RECOGNISING ACHIEVEMENT

ADVANCED GCE **4757/01**

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

FRIDAY 6 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Option 1: Vectors

1 A tetrahedron ABCD has vertices A (−3, 5, 2), B (3, 13, 7), C (7, 0, 3) and D (5, 4, 8).

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$, and hence find the equation of the plane ABC. [4]

(ii) Find the shortest distance from D to the plane ABC. [3]

(iii) Find the shortest distance between the lines AB and CD. [4]

(iv) Find the volume of the tetrahedron ABCD. [4]

The plane P with equation $3x - 2z + 5 = 0$ contains the point B, and meets the lines AC and AD at E and F respectively.

(v) Find λ and μ such that $\overrightarrow{AE} = \lambda\overrightarrow{AC}$ and $\overrightarrow{AF} = \mu\overrightarrow{AD}$. Deduce that E is between A and C, and that F is between A and D. [5]

(vi) Hence, or otherwise, show that P divides the tetrahedron ABCD into two parts having volumes in the ratio 4 to 17. [4]

Option 2: Multi-variable calculus

2 You are given $g(x, y, z) = 6xz - (x + 2y + 3z)^2$.

(i) Find $\dfrac{\partial g}{\partial x}$, $\dfrac{\partial g}{\partial y}$ and $\dfrac{\partial g}{\partial z}$. [4]

A surface S has equation $g(x, y, z) = 125$.

(ii) Find the equation of the normal line to S at the point P (7, −7.5, 3). [3]

(iii) The point Q is on this normal line and is close to P. At Q, $g(x, y, z) = 125 + h$, where h is small. Find the vector $\mathbf{n}$ such that $\overrightarrow{PQ} = h\mathbf{n}$ approximately. [5]

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. [4]

(v) Find the two points on S at which the tangent plane is parallel to $x + 5y = 0$. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 8t^3$, $y = 9t^2 - 2t^4$, for $t \geqslant 0$.

(i) Show that $\dot{x}^2 + \dot{y}^2 = (18t + 8t^3)^2$. Find the length of the arc of C for which $0 \leqslant t \leqslant 2$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 2$ is rotated through 2π radians about the x-axis. [6]

(iii) Show that the curvature at a general point on C is $\dfrac{-6}{t(4t^2 + 9)^2}$. [5]

(iv) Find the coordinates of the centre of curvature corresponding to the point on C where $t = 1$. [7]

Option 4: Groups

4 A binary operation $*$ is defined on real numbers x and y by

$$x * y = 2xy + x + y.$$

You may assume that the operation $*$ is commutative and associative.

(i) Explain briefly the meanings of the terms 'commutative' and 'associative'. [3]

(ii) Show that $x * y = 2\left(x + \frac{1}{2}\right)\left(y + \frac{1}{2}\right) - \frac{1}{2}$. [1]

The set S consists of all real numbers greater than $-\frac{1}{2}$.

(iii) (A) Use the result in part **(ii)** to show that S is closed under the operation $*$.

(B) Show that S, with the operation $*$, is a group. [9]

(iv) Show that S contains no element of order 2. [3]

The group $G = \{0, 1, 2, 4, 5, 6\}$ has binary operation $\circ$ defined by

$x \circ y$ is the remainder when $x * y$ is divided by 7.

(v) Show that $4 \circ 6 = 2$. [2]

The composition table for G is as follows.

$\circ$	0	1	2	4	5	6
0	0	1	2	4	5	6
1	1	4	0	6	2	5
2	2	0	5	1	6	4
4	4	6	1	5	0	2
5	5	2	6	0	4	1
6	6	5	4	2	1	0

(vi) Find the order of each element of G. [3]

(vii) List all the subgroups of G. [3]

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Every day, a security firm transports a large sum of money from one bank to another. There are three possible routes A, B and C. The route to be used is decided just before the journey begins, by a computer programmed as follows.

On the first day, each of the three routes is equally likely to be used.

If route A was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.4, 0.5 respectively.

If route B was used on the previous day, route A, B or C will be used, with probabilities 0.7, 0.2, 0.1 respectively.

If route C was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.6, 0.3 respectively.

The situation is modelled as a Markov chain with three states.

(i) Write down the transition matrix **P**. [2]

(ii) Find the probability that route B is used on the 7th day. [4]

(iii) Find the probability that the same route is used on the 7th and 8th days. [3]

(iv) Find the probability that the route used on the 10th day is the same as that used on the 7th day. [4]

(v) Given that $\mathbf{P}^n \to \mathbf{Q}$ as $n \to \infty$, find the matrix **Q** (give the elements to 4 decimal places). Interpret the probabilities which occur in the matrix **Q**. [4]

The computer program is now to be changed, so that the long-run probabilities for routes A, B and C will become 0.4, 0.2 and 0.4 respectively. The transition probabilities after routes A and B remain the same as before.

(vi) Find the new transition probabilities after route C. [4]

(vii) A long time after the change of program, a day is chosen at random. Find the probability that the route used on that day is the same as on the previous day. [3]