ $=(a^2+b^2)(c^2+d^2)\neq 0$	B1 M1 A1 M1 M1 A1 **[6]**	for M1, must at least get matrix in form $\begin{pmatrix}x&-y\\y&x\end{pmatrix}$, or state existence of inverse due to non-singularity Must not attempt to prove commutativity in (i)	
8	**(ii)**		$\begin{pmatrix}c&-d\\d&c\end{pmatrix}\begin{pmatrix}a&-b\\b&a\end{pmatrix}=\begin{pmatrix}ac-bd&-bc-ad\\bc+ad&ac-bd\end{pmatrix}$ $=\begin{pmatrix}a&-b\\b&a\end{pmatrix}\begin{pmatrix}c&-d\\d&c\end{pmatrix}$ so commutative	M1 A1 **[2]**		must also consider matrices reversed, but may be seen in (i)
8	**(iii)**		$\begin{pmatrix}0&-1\\1&0\end{pmatrix}^2=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$ $\begin{pmatrix}-1&0\\0&-1\end{pmatrix}^2=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ order 4	M1 M1 A1 **[3]**	g^2 must be correct allow 1 error in getting g^4	