OCR
RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Three points have coordinates $A(3,\ 2,\ 10)$, $B(11,\ 0,\ -3)$, $C(5,\ 18,\ 0)$, and L is the straight line through A with equation

$$\frac{x-3}{-1} = \frac{y-2}{4} = \frac{z-10}{1}.$$

(i) Find the shortest distance between the lines L and BC. **[5]**

(ii) Find the shortest distance from A to the line BC. **[6]**

A straight line passes through B and the point $P(5,\ 18,\ k)$, and intersects the line L.

(iii) Find k, and the point of intersection of the lines BP and L. **[7]**

The point D is on the line L, and AD has length 12.

(iv) Find the volume of the tetrahedron ABCD. **[6]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 2(x^3 + y^3) + 3(x^2 + y^2) + 12xy$.

(i) For a point on the surface at which $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$, show that either $y = x$ or $y = 1 - x$. **[5]**

(ii) Show that there are exactly two stationary points on the surface, and find their coordinates. **[7]**

(iii) The point $P(\frac{1}{2},\ \frac{1}{2},\ 5)$ is on the surface, and $Q(\frac{1}{2} + h,\ \frac{1}{2} + h,\ 5 + w)$ is a point on the surface close to P. Find an approximate expression for h in terms of w. **[5]**

(iv) Find the four points on the surface at which the normal line is parallel to the vector $24\mathbf{i} + 24\mathbf{j} - \mathbf{k}$. **[7]**

Option 3: Differential geometry

3 **(a)** Find the length of the arc of the polar curve $r = a(1 + \cos\theta)$ for which $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. **[6]**

(b) A curve C has cartesian equation $y = \frac{x^3}{6} + \frac{1}{2x}$.

(i) The arc of C for which $1 \leqslant x \leqslant 2$ is rotated through 2π radians about the x-axis to form a surface of revolution. Find the area of this surface. **[8]**

For the point on C at which $x = 2$,

(ii) show that the radius of curvature is $\frac{289}{64}$, **[5]**

(iii) find the coordinates of the centre of curvature. **[5]**

Option 4: Groups

4 **(a)** The composition table for a group G of order 8 is given below.

	a	b	c	d	e	f	g	h
a	c	e	b	f	a	h	d	g
b	e	c	a	g	b	d	h	f
c	b	a	e	h	c	g	f	d
d	f	g	h	a	d	c	e	b
e	a	b	c	d	e	f	g	h
f	h	d	g	c	f	b	a	e
g	d	h	f	e	g	a	b	c
h	g	f	d	b	h	e	c	a

(i) State which is the identity element, and give the inverse of each element of G. **[3]**

(ii) Show that G is cyclic. **[4]**

(iii) Specify an isomorphism between G and the group H consisting of $\{0, 2, 4, 6, 8, 10, 12, 14\}$ under addition modulo 16. **[3]**

(iv) Show that G is not isomorphic to the group of symmetries of a square. **[2]**

(b) The set S consists of the functions $\mathrm{f}_n(x) = \dfrac{x}{1 + nx}$, for all integers n, and the binary operation is composition of functions.

(i) Show that $\mathrm{f}_m \mathrm{f}_n = \mathrm{f}_{m+n}$. **[2]**

(ii) Hence show that the binary operation is associative. **[2]**

(iii) Prove that S is a group. **[6]**

(iv) Describe one subgroup of S which contains more than one element, but which is not the whole of S. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A contestant in a game-show starts with one, two or three 'lives', and then performs a series of tasks. After each task, the number of lives either decreases by one, or remains the same, or increases by one. The game ends when the number of lives becomes either four or zero. If the number of lives is four, the contestant wins a prize; if the number of lives is zero, the contestant loses and leaves with nothing.

At the start, the number of lives is decided at random, so that the contestant is equally likely to start with one, two or three lives. The tasks do not involve any skill, and after every task:

- the probability that the number of lives decreases by one is 0.5,
- the probability that the number of lives remains the same is 0.05,
- the probability that the number of lives increases by one is 0.45.

This is modelled as a Markov chain with five states corresponding to the possible numbers of lives. The states corresponding to zero lives and four lives are absorbing states.

(i) Write down the transition matrix **P**. **[3]**

(ii) Show that, after 8 tasks, the probability that the contestant has three lives is 0.0207, correct to 4 decimal places. **[2]**

(iii) Find the probability that, after 10 tasks, the game has not yet ended. **[3]**

(iv) Find the probability that the game ends after exactly 10 tasks. **[3]**

(v) Find the smallest value of N for which the probability that the game has not yet ended after N tasks is less than 0.01. **[3]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity. **[2]**

(vii) Find the probability that the contestant wins a prize. **[3]**

The beginning of the game is now changed, so that the probabilities of starting with one, two or three lives can be adjusted.

(viii) State the maximum possible probability that the contestant wins a prize, and how this can be achieved. **[2]**

(ix) Given that the probability of starting with one life is 0.1, and the probability of winning a prize is 0.6, find the probabilities of starting with two lives and starting with three lives. **[3]**