OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2014

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation	Meaning of annotation
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

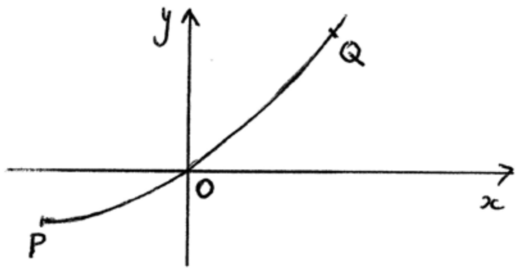
Question			Answer	Marks	Guidance	
1	(i)		$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 1 \\ -6 \\ 16 \end{pmatrix} \times \begin{pmatrix} 9 \\ -12 \\ -3 \end{pmatrix} = \begin{pmatrix} 210 \\ 147 \\ 42 \end{pmatrix} \quad [= 21\begin{pmatrix} 10 \\ 7 \\ 2 \end{pmatrix}]$	M1 A2	Evaluation of vector product Give A1 for one correct element	One correct element (FT) Give A1 for a non-zero multiple
			Equation of P is $10x + 7y + 2z = d$ $10x + 7y + 2z = 40$	M1 A1 **[5]**	*Accept* $210x + 147y + 42z - 840 = 0$ *etc*	
1	(ii)		$\text{AD} = \left\lvert \dfrac{6(-3) + 3(12) + 2(-7) - 32}{\sqrt{6^2 + 3^2 + 2^2}} \right\rvert$	M1 M1	For numerator For denominator	*M0 if constant term omitted*
			$= \dfrac{28}{7} = 4$	A1 **[3]**		
		OR	$6(-3 + 6\lambda) + 3(12 + 3\lambda) + 2(-7 + 2\lambda) = 32$		M1 Equation for λ	
			$\lambda = \dfrac{4}{7}, \quad \text{AD} = \left\lvert \lambda \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \right\rvert = \dfrac{4}{7}\sqrt{6^2 + 3^2 + 2^2}$		M1 Using λ to find the distance AD *Independent of previous M1*	*But M0 if* $\lambda = \pm 1$ *or* $\lambda = 0$
			$= 4$		A1	

Question			Answer	Marks	Guidance	
1	(iii)		When $x=0$, $\begin{matrix} 7y+2z=40 \\ 3y+2z=32 \end{matrix}$ $y=2,\ z=13$	M1 A1	Finding a point on L One correct point e.g. $(1,\ 1,\ 11.5)$	e.g. $(2,\ 0,\ 10)$, $(\frac{26}{3},\ -\frac{20}{3},\ 0)$
			$\begin{pmatrix} 10 \\ 7 \\ 2 \end{pmatrix} \times \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \\ -12 \end{pmatrix}$	M1 A1	Vector product of direction vectors Direction of L correct	OR Finding a second point on L and using 2 points to find direction
			Equation of L is $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$	A1 FT **[5]**	Any correct form *Dependent on M1M1*	Condone omission of ‘ $\mathbf{r}$ = ‘
		OR	Eliminating z, $4x+4y=8$ $x=\lambda,\ y=2-\lambda,\ z=13-\frac{3}{2}\lambda$		M1 Eliminating one variable A1 M1 Finding (e.g.) y and z in terms of x A1A1	Or $6y-4z=-40$ or $12x+8z=104$ Or A1 FT *dependent on M1M1*
1	(iv)		$\left[\begin{pmatrix} -3 \\ 12 \\ -7 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 13 \end{pmatrix}\right] \times \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ 10 \\ -20 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -70 \\ -49 \\ -14 \end{pmatrix}$	M1 A2 FT	Appropriate vector product Give A1 if one error	
			Shortest distance is $\dfrac{\sqrt{70^2+49^2+14^2}}{\sqrt{2^2+2^2+3^2}} = \sqrt{\dfrac{7497}{17}}$	M1 M1	Finding magnitude of vector product Complete method for finding distance	*Dependent on previous M1* *Dependent on previous M1M1*
			Shortest distance is 21	A1 **[6]**	*A0 for 21 resulting from wrong v.p.*	
		OR	$\left[\begin{pmatrix} 2\lambda \\ 2-2\lambda \\ 13-3\lambda \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ -7 \end{pmatrix}\right] . \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix} = 0$		M1 Allow one error A1 FT	
			$\lambda = 2$		M1 Obtaining a value of λ A1 FT	*Dependent on previous M1*
			Shortest distance is $\sqrt{(7)^2+(-14)^2+(14)^2}$ Shortest distance is 21		M1 A1	*Dependent on previous M1M1*

Question			Answer	Marks	Guidance	
1	(v)		$\overrightarrow{AD} = (\pm)\ \frac{4}{7}\begin{pmatrix}6\\3\\2\end{pmatrix}$	M1 A1 FT	$\overrightarrow{AD}$ is a multiple of $\begin{pmatrix}6\\3\\2\end{pmatrix}$ FT from (ii)	*M1 for* $\overrightarrow{AD} = \begin{pmatrix}6\\3\\2\end{pmatrix}$
			Volume is $\frac{1}{6}(\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{AD}$	M1	Appropriate scalar triple product	*Just stated.* $\frac{1}{6}$ *not needed*
			$= \frac{1}{6}\times 21\begin{pmatrix}10\\7\\2\end{pmatrix}.\frac{4}{7}\begin{pmatrix}6\\3\\2\end{pmatrix} = 2(60+21+4)$	M1	Evaluation of scalar triple product	*Independent of previous M's, but must be numerical*
			$= 170$	A1 **[5]**		
2	(i)		$\frac{\partial g}{\partial x} = 2x + 6z - 4y$	B1		
			$\frac{\partial g}{\partial y} = 6y + 2z - 4x$	B1		
			$\frac{\partial g}{\partial z} = 4z + 2y + 6x$	B1 **[3]**		
2	(ii)		At P, $\frac{\partial g}{\partial x} = -32,\ \frac{\partial g}{\partial y} = 24,\ \frac{\partial g}{\partial z} = 16$	B1		
			Normal line is $\mathbf{r} = \begin{pmatrix}2\\6\\-2\end{pmatrix} + \lambda\begin{pmatrix}-4\\3\\2\end{pmatrix}$	M1 A1 **[3]**	Direction of normal line FT	Condone omission of ' **r** = '

Question			Answer	Marks	Guidance	
2	(iii)		$(h = \delta g \approx)\dfrac{\partial g}{\partial x}\delta x + \dfrac{\partial g}{\partial y}\delta y + \dfrac{\partial g}{\partial z}\delta z$	M1		
			$h = (-32)(-4\lambda) + (24)(3\lambda) + (16)(2\lambda) \quad (= 232\lambda)$	A1 FT		
			Approx distance is $\lvert \lambda \rvert \sqrt{4^2 + 3^2 + 2^2}$	M1		
			$= \sqrt{29}\,\lvert \lambda \rvert = \dfrac{\sqrt{29}\,\lvert h \rvert}{232}$	A1	Accept $\dfrac{h}{8\sqrt{29}}$, $\dfrac{h}{43.1}$, $0.023h$ etc	
				[4]		
2	(iv)		Require $\dfrac{\partial g}{\partial x} = \dfrac{\partial g}{\partial z} = 0$	M1		
			$2x + 6z - 4y = 0$ and $4z + 2y + 6x = 0$			
			$y = -x, \quad z = -x$	M1	For (e.g.) y and z as multiples of x	
			$x^2 + 3x^2 + 2x^2 + 2x^2 - 6x^2 + 4x^2 - 24 = 0$	M1	Quadratic in one variable	
			$6x^2 - 24 = 0$	A1	In simplified form	
			Points $(2,\ -2,\ -2)$ and $(-2,\ 2,\ 2)$	A1A1	If neither point correct, give A1 for any four correct coordinates	
				[6]		

Question			Answer	Marks	Guidance	
2	(v)		$\begin{pmatrix} 2x+6z-4y \\ 6y+2z-4x \\ 4z+2y+6x \end{pmatrix} = \lambda \begin{pmatrix} 10 \\ -1 \\ 2 \end{pmatrix}$	M1 A1 FT	Allow M1 even if $\lambda = 1$	
			$y = 3x,\ z = -5x$	M1	For (e.g.) y and z as multiples of x	Or $x = -\frac{1}{4}\lambda,\ y = -\frac{3}{4}\lambda,\ z = \frac{5}{4}\lambda$
			$x^2 + 27x^2 + 50x^2 - 30x^2 - 30x^2 - 12x^2 - 24 = 0$	M1	Quadratic in one variable	
			$6x^2 - 24 = 0$	A1	Or $y^2 - 36 = 0$ or $z^2 - 100 = 0$	Or $\lambda^2 - 64 = 0$
			Possible points $(2,\ 6,\ -10)$ and $(-2,\ -6,\ 10)$	A1	For one correct point	
			At $(2,\ 6,\ -10)$, $10x - y + 2z = -6$ At $(-2,\ -6,\ 10)$, $10x - y + 2z = 6$	M1	Checking at least one point	
			It is the tangent plane at $(-2,\ -6,\ 10)$	A1 **[8]**		
		OR	$10x - (3x) + 2(-5x) = 6$ $x = -2$ It is the tangent plane at $(-2,\ -6,\ 10)$		M1 Equation in one variable A1 Or $y = -6$ or $z = 10$ or $\lambda = 8$ M1 Using this value to obtain at least two coordinates A2 Give A1 for two coordinates correct	

Question			Answer	Marks	Guidance	
3	(a)	(i)	$\psi = \frac{1}{3}\pi(1 - e^{-\frac{s}{2}})$ 	B1 B1 B1 B1 **[4]**	 Positive increasing gradient through O Zero gradient at P Q marked in first quadrant	
3	(a)	(ii)	At O ($\psi = \frac{1}{6}\pi$), $s = 2\ln 2$ At Q ($\psi = \frac{3}{10}\pi$), $s = 2\ln 10$ Arc length OQ is $2\ln 10 - 2\ln 2 = 2\ln 5$	M1 M1 A1 **[3]**	 *Or* $\ln 25$ *or* 3.22 *(only)*	
3	(a)	(iii)	$\rho = \dfrac{ds}{d\psi}$ $= \dfrac{6}{\pi - 3\psi}$ At O ($\psi = \frac{1}{6}\pi$), radius of curvature is $\rho = \dfrac{12}{\pi}$	M1 A1 A1 **[3]**	*Or* $(\kappa =)\dfrac{d\psi}{ds}$ *Or* $\kappa = \dfrac{\pi}{6}e^{-\frac{s}{2}}$ *and* $s = 2\ln 2$ Accept 3.82	 *All 3 marks can be awarded in (iv)*
3	(a)	(iv)	Centre of curvature is $(-\rho\sin\psi,\ \rho\cos\psi)$ $\left(-\dfrac{6}{\pi},\ \dfrac{6\sqrt{3}}{\pi}\right)$	M1 A1A1 **[3]**	 FT is $\left(-\frac{1}{2}\lvert\rho\rvert,\ \frac{\sqrt{3}}{2}\lvert\rho\rvert\right)$	 Accept $(-1.91,\ 3.31)$

Question			Answer	Marks	Guidance	
3	(b)	(i)	$1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{1}{2}x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}\right)^2$	B1		
			$=1+\frac{1}{4}x-\frac{1}{2}+\frac{1}{4}x^{-1}=\frac{1}{4}x+\frac{1}{2}+\frac{1}{4}x^{-1}$	M1		
			$=\left(\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}\right)^2$	A1	*or* $\frac{(x+1)^2}{4x}$	*Condone correct answer from inaccurate working*
			Area is $\int 2\pi x\,ds$	M1		
			$=\int_1^4 2\pi x\left(\frac{1}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{-\frac{1}{2}}\right)dx$	A1	Any correct form	
			$=\pi\left[\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{3}x^{\frac{3}{2}}\right]_1^4$	A1		
			$=\frac{256}{15}\pi$	A1	*Exact answer only*	
				[7]		
3	(b)	(ii)	Differentiating partially with respect to λ	M1		
			$0=\frac{1}{9}x^{\frac{3}{2}}-\frac{2}{9}\lambda x^{\frac{1}{2}}$	A1	*For RHS*	
			$\lambda=\frac{1}{2}x$, so $y=\frac{1}{9}(\frac{1}{2}x)x^{\frac{3}{2}}-\frac{1}{9}(\frac{1}{4}x^2)x^{\frac{1}{2}}$	M1	Eliminating λ	
			$y=\frac{1}{36}x^{\frac{5}{2}}$	A1	*Must be simplified*	
				[4]		

Question			Answer	Marks	Guidance	
4	(i)		$(a^2b)^2 = a^4b^2 = a^4$ $(a^2b)^3 = b,\ (a^2b)^4 = a^2,\ (a^2b)^5 = a^4b$ $(a^2b)^6 = e$ Hence a^2b has order 6	M1 A1 E1 **[3]**	Finding one power Three powers correct Fully correct explanation	*No need to state conclusion, provided it has been fully justified*
4	(ii)		$\begin{array}{c\|cccc} & e & a^3 & b & a^3b \\ \hline e & e & a^3 & b & a^3b \\ a^3 & a^3 & e & a^3b & b \\ b & b & a^3b & e & a^3 \\ a^3b & a^3b & b & a^3 & e \end{array}$ The set is closed; hence it is a subgroup of G	B2 B1 **[3]**	Give B1 for no more than three errors or omissions *'Closed' (or equivalent) is required*	
4	(iii)		$\{e,\ a^3\},\ \{e,\ b\},\ \{e,\ a^3b\}$ $\{e,\ a^2,\ a^4\}$ $\{e,\ a,\ a^2,\ a^3,\ a^4,\ a^5\}$ $\{e,\ a^2b,\ a^4,\ b,\ a^2,\ a^4b\}$ $\{e,\ ab,\ a^2,\ a^3b,\ a^4,\ a^5b\}$	B2 B1 B1 B1 B1 **[6]**	Give B1 for one correct B0 if any other set of order 3 *No mark for $\{e\}$. Deduct one mark (out of B6) for each set (including G) of order other than 1, 2, 3, 6*	*Deduct one mark (out of B2) for each set of order 2 in excess of 3* *Deduct one mark (out of B3) for each set of order 6 in excess of 3*

Question			Answer	Marks	Guidance	
4	(iv)		$11^2=31,\ 11^3=71,\ 11^4=61,\ 11^5=41,\ 11^6=1$ $17^2=19,\ 17^3=53,\ 17^4=1$ 11 has order 6 17 has order 4 $19^2=1$; 19 has order 2	M1 A1 A1 B1 **[4]**	Finding at least two powers of 11 (or 17) *Either correct implies M1*	
4	(v)		$\{1,\ 17,\ 19,\ 53\}$	M1 A1 **[2]**	Selecting powers of 17 Or B2 for $\{1,\ 37,\ 19,\ 73\}$	
4	(vi)	(*A*)	Taking $a=11,\ b=19$ $1, 11, 11^2, \dots, 11^5, 19, 11\times19, 11^2\times19, \dots, 11^5\times19$ $\{1, 11, 31, 71, 61, 41, 19, 29, 49, 89, 79, 59\}$ i.e. $\{1, 11, 19, 29, 31, 41, 49, 59, 61, 71, 79, 89\}$	B1 M1 A1 **[3]**	*There are (many) other possibilities* Finding elements of *G* using their *a, b*	
4	(vi)	(*B*)	$1,\ 11^3,\ 19,\ 11^3\times19$ $\{1,\ 71,\ 19,\ 89\}$	M1 M1 A1 **[3]**	Reference to group in (ii) Finding group in (ii) with their *a, b*	
5			***Pre-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	*Do not penalise answers given to more than 4 dp*
5	(i)		$\mathbf{P}=\begin{pmatrix}0.9 & 0.3 & 0.1\\ 0.07 & 0.6 & 0.7\\ 0.03 & 0.1 & 0.2\end{pmatrix}$	B2 **[2]**	Give B1 for two columns correct	

Question			Answer	Marks	Guidance	
5	(ii)		$\mathbf{P}^9\begin{pmatrix}1\\0\\0\end{pmatrix}$ P(A) = 0.7268 P(B) = 0.2189 P(C) = 0.0544	M1 M1 A1 **[3]**	For $\mathbf{P}^9$ (allow $\mathbf{P}^{10}$) For initial column matrix (or first column of $\mathbf{P}^9$)	 *Dependent on previous M1*
5	(iii)		$\mathbf{P}^{11}=\begin{pmatrix}0.7242 & \dots & \dots\\0.2211 & \dots & \dots\\0.0547 & \dots & \dots\end{pmatrix}$ $0.7242\times0.9+0.2211\times0.6+0.0547\times0.2$ $=0.7954$	M1 M1 M1 A1 **[4]**	Appropriate elements from $\mathbf{P}^{11}$ Diagonal elements from **P** *Dependent on previous M1M1*	(allow $\mathbf{P}^{12}$)
5	(iv)		$\begin{pmatrix}0.9 & 0.6 & 0.2\end{pmatrix}\mathbf{P}^{n-1}\begin{pmatrix}1\\0\\0\end{pmatrix}$ $=(0.8009)$ when $n=7$ $=(0.7986)$ when $n=8$ Smallest value is $n=8$ Probability is 0.7986	M1 M1 B1 A1 **[4]**	Repeating (iii) for another value of n Obtaining values both sides of 0.8 *0.7986 implies M1M1*	 Valid method required here
5	(v)		Expected run length is $\dfrac{1}{1-0.9}$ $= 10$	M1 A1 **[2]**	Using $\dfrac{1}{1-p}$ or $\dfrac{p}{1-p}$ with $p=0.9$	
5	(vi)		$\mathbf{P}^n\rightarrow\begin{pmatrix}0.7225 & 0.7225 & 0.7225\\0.2225 & 0.2225 & 0.2225\\0.0549 & 0.0549 & 0.0549\end{pmatrix}$ P(A) = 0.7225 P(B) = 0.2225 P(C) = 0.0549	B2 B1 **[3]**	Give B1 for 6 elements correct to 3 dp FT if columns agree to 4 dp	

Question			Answer	Marks	Guidance	
5	(vii)		$0.7225 \times 0.9 \times 0.9$ $= 0.5853$	M1 A1 **[2]**	FT P(A) × 0.81	
5	viii		$\begin{pmatrix} 1-2x & y & 0.1 \\ x & 1-2y & 0.7 \\ x & y & 0.2 \end{pmatrix}\begin{pmatrix} 0.5 \\ 0.4 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.4 \\ 0.1 \end{pmatrix}$ $0.5(1-2x) + 0.4y + 0.01 = 0.5$ $0.5x + 0.4(1-2y) + 0.07 = 0.4$ $0.5x + 0.4y + 0.02 = 0.1$ $x = 0.06, \quad y = 0.125$ Transition matrix is $\begin{pmatrix} 0.88 & 0.125 & 0.1 \\ 0.06 & 0.75 & 0.7 \\ 0.06 & 0.125 & 0.2 \end{pmatrix}$	M1 A1 M1 A1 **[4]**	First or second column correct Obtaining values for x and y	
5			***Post-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	*Do not penalise answers given to more than 4 dp*
5	(i)		$\mathbf{P} = \begin{pmatrix} 0.9 & 0.07 & 0.03 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.7 & 0.2 \end{pmatrix}$	B2 **[2]**	Give B1 for two rows correct	
5	(ii)		$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\mathbf{P}^9$ P(A) = 0.7268 P(B) = 0.2189 P(C) = 0.0544	M1 M1 A1 **[3]**	For $\mathbf{P}^9$ (allow $\mathbf{P}^{10}$) For initial row matrix (or first row of $\mathbf{P}^9$)	*Dependent on previous M1*

Question			Answer	Marks	Guidance	
5	(iii)		$\mathbf{P}^{11} = \begin{pmatrix} 0.7242 & 0.2211 & 0.0547 \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$ $0.7242 \times 0.9 + 0.2211 \times 0.6 + 0.0547 \times 0.2$ $= 0.7954$	M1 M1 M1 A1 **[4]**	Appropriate elements from $\mathbf{P}^{11}$ Diagonal elements from **P** *Dependent on previous M1M1*	(allow $\mathbf{P}^{12}$)
5	(iv)		$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{P}^{n-1} \begin{pmatrix} 0.9 \\ 0.6 \\ 0.2 \end{pmatrix}$ $= (0.8009)$ when $n = 7$; $= (0.7986)$ when $n = 8$ Smallest value is $n = 8$ Probability is 0.7986	M1 M1 B1 A1 **[4]**	Repeating (iii) for another value of n Obtaining values both sides of 0.8 *0.7986 implies M1M1*	 Valid method required here
5	(v)		Expected run length is $\frac{1}{1-0.9}$ $= 10$	M1 A1 **[2]**	Using $\frac{1}{1-p}$ or $\frac{p}{1-p}$ with $p = 0.9$	
5	(vi)		$\mathbf{P}^{n} \to \begin{pmatrix} 0.7225 & 0.2225 & 0.0549 \\ 0.7225 & 0.2225 & 0.0549 \\ 0.7225 & 0.2225 & 0.0549 \end{pmatrix}$ P(A) = 0.7225 P(B) = 0.2225 P(C) = 0.0549	B2 B1 **[3]**	Give B1 for 6 elements correct to 3 dp FT if rows agree to 4 dp	
5	(vii)		$0.7225 \times 0.9 \times 0.9$ $= 0.5853$	M1 A1 **[2]**	 FT P(A) × 0.81	

Question			Answer	Marks	Guidance	
5	viii		$\begin{pmatrix}0.5 & 0.4 & 0.1\end{pmatrix}\begin{pmatrix}1-2x & x & x \\ y & 1-2y & y \\ 0.1 & 0.7 & 0.2\end{pmatrix}$	M1	First or second row correct	
			$=\begin{pmatrix}0.5 & 0.4 & 0.1\end{pmatrix}$	A1		
			$0.5(1-2x)+0.4y+0.01=0.5$ $0.5x+0.4(1-2y)+0.07=0.4$ $0.5x+0.4y+0.02=0.1$			
			$x=0.06, \quad y=0.125$	M1	Obtaining values for x and y	
			Transition matrix is $\begin{pmatrix}0.88 & 0.06 & 0.06 \\ 0.125 & 0.75 & 0.125 \\ 0.1 & 0.7 & 0.2\end{pmatrix}$	A1		
				[4]		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553