OCR

Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Three points have coordinates $\mathrm{A}(-3, 12, -7)$, $\mathrm{B}(-2, 6, 9)$, $\mathrm{C}(6, 0, -10)$. The plane P passes through the points A, B and C.

(i) Find the vector product $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. Hence or otherwise find an equation for the plane P in the form $ax + by + cz = d$. **[5]**

The plane Q has equation $6x + 3y + 2z = 32$. The perpendicular from A to the plane Q meets Q at the point D. The planes P and Q intersect in the line L.

(ii) Find the distance AD. **[3]**

(iii) Find an equation for the line L. **[5]**

(iv) Find the shortest distance from A to the line L. **[6]**

(v) Find the volume of the tetrahedron ABCD. **[5]**

Option 2: Multi-variable calculus

2 A surface S has equation $\mathrm{g}(x, y, z) = 0$, where $\mathrm{g}(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24$.
$\mathrm{P}(2, 6, -2)$ is a point on the surface S.

(i) Find $\dfrac{\partial \mathrm{g}}{\partial x}$, $\dfrac{\partial \mathrm{g}}{\partial y}$ and $\dfrac{\partial \mathrm{g}}{\partial z}$. **[3]**

(ii) Find the equation of the normal line to the surface S at the point P. **[3]**

(iii) The point Q is on this normal line and close to P. At Q, $\mathrm{g}(x, y, z) = h$, where h is small. Find, in terms of h, the approximate perpendicular distance from Q to the surface S. **[4]**

(iv) Find the coordinates of the two points on the surface at which the normal line is parallel to the y-axis. **[6]**

(v) Given that $10x - y + 2z = 6$ is the equation of a tangent plane to the surface S, find the coordinates of the point of contact. **[8]**

Option 3: Differential geometry

3 **(a)** A curve has intrinsic equation $s = 2\ln\left(\frac{\pi}{\pi - 3\psi}\right)$ for $0 \leqslant \psi < \frac{1}{3}\pi$, where s is the arc length measured from a fixed point P and $\tan\psi = \frac{\mathrm{d}y}{\mathrm{d}x}$. P is in the third quadrant. The curve passes through the origin O, at which point $\psi = \frac{1}{6}\pi$. Q is the point on the curve at which $\psi = \frac{3}{10}\pi$.

(i) Express ψ in terms of s, and sketch the curve, indicating the points O, P and Q. **[4]**

(ii) Find the arc length OQ. **[3]**

(iii) Find the radius of curvature at the point O. **[3]**

(iv) Find the coordinates of the centre of curvature corresponding to the point O. **[3]**

(b) **(i)** Find the surface area of revolution formed when the curve $y = \frac{1}{3}\sqrt{x}(x-3)$ for $1 \leqslant x \leqslant 4$ is rotated through 2π radians about the y-axis. **[7]**

(ii) The curve in part **(b)(i)** is one member of the family $y = \frac{1}{9}\lambda\sqrt{x}(x-\lambda)$, where λ is a positive parameter. Find the equation of the envelope of this family of curves. **[4]**

Option 4: Groups

4 The twelve distinct elements of an abelian multiplicative group G are

$$e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b$$

where e is the identity element, $a^6 = e$ and $b^2 = e$.

(i) Show that the element a^2b has order 6. **[3]**

(ii) Show that $\{e, a^3, b, a^3b\}$ is a subgroup of G. **[3]**

(iii) List all the cyclic subgroups of G. **[6]**

You are given that the set

$$H = \{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89\}$$

with binary operation multiplication modulo 90 is a group.

(iv) Determine the order of each of the elements 11, 17 and 19. **[4]**

(v) Give a cyclic subgroup of H with order 4. **[2]**

(vi) By identifying possible values for the elements a and b above, or otherwise, give one example of each of the following:

(A) a non-cyclic subgroup of H with order 12, **[3]**

(B) a non-cyclic subgroup of H with order 4. **[3]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

The speeds of vehicles are measured on a busy stretch of road and are categorised as A (not more than 30 mph), B (more than 30 mph but not more than 40 mph) or C (more than 40 mph).

- Following a vehicle in category A, the probabilities that the next vehicle is in categories A, B, C are 0.9, 0.07, 0.03 respectively.
- Following a vehicle in category B, the probabilities that the next vehicle is in categories A, B, C are 0.3, 0.6, 0.1 respectively.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

This is modelled as a Markov chain with three states corresponding to the categories A, B, C. The speed of the first vehicle is measured as 28 mph.

(i) Write down the transition matrix $\mathbf{P}$. **[2]**

(ii) Find the probabilities that the 10th vehicle is in each of the three categories. **[3]**

(iii) Find the probability that the 12th and 13th vehicles are in the same category. **[4]**

(iv) Find the smallest value of n for which the probability that the nth and $(n+1)$th vehicles are in the same category is less than 0.8, and give the value of this probability. **[4]**

(v) Find the expected number of vehicles (including the first vehicle) in category A before a vehicle in a different category. **[2]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity, and hence write down the equilibrium probabilities for the three categories. **[3]**

(vii) Find the probability that, after many vehicles have passed by, the next three vehicles are all in category A. **[2]**

On a new stretch of road, the same categories are used but some of the transition probabilities are different.

- Following a vehicle in category A, the probability that the next vehicle is in category B is equal to the probability that it is in category C.
- Following a vehicle in category B, the probability that the next vehicle is in category A is equal to the probability that it is in category C.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

In the long run, the proportions of vehicles in categories A, B, C are 50%, 40%, 10% respectively.

(viii) Find the transition matrix for the new stretch of road. **[4]**

END OF QUESTION PAPER

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