GCE

Mathematics (MEI)

Advanced GCE

Unit **4757:** Further Applications of Advanced Mathematics

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}-90\\160\\20\end{pmatrix}\times\begin{pmatrix}3\\198\\15.6\end{pmatrix}=\begin{pmatrix}-1464\\1464\\-18300\end{pmatrix}$ $\left[=-732\begin{pmatrix}2\\-2\\25\end{pmatrix}\right]$	M1 A2	Evaluation of vector product Give A1 for one component correct	A1 for any non-zero multiple correctly obtained
			$2x-2y+25z=2(15)-2(-60)+25(20)$	M1	For $2x-2y+25z=d$	
			Equation of ABC is $2x-2y+25z=650$	E1 **[5]**	Evidence of substitution required	
1	(ii)		$\overrightarrow{AB}\times\mathbf{d}_A=\begin{pmatrix}-90\\160\\20\end{pmatrix}\times\begin{pmatrix}15\\14\\-2\end{pmatrix}=\begin{pmatrix}-600\\120\\-3660\end{pmatrix}$	M1 A2	Appropriate vector product Give A1 if one error	
			$\lvert\overrightarrow{AB}\times\mathbf{d}_A\rvert=\sqrt{600^2+120^2+3660^2}$	M1		
			$\lvert\mathbf{d}_A\rvert=\sqrt{15^2+14^2+2^2}$			
			Distance is $\dfrac{\lvert\overrightarrow{AB}\times\mathbf{d}_A\rvert}{\lvert\mathbf{d}_A\rvert}$	M1		
			Distance is 180 m	A1 **[6]**		
		OR	$\left[\begin{pmatrix}15+15\lambda\\-60+14\lambda\\20-2\lambda\end{pmatrix}-\begin{pmatrix}-75\\100\\40\end{pmatrix}\right]\bullet\begin{pmatrix}15\\14\\-2\end{pmatrix}=0$		M1A1	
			$\lambda=2$		M1 Obtaining a value of λ A1	
			Distance is $\sqrt{(120)^2+(-132)^2+(-24)^2}$		M1	
			Distance is 180 m		A1	

Question			Answer	Marks	Guidance	
1	(iii)		$\mathbf{d}=\begin{pmatrix}15\\14\\-2\end{pmatrix}\times\begin{pmatrix}8\\7\\-2\end{pmatrix}=\begin{pmatrix}-14\\14\\-7\end{pmatrix}\quad[=7\begin{pmatrix}-2\\2\\-1\end{pmatrix}]$	M1 A2	Vector product of direction vectors Give A1 if one error	
			$\overrightarrow{AC}\cdot\hat{\mathbf{d}}=\dfrac{\begin{pmatrix}3\\198\\15.6\end{pmatrix}\bullet\begin{pmatrix}-2\\2\\-1\end{pmatrix}}{\sqrt{2^2+2^2+1^2}}=\dfrac{374.4}{3}$	M1 M1	Appropriate scalar product Fully correct method for finding distance	
			Distance is 124.8 m	A1 **[6]**		
1	(iv)		$\begin{pmatrix}15\\-60\\20\end{pmatrix}+\lambda\begin{pmatrix}15\\14\\-2\end{pmatrix}=\begin{pmatrix}-75\\100\\40\end{pmatrix}+\mu\begin{pmatrix}93\\38\\p-40\end{pmatrix}$	M1	Must use different parameters	
			$15+15\lambda=-75+93\mu$ $-60+14\lambda=100+38\mu$	A1	Both equations correct	
			$\lambda=25,\quad\mu=5$	M1	Obtaining value of λ or μ	
			$20-50=40+5(p-40)$	M1	*Or other method for finding p*	
			$p=26$	A1		
			Q is $(15+375,\ -60+350,\ 20-50)$	M1		
			Q is $(390,\ 290,\ -30)$	A1 **[7]**		
2	(i)		$\dfrac{\partial g}{\partial x}=2x+2z$	B1		
			$\dfrac{\partial g}{\partial y}=4y+2z$	B1		
			$\dfrac{\partial g}{\partial z}=-2z+2x+2y+4$	B1 **[3]**		

Question			Answer	Marks	Guidance	
2	(ii)		At P, $\frac{\partial g}{\partial x}=-2,\ \frac{\partial g}{\partial y}=-2,\ \frac{\partial g}{\partial z}=-4$	B1		
			Normal line is $\mathbf{r}=\begin{pmatrix}-2\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$	M1 A1 **[3]**	For direction of normal line FT	*Condone omission of* $\mathbf{r}=$
2	(iii)		If Q is $(-2+\lambda,\ -1+\lambda,\ 1+2\lambda)$			
			$h=\delta g\approx\frac{\partial g}{\partial x}\delta x+\frac{\partial g}{\partial y}\delta y+\frac{\partial g}{\partial z}\delta z$	M1	*Requires some substitution in RHS or h on LHS*	Allow M1 for $\delta x=-2+\lambda$ etc
			$=(-2)(\lambda)+(-2)(\lambda)+(-4)(2\lambda)\quad[=-12\lambda]$	A1	FT	
			$PQ=\sqrt{(\lambda)^2+(\lambda)^2+(2\lambda)^2}$	M1		
			$=\sqrt{6}\,\lvert\lambda\rvert$	A1	FT Allow $\sqrt{6}\,\lambda$	
			$c=\frac{\sqrt{6}}{12}$	A1 **[5]**	A0 for $c=-\frac{\sqrt{6}}{12}$	
2	(iv)		Require $\frac{\partial g}{\partial x}=\frac{\partial g}{\partial y}=0$	M1		
			$x=-z,\quad y=-\frac{1}{2}z$			
			$z^2+\frac{1}{2}z^2-z^2-2z^2-z^2+4z-3=0$	M1	Obtaining equation in one variable	
			$5z^2-8z+6=0$	A1	Or $5x^2+8x+6=0$ or $10y^2+8y+3=0$ or $\lambda^2+14=0$	($\lambda=\frac{\partial g}{\partial z}$)
			Discriminant is $64-120=-56<0$	M1	*Dependent on quadratic with negative discriminant*	
			Hence there are no such points	E1 **[5]**	Correctly shown	

Question			Answer	Marks	Guidance	
2	(v)		Require $\frac{\partial g}{\partial x}=\frac{\partial g}{\partial y}=\frac{\partial g}{\partial z}$ $(=\lambda)$	M1	*Allow M1 if $\lambda=1$*	
			$2x+2z=4y+2z=-2z+2x+2y+4$ $x=2y,\quad y=2z-2$	A1	FT	
			$(4z-4)^2+2(2z-2)^2-z^2+\ldots+4z-3=0$	M1	Obtaining equation in one variable	
			$5z^2-8z+3=0$	A1	Or $5x^2+8x=0$ or $5y^2+4y=0$	Or $\lambda^2-4=0$
			Points $(0,\ 0,\ 1)$ and $(-1.6,\ -0.8,\ 0.6)$	M1	Obtaining at least one point	
			$k=0+0+1$ or $k=-1.6-0.8+0.6$	M1	Obtaining a value of k	*Implies previous M1 if values of x, y, z not seen*
			$k=1,\ \ -1.8$	A1A1 **[8]**		
3	(i)		$\frac{dx}{d\theta}=3a\cos^2\theta\sin\theta,\quad \frac{dy}{d\theta}=3a\sin^2\theta\cos\theta$	B1		
			$\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2=(3a\sin\theta\cos\theta)^2(\cos^2\theta+\sin^2\theta)$	M1		
			$=(3a\sin\theta\cos\theta)^2$	A1		
			$s=\int 3a\sin\theta\cos\theta\,d\theta$	M1A1	FT	A1 requires workable integral form
			$=\frac{3}{2}a\sin^2\theta\quad(+c)$	A1		
			$s=0$ when $\theta=0 \Rightarrow c=0$		Or $\int_0^\theta\ldots$ used	Required for final E1
			$\tan\psi=\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta}$	M1		
			$=\tan\theta$	A1		
			Hence $\psi=\theta$ and $s=\frac{3}{2}a\sin^2\psi$	E1 **[9]**	Correctly shown	

Question			Answer	Marks	Guidance	
3	(ii)		$\rho = \dfrac{ds}{d\psi} = 3a\sin\psi\cos\psi$	M1A1		
			When $\theta = \dfrac{\pi}{6}$, $\psi = \dfrac{\pi}{6}$, $\rho = 3a\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$	M1		
			Radius of curvature is $\dfrac{3\sqrt{3}}{4}a$	A1		*Condone use of $\psi = \dfrac{\pi}{6}$ even if $\psi = \theta$ not established in (i)*
		OR	When $\theta = \dfrac{\pi}{6}$, $\dot{x} = \dfrac{9a}{8}$, $\dot{y} = \dfrac{3\sqrt{3}a}{8}$ $\ddot{x} = \dfrac{3\sqrt{3}a}{8}$, $\ddot{y} = \dfrac{15a}{8}$		M1 Obtaining second derivatives A1	 *May be implied by later work*
			$\rho = \dfrac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}-\ddot{x}\dot{y}} = \dfrac{\left(\frac{81a^2}{64}+\frac{27a^2}{64}\right)^{\frac{3}{2}}}{\left(\frac{9a}{8}\right)\left(\frac{15a}{8}\right)-\left(\frac{3\sqrt{3}a}{8}\right)\left(\frac{3\sqrt{3}a}{8}\right)}$		M1 Applying formula for ρ or κ	
			Radius of curvature is $\dfrac{3\sqrt{3}}{4}a$		A1	
			Normal vector is $\hat{\mathbf{n}} = \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$	M1 A1	Obtaining gradient or normal vector Correct normal vector	 Not necessarily unit vector
			$\mathbf{c} = \begin{pmatrix} a\left(1-\frac{3\sqrt{3}}{8}\right) \\ \frac{1}{8}a \end{pmatrix} + \dfrac{3\sqrt{3}}{4}a\begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$	M1	Must use unit normal here	
			Centre of curvature is $\left(a\left(1-\dfrac{3\sqrt{3}}{4}\right),\ \dfrac{5a}{4} \right)$	A1A1 **[9]**		

Question			Answer	Marks	Guidance	
3	(iii)		Surface area is $\int 2\pi x \, ds$ $= \int_0^{\frac{\pi}{3}} 2\pi a(1-\cos^3\theta)(3a\sin\theta\cos\theta)\,d\theta$ $= 6\pi a^2 \int_0^{\frac{\pi}{3}} (\sin\theta\cos\theta - \sin\theta\cos^4\theta)\,d\theta$ $= 6\pi a^2 \left[\frac{1}{2}\sin^2\theta + \frac{1}{5}\cos^5\theta \right]_0^{\frac{\pi}{3}}$ $= \dfrac{87\pi a^2}{80}$	M1 A1 M1 A1A1 A1 **[6]**	 FT Integrating For $\frac{1}{2}\sin^2\theta$ and $\frac{1}{5}\cos^5\theta$	 *Correct limits required* At least one trigonometric term Or equivalent
4	(i)		$\begin{array}{c\|cccc} P & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$ Table shows closure Identity is 1 All elements are self-inverse	B1 B1 B1 B1 **[4]**	 *Condone no mention of inverse of 1*	
4	(ii)		$(xy)(y^{-1}x^{-1})$ $= x(yy^{-1})x^{-1} = xex^{-1} = xx^{-1} = e$ So $y^{-1}x^{-1}$ is the inverse of xy	M1 E1 **[2]**	Or $(y^{-1}x^{-1})(xy)$	
4	(iii)		$a^{-1} = a,\ b^{-1} = b,\ c^{-1} = c,\ c^{-1} = (ab)^{-1} = b^{-1}a^{-1}$ Hence $c = ba$	M1 E1 **[2]**	For any one of these	

Question			Answer	Marks	Guidance	
4	(iv)		$bc = b(ba)$ $bc = ea = a$ $ac = a(ab) = eb = b$ $cb = a, \quad ca = b$	M1 E1 E1 B1 **[4]**	Or $ba = c \Rightarrow a = b^{-1}c$	Any correct first step
4	(v)		$\begin{array}{c\|cccc} R & e & a & b & c \\ \hline e & e & a & b & c \\ a & a & e & c & b \\ b & b & c & e & a \\ c & c & b & a & e \end{array}$ *R* is closed Hence *R* is a subgroup Same pattern as *P*; hence *R* and *P* are isomorphic	B1 M1 E1 E1 **[4]**	 No need to mention identity or inverses *Dependent on B1 (only)*	
4	(vi)		$\begin{array}{c\|cccccccc} \text{Element} & A & B & C & D & E & F & G & H \\ \hline \text{Order} & 2 & 2 & 2 & 2 & 1 & 4 & 2 & 4 \end{array}$	B3 **[3]**	Give B1 for 3 correct; B2 for 6 correct	
4	(vii)		$\{E, A\}, \{E, B\}, \{E, C\}, \{E, D\}, \{E, G\}$ $\{E, F, G, H\}$ $\{E, A, B, G\}$ $\{E, C, D, G\}$	B2 B1 B1 B1 **[5]**	Ignore $\{E\}$ and *T* in the marking Give B1 for 3 correct	Deduct 1 mark (from this B2) for each subgroup of order 2 given in excess of five Deduct 1 mark (from this B1B1B1) for each subgroup of order 3 or more given in excess of three

Question			Answer	Marks	Guidance	
5			***Pre-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
	(i)		*A* and *E* are reflecting barriers	B1 **[1]**		
5	(ii)		$\mathbf{P} = \begin{pmatrix} 0 & 0.4 & 0 & 0 & 0 \\ 1 & 0 & 0.4 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 \\ 0 & 0 & 0.6 & 0 & 1 \\ 0 & 0 & 0 & 0.6 & 0 \end{pmatrix}$	B2 **[2]**	Give B1 for three columns correct	
5	(iii)		$\mathbf{P}^{10} = \begin{pmatrix} 0.1378 & . & . & . & . \\ 0 & . & . & . & . \\ 0.4689 & . & . & . & . \\ 0 & . & . & . & . \\ 0.3933 & . & . & . & . \end{pmatrix}$	M1 M1	For $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$ Using first or second column of $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$	
			Possible positions are $A,\ C,\ E$ $\mathrm{P}(A) = 0.1378,\ \mathrm{P}(C) = 0.4689,\ \mathrm{P}(E) = 0.3933$	B1 A1 **[4]**		
5	(iv)		$\mathbf{P}^{13} = \begin{pmatrix} 0 & \ldots \\ 0.3162 & \ldots \\ 0 & \ldots \\ 0.6838 & \ldots \\ 0 & \ldots \end{pmatrix},\ \mathbf{P}^{2} = \begin{pmatrix} . & . & . & . & . \\ . & 0.64 & . & . & . \\ . & . & . & . & . \\ . & . & . & 0.84 & . \\ . & . & . & . & . \end{pmatrix}$	M1	Using first or second column of $\mathbf{P}^{13}$ or $\mathbf{P}^{12}$, and diagonal elements from $\mathbf{P}^{2}$	
			$0.3162 \times 0.64 + 0.6838 \times 0.84$	M1	Allow M1 for $0.1301 \times 0.4 + 0.4651 \times 0.48 + 0.4048 \times 0.6$	$(= 0.5182)$
			Probability is 0.7768	A1 **[3]**		

Question			Answer	Marks	Guidance	
5	(v)		$\mathbf{P}^{15}=\begin{pmatrix} . & \dots \\ . & \dots \\ . & \dots \\ 0.6882 & \dots \\ . & \dots \end{pmatrix}$, $\mathbf{P}^{17}=\begin{pmatrix} . & \dots \\ . & \dots \\ . & \dots \\ 0.6904 & \dots \\ . & \dots \end{pmatrix}$ After 17 steps	M1 M1 A1 **[3]**	Considering powers of **P** Appropriate element exceeding 0.69	
5	(vi)		Limit of $\mathbf{P}^{2n}$ is $\begin{pmatrix} 0.1231 & 0 & 0.1231 & 0 & 0.1231 \\ 0 & 0.3077 & 0 & 0.3077 & 0 \\ 0.4615 & 0 & 0.4615 & 0 & 0.4615 \\ 0 & 0.6923 & 0 & 0.6923 & 0 \\ 0.4154 & 0 & 0.4154 & 0 & 0.4154 \end{pmatrix}$ Limit of $\mathbf{P}^{2n+1}$ is $\begin{pmatrix} 0 & 0.1231 & 0 & 0.1231 & 0 \\ 0.3077 & 0 & 0.3077 & 0 & 0.3077 \\ 0 & 0.4615 & 0 & 0.4615 & 0 \\ 0.6923 & 0 & 0.6923 & 0 & 0.6923 \\ 0 & 0.4154 & 0 & 0.4154 & 0 \end{pmatrix}$	B2 B2 **[4]**	Give B1 for any non-zero element correct to 3 dp Give B1 for any non-zero element correct to 3 dp	*SC* If $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ interchanged, award B1B1
5	(vii)		Expected time at *A* is 50×0.1231 *A*: 6.2 *B*: 15.4 *C*: 23.1 *D*: 34.6 *E*: 20.8 (min)	M1 A2 **[3]**	Condone 100×0.1231 for M1 Give A1 for one correct	
5	(viii)		Expected number is $\frac{1}{0.4}-1$ $=1.5$ Expected time is $1+1.5\times 2$ $=4$ minutes	M1 A1 M1 A1 **[4]**	For $\frac{1}{0.4}$ For 1.5×2	

Question			Answer	Marks	Guidance	
5			***Post-multiplication by transition matrix***		Allow tolerance of ± 0.0001 in probabilities throughout this question	
	(i)		A and E are reflecting barriers	B1 **[1]**		
5	(ii)		$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 & 0 \\ 0 & 0.4 & 0 & 0.6 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	B2 **[2]**	Give B1 for three rows correct	
5	(iii)		$\mathbf{P}^{10} = \begin{pmatrix} 0.1378 & 0 & 0.4689 & 0 & 0.3933 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$	M1	For $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$	
				M1	Using first or second row of $\mathbf{P}^{10}$ or $\mathbf{P}^{9}$	
			Possible positions are $A,\ C,\ E$	B1		
			$\mathrm{P}(A) = 0.1378,\ \mathrm{P}(C) = 0.4689,\ \mathrm{P}(E) = 0.3933$	A1 **[4]**		

Question			Answer	Marks	Guidance	
5	(iv)		$\mathbf{P}^{13} = \begin{pmatrix} 0 & 0.3162 & 0 & 0.6838 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$ $\mathbf{P}^{2} = \begin{pmatrix} . & . & . & . & . \\ . & 0.64 & . & . & . \\ . & . & . & . & . \\ . & . & . & 0.84 & . \\ . & . & . & . & . \end{pmatrix}$	M1	Using first or second row of $\mathbf{P}^{13}$ or $\mathbf{P}^{12}$, and diagonal elements from $\mathbf{P}^{2}$	
			$0.3162 \times 0.64 + 0.6838 \times 0.84$	M1	Allow M1 for $0.1301 \times 0.4 + 0.4651 \times 0.48 + 0.4048 \times 0.6$	($= 0.5182$)
			Probability is 0.7768	A1 **[3]**		
5	(v)		$\mathbf{P}^{15} = \begin{pmatrix} . & . & . & 0.6882 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$, $\mathbf{P}^{17} = \begin{pmatrix} . & . & . & 0.6904 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$	M1 M1	Considering powers of **P** Appropriate element exceeding 0.69	
			After 17 steps	A1 **[3]**		

Question			Answer	Marks	Guidance	
5	(vi)		Limit of $\mathbf{P}^{2n}$ is $\begin{pmatrix} 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \end{pmatrix}$	B2	Give B1 for any non-zero element correct to 3 dp	
			Limit of $\mathbf{P}^{2n+1}$ is $\begin{pmatrix} 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \\ 0.1231 & 0 & 0.4615 & 0 & 0.4154 \\ 0 & 0.3077 & 0 & 0.6923 & 0 \end{pmatrix}$	B2 **[4]**	Give B1 for any non-zero element correct to 3 dp	*SC* If $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ interchanged, award B1B1
5	(vii)		Expected time at *A* is 50×0.1231 *A*: 6.2 *B*: 15.4 *C*: 23.1 *D*: 34.6 *E*: 20.8 (min)	M1 A2 **[3]**	Condone 100×0.1231 for M1 Give A1 for one correct	
5	(viii)		Expected number is $\frac{1}{0.4} - 1$ $= 1.5$ Expected time is $1 + 1.5 \times 2$ $= 4$ minutes	M1 A1 M1 A1 **[4]**	For $\frac{1}{0.4}$ For 1.5×2	

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