



ADVANCED GCE

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

4757

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Monday 1 June 2009

Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Option 1: Vectors

1 The point A $(-1, 12, 5)$ lies on the plane P with equation $8x - 3y + 10z = 6$. The point B $(6, -2, 9)$ lies on the plane Q with equation $3x - 4y - 2z = 8$. The planes P and Q intersect in the line L .

(i) Find an equation for the line L . [5]

(ii) Find the shortest distance between L and the line AB. [6]

The lines M and N are both parallel to L , with M passing through A and N passing through B.

(iii) Find the distance between the parallel lines M and N . [5]

The point C has coordinates $(k, 0, 2)$, and the line AC intersects the line N at the point D.

(iv) Find the value of k , and the coordinates of D. [8]

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x(x + y)^3 - 2x^3 + 24x$.

(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [4]

(ii) Find the coordinates of the three stationary points on the surface. [7]

(iii) Find the equation of the normal line at the point P $(1, -2, 19)$ on the surface. [3]

(iv) The point Q $(1 + k, -2 + h, 19 + 3h)$ is on the surface and is close to P. Find an approximate expression for k in terms of h . [4]

(v) Show that there is only one point on the surface at which the tangent plane has an equation of the form $27x - z = d$. Find the coordinates of this point and the corresponding value of d . [6]

Option 3: Differential geometry

3 A curve has parametric equations $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, for $0 \leq \theta \leq \pi$, where a is a positive constant.

(i) Show that the arc length s from the origin to a general point on the curve is given by $s = 4a \sin \frac{1}{2}\theta$. [6]

(ii) Find the intrinsic equation of the curve giving s in terms of a and ψ , where $\tan \psi = \frac{dy}{dx}$. [4]

(iii) Hence, or otherwise, show that the radius of curvature at a point on the curve is $4a \cos \frac{1}{2}\theta$. [3]

(iv) Find the coordinates of the centre of curvature corresponding to the point on the curve where $\theta = \frac{2}{3}\pi$. [6]

(v) Find the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [5]

Option 4: Groups

4 The group $G = \{1, 2, 3, 4, 5, 6\}$ has multiplication modulo 7 as its operation. The group $H = \{1, 5, 7, 11, 13, 17\}$ has multiplication modulo 18 as its operation.

- (i) Show that the groups G and H are both cyclic. [4]
- (ii) List all the proper subgroups of G . [3]
- (iii) Specify an isomorphism between G and H . [4]

The group $S = \{a, b, c, d, e, f\}$ consists of functions with domain $\{1, 2, 3\}$ given by

| | | |
|------------|------------|------------|
| $a(1) = 2$ | $a(2) = 3$ | $a(3) = 1$ |
| $b(1) = 3$ | $b(2) = 1$ | $b(3) = 2$ |
| $c(1) = 1$ | $c(2) = 3$ | $c(3) = 2$ |
| $d(1) = 3$ | $d(2) = 2$ | $d(3) = 1$ |
| $e(1) = 1$ | $e(2) = 2$ | $e(3) = 3$ |
| $f(1) = 2$ | $f(2) = 1$ | $f(3) = 3$ |

and the group operation is composition of functions.

- (iv) Show that $ad = c$ and find da . [4]
- (v) Give a reason why S is not isomorphic to G . [1]
- (vi) Find the order of each element of S . [4]
- (vii) List all the proper subgroups of S . [4]

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

- 5 Each level of a fantasy computer game is set in a single location, Alphaworld, Betaworld, Chiworld or Deltaworld. After completing a level, a player goes on to the next level, which could be set in the same location as the previous level, or in a different location.

In the first version of the game, the initial and transition probabilities are as follows.

Level 1 is set in Alphaworld or Betaworld, with probabilities 0.6, 0.4 respectively.

After a level set in Alphaworld, the next level will be set in Betaworld, Chiworld or Deltaworld, with probabilities 0.7, 0.1, 0.2 respectively.

After a level set in Betaworld, the next level will be set in Alphaworld, Betaworld or Deltaworld, with probabilities 0.1, 0.8, 0.1 respectively.

After a level set in Chiworld, the next level will also be set in Chiworld.

After a level set in Deltaworld, the next level will be set in Alphaworld, Betaworld or Chiworld, with probabilities 0.3, 0.6, 0.1 respectively.

The situation is modelled as a Markov chain with four states.

- (i) Write down the transition matrix. [2]
- (ii) Find the probabilities that level 14 is set in each location. [3]
- (iii) Find the probability that level 15 is set in the same location as level 14. [3]
- (iv) Find the level at which the probability of being set in Chiworld first exceeds 0.5. [3]
- (v) Following a level set in Betaworld, find the expected number of further levels which will be set in Betaworld before changing to a different location. [3]

In the second version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are all the same as in the first version; but after a level set in Chiworld, the next level will be set in Chiworld or Deltaworld, with probabilities 0.9, 0.1 respectively.

- (vi) By considering powers of the new transition matrix, or otherwise, find the equilibrium probabilities for the four locations. [5]

In the third version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are again all the same as in the first version; but the transition probabilities after Chiworld have changed again. The equilibrium probabilities for Alphaworld, Betaworld, Chiworld and Deltaworld are now 0.11, 0.75, 0.04, 0.1 respectively.

- (vii) Find the new transition probabilities after a level set in Chiworld. [5]

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